

# MEASUREMENTS OF HIGH VALUES OF DIELECTRIC PERMITTIVITY USING TRANSMISSION LINES

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## Abstract

Usage of lossy materials is necessary for absorption of higher order modes excited in the RF cavities. Presently, measurements of lossy materials with usage of transmission lines give errors rapidly increasing with increase of the dielectric permittivity. A method is presented for measurements of high values of dielectric permittivity epsilon in a waveguide at high frequencies with lower errors. This method supplements the method of measurements evolved for low values of epsilon and is close to resonant methods, when a sample is placed into a cavity and the measurement is done at one only frequency. The new approach with use of Microwave Studio simulations makes possible to measure this value in several frequency points at one measurement.

## INTRODUCTION

The method to compute complex  $\epsilon$  and  $\mu$  with help of analytical formulas from the measured complex reflection ( $S_{11}$ ) and transmission ( $S_{21}$ ), when a sample of a material is placed into a transmission line, was proposed by W. Weir [1] and has been detailed in other publications [2 - 4].

The sample geometry for measurements with waveguides is a rectangular plate with two dimensions close to dimensions of the waveguide cross-section and with a thickness that can be different.

In [4] it was shown that measurement of high values of  $\epsilon$  requires very high accuracy of measurement of the S-parameters. The commonly used Network Analyzers cannot guarantee this accuracy better than 0.01. Such an error leads to errors in  $\text{Re } \epsilon$  about 20 % if  $\epsilon$  is about 30, and +200/-20 % if  $\epsilon$  is 80.

Moreover, these errors increase dramatically if there are gaps between the sample and the waveguide where this sample is inserted. In the theoretical approach no transformation into higher order modes were considered because their excitation needs a coupling with these modes. This coupling can be done by the gaps.

Measurements are performed with the wave TE<sub>10</sub> in a rectangular waveguide. In this case the electric field in the gap is  $\epsilon$  times higher than in the sample, so the most part of power leaks through this gap if the gap is along the broader side of the waveguide, the size of the gap is bigger than 1 % of the waveguide size, and  $\epsilon$  is bigger than 10.

It is impossible to insert a sample into the holder – a part of the line where the sample is placed - if there are no gaps between them. So, the situation seems hopeless and this method can only be used for  $\epsilon$  about 10 or less.

However, real measurements give us a hint, how to improve measurement accuracy, at least partly.

## HALF-WAVE RESONANCE

In Fig. 1 results of measurements of a sample of ZrO<sub>2</sub> are shown. The sample, 3 mm thick, was measured in a WR90 waveguide (22.86 × 10.16 mm) in the frequency range from 8.2 to 12.4 GHz. We can see two big resonances; their frequencies are  $f_1 = 8.956$  and  $f_2 = 10.33$  GHz. There are also several peaks above 10 GHz.

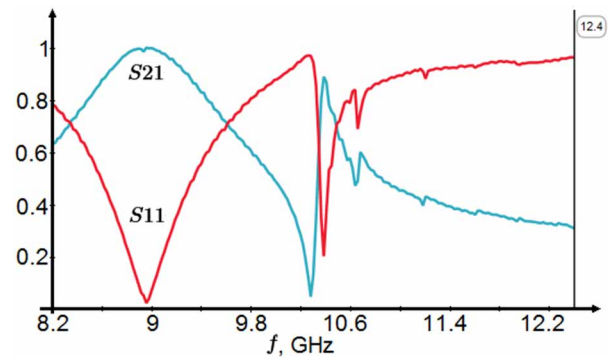


Figure 1: Measured S-parameters of a ZrO<sub>2</sub> sample.

Values of  $\epsilon$  and  $\mu$ , corresponding to the measured S-parameters, calculated by formulas from [1 - 4], are presented in Fig. 2. We can see that the theory doesn't work in this example near the resonances, and only at the higher frequencies the values become having some physical sense, though the imaginary part of  $\epsilon$  should be negative in this case to correspond to losses in the material.

We simulated this example with help of CST Microwave Studio. The model used for the simulation is shown in Fig. 3.

In Fig. 4 one can see a resonance at 9.2 GHz like in Fig. 1, but no more resonances appear. The resonance at 9.2 GHz is a so-called half-wave-length resonance. The half-wave ceramic windows are used in RF technique in transitions between vacuum and air-filled waveguides. They make possible to transfer RF power in these waveguides without reflection if the thickness of the window  $d = \Lambda/2$ , where  $\Lambda$  is the wave length in the waveguide filled with the dielectric.

For a rectangular waveguide with cross-section  $a \times b$

$$\Lambda = \frac{\lambda}{\sqrt{\epsilon - (\lambda/2a)^2}},$$

$$\text{then } \epsilon = \frac{\lambda}{4} \left( \frac{1}{a^2} + \frac{4}{\Lambda^2} \right) = \frac{c}{4f} \left( \frac{1}{a^2} + \frac{1}{d^2} \right), \quad (1)$$

where  $f$  is frequency,  $\lambda$  and  $c$  are the wave length and speed of light in free space.

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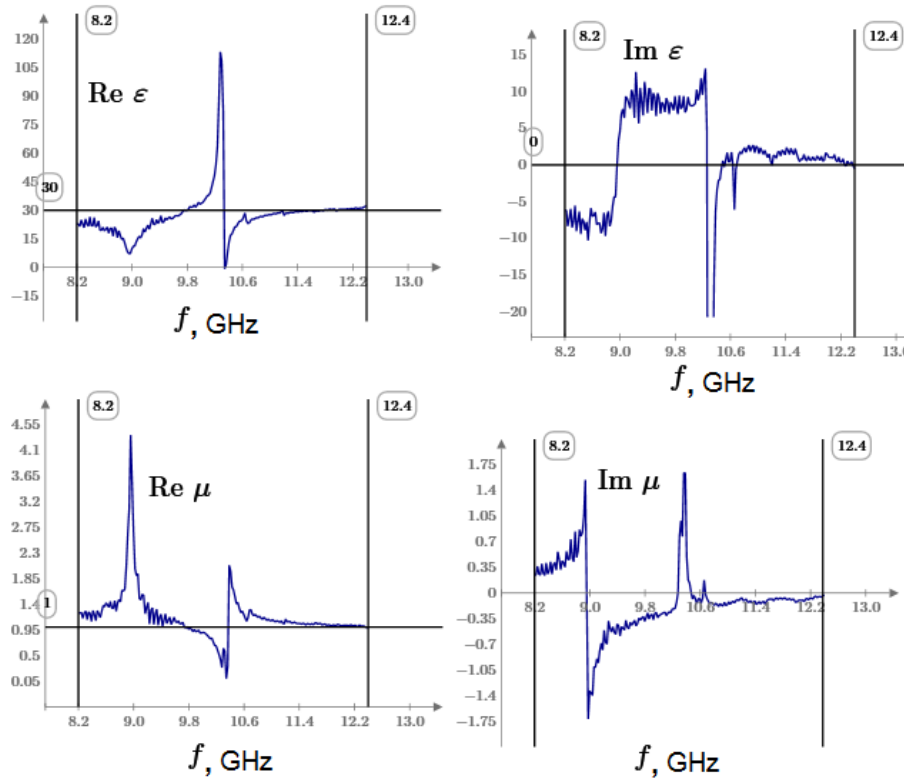


Figure 2: Results of calculations by theoretical formulas from [1 – 4].

### GAP ALONG THE LONGER SIDE OF THE SAMPLE

From (1) we can find  $\epsilon = 31.67$  for  $f_1 = 8.956$  GHz. The second resonance, at  $f_2 = 10.33$  GHz in measurements, appears in simulation if gaps  $\Delta b1$  and  $\Delta b2$  between the sample and the waveguide broader wall are introduced, Figs. 5 and 6. Let us to begin with the gaps along the narrow sides of the waveguide,  $\Delta a1$  and  $\Delta a2$ , equal to zero, and both gaps  $\Delta b1$  and  $\Delta b2$  be equal and consider the dependence of the resonances  $f_1$  and  $f_2$  on the gap width, Fig. 7. We can see that the  $\lambda/2$ -resonance corresponds to the zero gap, as it was supposed in derivation of (1), and the higher order mode resonance, let's name it HOM1, corresponds to  $\Delta b = \Delta b1 = \Delta b2 = 6.7$  micrometers. But the gaps are the same for both resonances!

Fitting values of  $\epsilon$  and the width of the gap, we can find that both resonances correspond to  $\epsilon = 33.1$  and  $\Delta b = 15.5$  micrometers, Fig. 8. (We believe that  $\epsilon$  does not

change substantially between  $f_1$  and  $f_2$ ). The point for HOM1 at  $\Delta b = 0$  is added as an extrapolation, actually this resonance is absent.

We can see that now found values of  $\epsilon$ , close to 30, correspond to the higher frequency end for  $\text{Re } \epsilon$  in Fig. 2. These values show that peaks of  $\epsilon$  shown in Fig. 2 (upper, left) do not exist in reality.

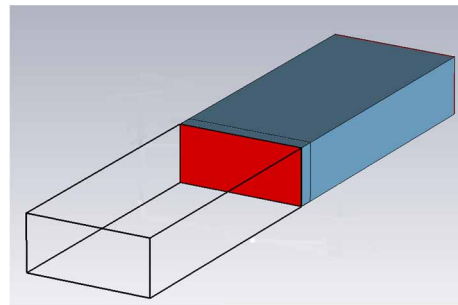


Figure 3: The model for simulations.

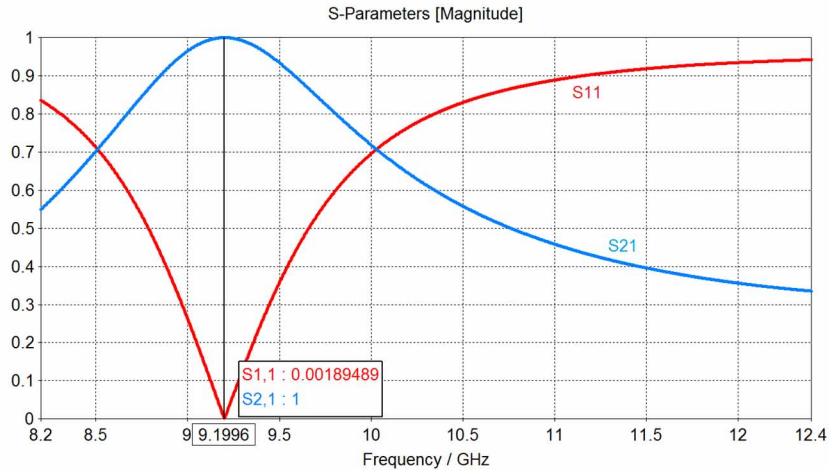


Figure 4: The half-wave resonance in a sample with  $\epsilon = 30$ .

More precise calculation should use a finer mesh because the gaps are very small, take into account gaps  $\Delta a1$  and  $\Delta a2$ , consider the gaps to be unequal, and consider losses, e.g., the imaginary part of  $\epsilon$  and dependence of  $\epsilon$  on frequency. The correction for  $\epsilon$  found even with a rough mesh shows that accuracy of its measurement can be significantly improved compared to the results presented in Fig. 2.

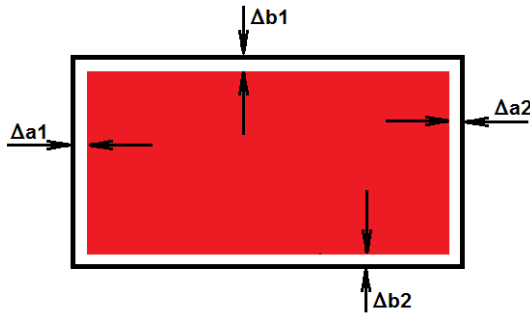


Figure 5: Gaps between the sample and waveguide walls.

The ideal fit of the measured complex values of  $S_{11}$  and  $S_{21}$ , Fig.1, and the simulated values of these parameters, Fig. 6, can be presented as a minimum of the objective function

$$F = \sum_n (|S_{11}_n^m - S_{11}_n^s|^2 + |S_{21}_n^m - S_{21}_n^s|^2),$$

where index  $m$  means measured values, index  $s$  means simulated values, and  $n$  runs through all the corresponding

points. Changeable parameters of minimization can be already mentioned values of gaps, real and imaginary parts of  $\epsilon$ , possibly depending on frequency, other deviations from the ideal shape, e.g., gaps with changing width, small rotations of the sample about the axes of coordinates, etc. Measurements of shape with accuracy better than 1 micrometer can be done using contemporary instruments like a Zeiss Acura Measuring Machine with a ruby probe [4].

## HIGHER ORDER MODES IN THE SAMPLE

Resonances in the measurements can be found in simulations and defined as eigenmodes of the dielectric sample. The simplest of them is the mode TM111, corresponding to zero reflection when  $d = \lambda/2$ .

The mode TM120 appears when there appears a coupling for it due to the gaps  $\Delta b1$  and  $\Delta b2$  along the longer side of the sample. Other modes are also defined by gaps.

One can see a small depression on the top of the  $S_{21}$  curve in Fig. 1. Simulation showed that this depression appears only if gaps  $\Delta b1$  and  $\Delta b2$  are not equal. The bigger is difference between these gaps, the deeper is the depression.

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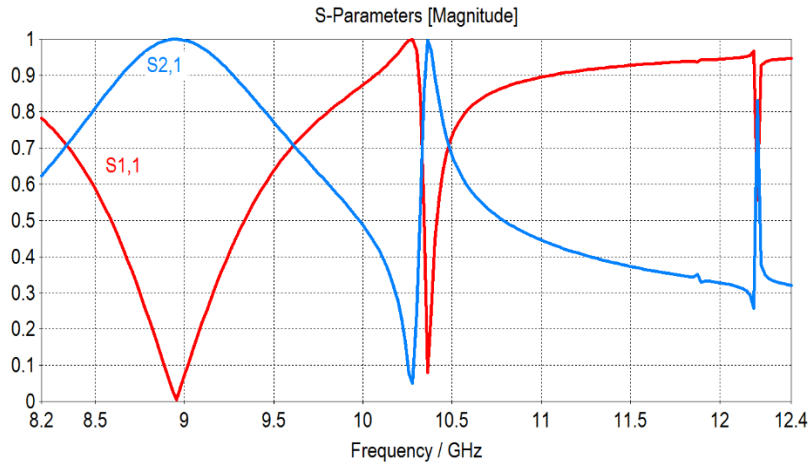


Figure 6: The second and other resonances appear in simulation when a gap along the long side of the sample is introduced.

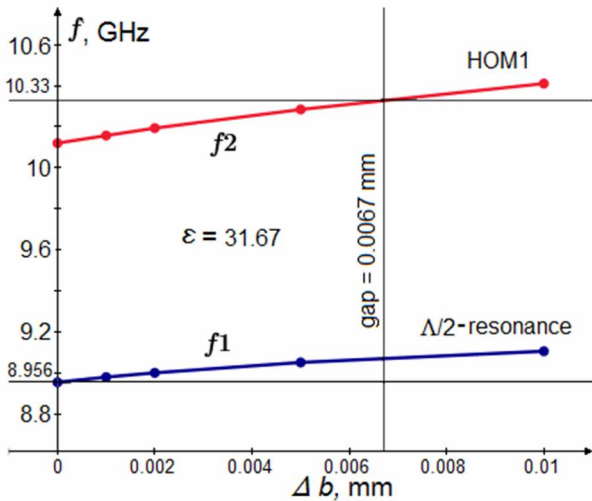


Figure 7: Gaps are different for  $\epsilon = 31.67$ .

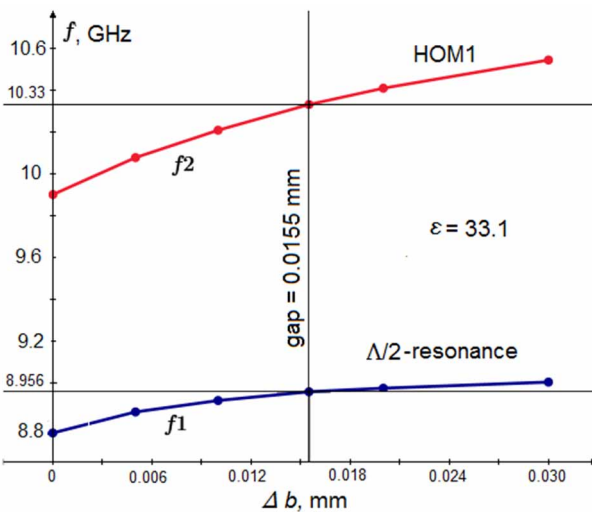


Figure 8: Corrected values of  $\epsilon$  and width of gap.

### OTHER POSITIONS OF THE SAMPLE IN THE WAVEGUIDE

Dimensions of the sample, as it was already mentioned, are close to the dimensions of the waveguide cross-section, so a well-defined position of it can be flat with the broad side of the waveguide across it, or along the waveguide, flush with the narrow wall, Fig. 9. Well-defined positions means that the gaps between the sample and the waveguide walls are minimal.

S-parameters measured for the sample across the waveguide have many complicated resonances and strongly depend both on the losses in the sample and gaps between the sample and the waveguide surface. This position can be possibly used for evaluation of the imaginary part of  $\epsilon$ .

A typical pattern of the S-parameters, when the sample is oriented along the waveguide and placed near the narrow wall is shown in Fig. 10. The number and positions of the peaks of  $S_{11}$  depend on  $\epsilon$  and dimensions of the sample and the waveguide. For a given waveguide WR90 and the sample thickness 3 mm, the positions of the  $S_{11}$  peaks are presented in Fig. 11. If we have a measured plot of the S-parameters, like in Fig. 10, we have to measure the frequencies of the peaks and to use Fig. 11 to define  $\epsilon$ .

For example, let the measured frequencies are 10 and 11 GHz and these are the second and the third peaks. We can find the position of the horizontal marker such that it crosses simultaneously the lines of the second and the third peaks and the vertical markers for these frequencies. On the ordinate axis we can see that these peaks position correspond to  $\epsilon = 26$ , see Fig. 11.

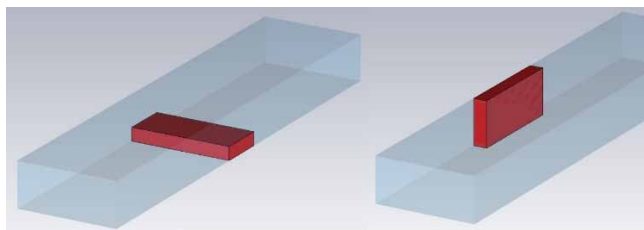


Figure 9: Two well-defined positions of a sample in the waveguide.

There is one only peak if  $\epsilon < 10$ . Some irregularities of the curves in Fig. 11 corresponding to the peaks near  $f = 8.6$  GHz,  $\epsilon$  from 20 to 30, and in the area near  $f = 12.0$  GHz,  $\epsilon$  from 45 to 50 are caused by other resonances close to these peaks.

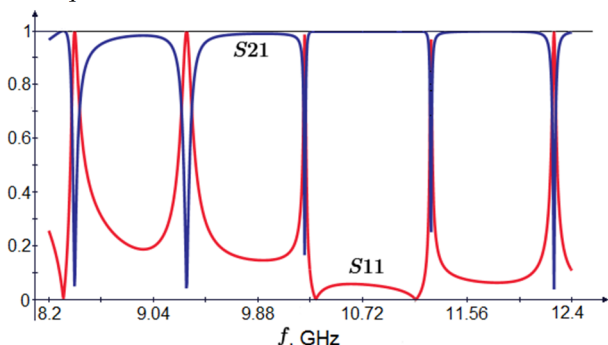


Figure 10: S-parameters for the along oriented sample,  $\epsilon = 30$ ,  $d = 3$  mm, waveguide  $22.86 \times 10.16$  mm.

The presented method of measurements of  $\epsilon$  with the sample placed along the waveguide is acceptable for low losses in the sample, when the loss tangent is less than approximately 0.003. When the losses are higher, the peaks become lower and slightly shift to the higher frequencies though the distances between peaks keep almost the same. Gaps between the sample and walls of the waveguide should be less than 10 micrometers for the same error as the error caused by  $\tan \delta \approx 0.003$ .

In these measurements, the position of the sample along the length of the waveguide does not matter because only absolute values of S-parameters are taken into account. We consider that the material of the sample is non-magnetic.

## CONCLUSION

A method is presented for measurements of high values of dielectric permittivity  $\epsilon$  in a waveguide at high frequencies. This method supplements the method of measurements evolved for low values of  $\epsilon$  [1 – 4], and is close to resonant methods [5], when a sample is placed into a cavity, and the measurement is done at one only

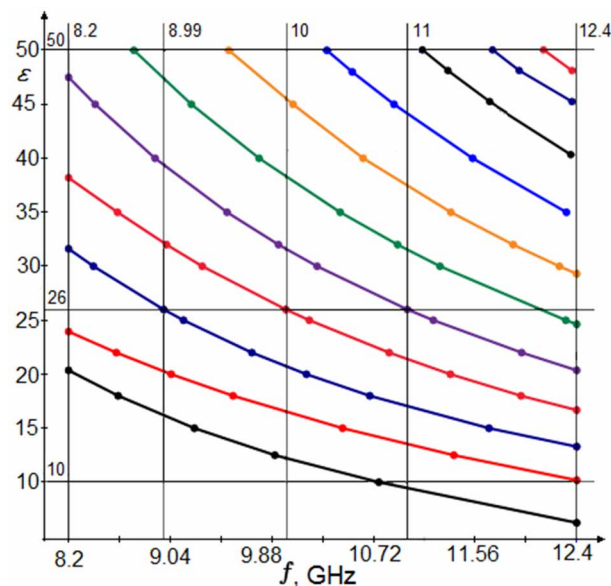


Figure 11: Position of  $S_{11}$ -peaks for different values of  $\epsilon$ .

frequency, but makes possible to measure this value in several frequency points at one measurement.

The special shape of the sample used for measurements in a waveguide makes possible to use well-defined positions of it and gives additional possibilities of measuring both real and imaginary part of dielectric permittivity.

## ACKNOWLEDGMENT

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