SUCCESSFUL SUPERHEATING FIELD FORMULAS FROM AN INTUITIVE MODEL*

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Abstract

To date, many theoretical formulas for superheating field on SRF cavity have been proposed based rather complicated calculations. This paper proposes the formulas of superheating field a very intuitive and simple model led from energy balance between RF magnetic energy and superconducting condensed one, and a condition of vanishing the mirror vortex line image. The penetration of a single vortex determines the superheating field for a type II superconductor. On the other hand, for type I superconductors, the surface flux penetration determines it. The formula fits very well quantitatively the results of niobium cavity and Nb3Sn one. In addition, it gives a nice guideline for new material beyond niobium.

SUPERHEATING OF SRF

The RF critical field of superconducting cavity is determined by the superheating field (Bsh), which is higher than the Hc (Type-I) or Hc1 (Type-II). So far, many models have been proposed for it [1-5]. In this paper, a plane flux penetration for Type-I SRF surface, and a vortex line penetration for Type-II SRF are assumed as the models [6]. By these models, the super-heating field is determined by an energy balance between the flux energy just before entering into SRF surface and the superconducting condensation energy. Combining to Abrikosov's theory, the temperature (T) dependence of superheating field are formalized, which are described by only the measurable characteristic parameters critical field Hc (T), and Landau-Ginsburg parameter κ (T). The forms derived here can estimate the number of vortex flux line per unit area, which gives a very intuitive understanding.

ABRIKOSOV'S EQUATIONS AND THE TEMPERATURE DEPENDENCIES

Abrikosov has derived relational expressions among Hc, H_{C2}, λ, and ξ, here λ is a field penetration depth, and ξ coherent length of a superconductor [5, 7. 8].

$$H_c = \frac{\phi_0}{2\pi\sqrt{2}\lambda\xi}, \tag{1}$$

$$H_{C2} = \frac{\phi_0}{2\pi\xi^2} , \tag{2}$$

from these equations, λ and ξ are expressed by H_C and H_{C2} as follows:

$$\lambda = \sqrt{\frac{\phi_0 H_{C2}}{4\pi H_C^2}}, \tag{3}$$

$$\xi = \sqrt{\frac{\phi_0}{2\pi H_C^2}}. \tag{4}$$

From the theory and expermental data, the temperature depence of λ(T) and H_C(T) are given as:

$$\lambda(T) = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_C}\right)^4}} \tag{5}$$

$$H_C(T) = H_C(0)\left[1-\left(\frac{T}{T_C}\right)^2\right] . \tag{6}$$

From these expressions, the temperature dependence of H_{C2} (T), ξ (T), and κ (T) can be driven as:

$$H_{C2}(T) = H_{C2}(0)\frac{1-\left(\frac{T}{T_C}\right)^2}{1+\left(\frac{T}{T_C}\right)^2} , \tag{7}$$

$$\xi(T) = \xi(0)\sqrt{\frac{1+\left(\frac{T}{T_C}\right)^2}{1-\left(\frac{T}{T_C}\right)^2}} , \tag{8}$$

$$\kappa(T) \equiv \frac{\lambda(T)}{\xi(T)} = \frac{\kappa(0)}{1+\left(\frac{T}{T_C}\right)^2} . \tag{9}$$

Figure 1 shows the temperature dependence H_C with niobium. H_C (0) is 1934.2 Gauss with very high purity Nb (RRR > 2000). Figure 2 shows the temperature dependence of κ with high purity niobium. T^2 dependence well hits the experimental data. κ (0) is 1.508 for high pure niobium.

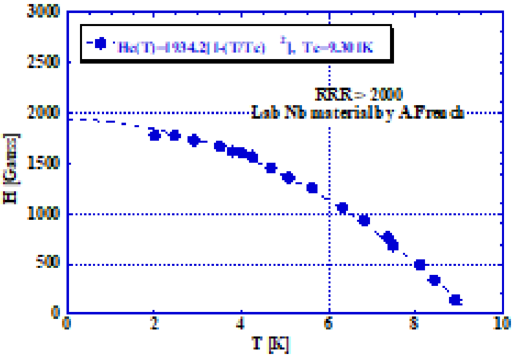

Figure 1: Temperature dependence of H_C with high purity niobium. The experimental data is from [9]. The T^2 dependence fits the experimental data. H_C (0) is 1900 Gauss.

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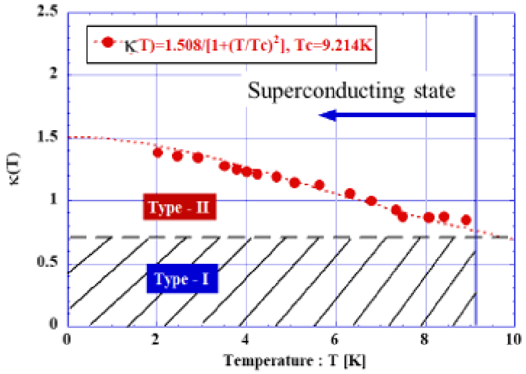

Figure 2: Temperature dependence of κ with high purity niobium is calculated from [9]. The T^2 dependence hits well the experimental data. κ (0) is 1.508 for high purity niobium.

SUPERHEATING FIELD BY FLUX PENETRATION MODEL

T. Yogi has proposed institutive models for the superheating field: a plane flux penetration for Type-I superconductor, and a vortex line penetration for Type-II [3]. Figure 3 illustrates the vortex line penetration model. From energy balance between RF magnetic field energy and superconducting condensed energy per unite length, one gets the superheating expression (11) for the vortex line penetration model.

$$\frac{1}{2}\mu_o H^2\lambda^2 - \frac{1}{2}\mu_o H_C^2\xi^2 = 0 \tag{10}$$

$$H_{sh} = \frac{H_C}{\kappa} \ . \tag{11}$$

By similar way, the expression (13) for the plane penetration is given for the energy balance:

$$\frac{1}{2}\mu_o H^2\lambda - \frac{1}{2}\mu_o H_C^2\xi = 0 \ . \tag{12}$$

For plane flux entry,

$$H_{sh} = \frac{H_C}{\sqrt{\kappa}} \ . \tag{13}$$

Explicit temperature dependence of H_{SH} for vortex line penetration is;

$$H_{SH}\,(T) = \frac{Hc(0)}{\kappa(0)}\left[1-\left(\frac{T}{T_C}\right)^4\right] \ . \tag{14}$$

For the plane penetration,

$$H_{SH}\,(T) = \frac{Hc(0)}{\sqrt{\kappa(0)}}\left[1-\left(\frac{T}{T_C}\right)^2\right] . \tag{15}$$

From Fig.1 and 2, H_C (0) = 1934.2 mT and κ (0) is 1.508, and $\frac{H_C(0)}{\kappa(0)} = 128.3\ mT$ for niobium. As seen in Fig 4, the experimental data can be fitted well by the T^4 temperature dependence. On the other hand, the experimental value at 0K is 1800 mT. Theoretical value is small by $\sqrt{2}$ than experimental data. This model does not produce the experimental result.

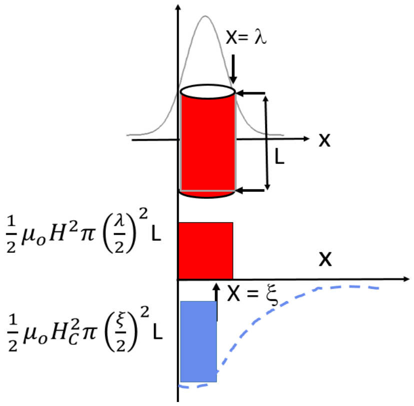

Figure 3: Vortex penetration model for Type-II.

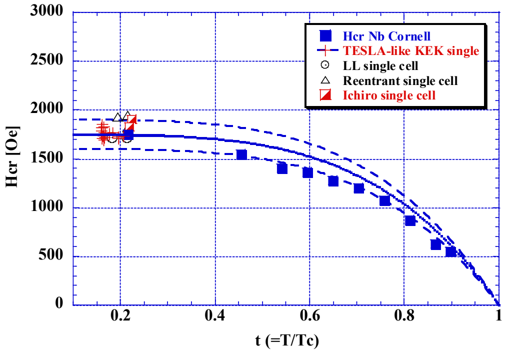

Figure 4: Temperature dependence of the superheating field of niobium cavity. The solid square date is the pulse measurement at Cornell University, and t= 0.2 (T=2 K) date is from all KEK CW measurement results.

UPDATED MODEL

The deference of $\sqrt{2}$ between the theory and experiment was bothering one of authors (K. Saito) since 2001. Finally, he has noticed that the model in Fig. 3 is a problem in the timing of the vortex entry, which is for after the vortex fully has entered into the top surface. HSH should be the value for when the vortex is about going to enter. The model should be change to Fig. 5.

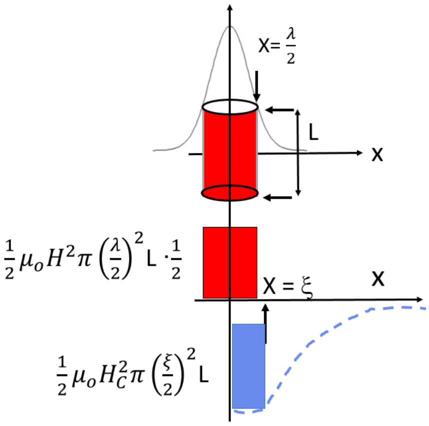

Figure 5: Updated vortex entry model for the superheating field.

The magnetic field energy is a half of Fig. 3. Thus, the matter of $\sqrt{2}$ can be resolved. Thus the corrected formulas for the superheating field are:

$$H_{SH}(T) = \sqrt{2}\frac{H_C(0)}{\kappa(0)} \qquad \text{for Type-II} \qquad (16),$$

$$H_{SH} = \sqrt{2}\frac{H_C(0)}{\sqrt{\kappa(0)}}\left[1-\left(\frac{T}{T_C}\right)^2\right] \qquad \text{for Type-I} \qquad (17).$$

From the formula (16), the theoretical superheating field is HSH (0) = 178.0 mT for niobium, very much agree with the experimental result 180 mT.

Usually, the vortex mirror image on the surface before vortex entry produces a repulsive force on the coming vortex line, and a barrier is produced against the vortex entry. However, this vortex mirror image will vanish in the condition of Fig. 5. So one needs not to bother the complex calculation on the barrier impact with the superheating field calculation.

T^2 DEPENDENCE OBSERVED AT CORNELL UNIVERSITY

Recently Cornell group has published the temperature dependence of superheating with niobium cavity and Nb3Sn cavity, re-measured by pulse measurement method, as shown in Fig. 6. As interesting, the temperature dependence of niobium cavity shows T2 dependence not T4. How to be explain this fact is an exciting. As shown in Fig. 7 left, usually the magnetic field behavior of niobium material shows a typical Type-II transition, which has a RRR= 250. However, it is well known that a very high purity niobium material (RRR= 2000) shows the Type-I-like sharp transition as seen in Fig .7 right. The Nb material of the Cornell cavity could be very high purity. In this case, the superheating field behavior switches to Type-I-like: plane entry, then the temperature dependence shows T2 dependence. In this case, from the formula (17), the superheating field is estimated as HSH = 218.7 mT.

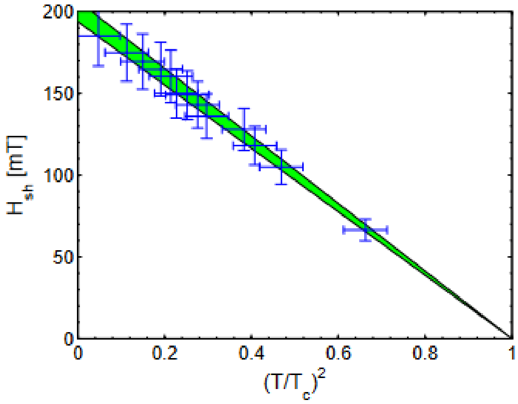

Figure 6: The temperature dependence of superheating field with niobium cavity at Cornell University, which was measured by pulse method. The temperature dependence with Nb cavity shows T2 dependence [10].

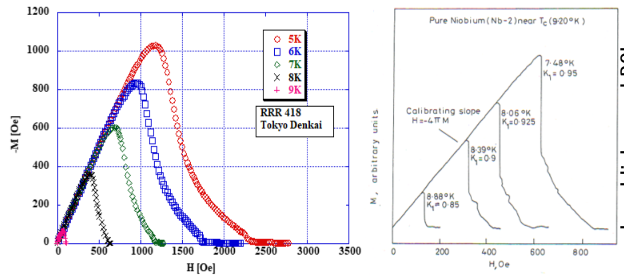

Figure 7: Behavior of the magnetic transition with niobium. The left shows with currently used high RRR grade material with RRR=250 produced by Tokyo Denkai. The right figure shows with very high purity niobium with RRR=2000 [8].

PENETRATION LENGTH AT SUPER HEATING FIELD

Let calculate the number of fluxoide penetrating the superheating area at the superheating field. The superheating field of 0K is 178mT for niobium as described by formula (17). If divided this number by $\phi o = 2.07 \times 10^{-15}$ Tm², (fluxoide), we can calculate the number of fluxoide per one m².

$$N = \frac{178\times10^{-3}}{2.07\times10^{-15}} = 8.66 \times 10^{13}/m^2.$$

We supposed the flux penetration is the λ/2 length in the niobium surface as seen in Fig. 5. The area is:

$$S = \frac{1}{2}\pi\cdot\left(\frac{\lambda}{2}\right)^2 .$$

With λ=438Å = 4.38 x10⁻⁸ m,

$$S = 7.53 \times 10^{-16}\ m^2 ,$$

$$N = 0.0652 .$$

Our expecting value is N=0.5, and the above number is one order of magnitude small. This will be problem in λ. If calculated to produce N = 0.5, λ= 1191 Å. Thus, we can understand the penetration depth might be 2.7 times longer than the experimented value [5] at such a high field.

SUPERHEATING FIELD OF Nb3Sn CAVITY

Figure 8 shows the data hitting result by formula (16) with Nb3Sn cavity at Cornell University, by pulse measurement method [11]. The superheating field is 99.8 mT. On the other hand, relatively new data at Cornell shows H_{SH} is 135.0 mT. From these data, achievable accelerating field would be ~35 MV/m for the ILC type cavity shape with Hp/Eacc = 42.6 Gauss/ [MV/m].

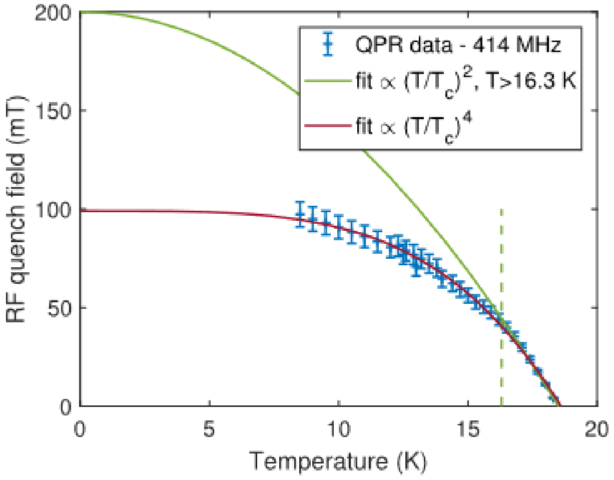

Figure 8: Temperature dependence of Nb_3Sn cavity [10].

BEYOND Nb MATERIAL

These superheating field formulas give us a clear direction for searching new materials beyond Nb. Without saying, the conclusion is a high H_C material with low κ value for higher gradient. However, generally saying, H_C and κ conflict each other in Type-II superconductors, for example A-15. Niobium is a best well balanced material between H_C and κ value. Nb3Sn would have no hope such high gradient over 50 MV/m because of high κ value (~35). Other A-15 type-II with higher transition temperature also could have a same problem as Nb3Sn. Of course, these material with higher T_C than Nb will be useful for high Q operation at CW medium acceleration gradients, and application at 4.3K operation is very much beneficial.

SUMMARY

The exact formula for superheating field was derived for SRF cavities which fabricated by Type-I or Type-II superconducting material. The temperature dependence is T^2 for Type-I and T^4 for Type-II. These formulas can reproduce the absolute value.

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