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EQUIDISTANT OPTIMIZATION OF ELLIPTICAL SRF STANDING WAVE CAVITIES

V. Shemelin*, LEPP, Physics Dept., Newman Laboratory, Cornell University, Ithaca, NY, USA
V. Yakovlev, Fermi National Accelerator Laboratory, Batavia, IL, USA

Abstract

A record accelerating rate was achieved earlier in standing wave (SW) SRF cavities when their shape was optimized for lower peak surface magnetic field. In view of new materials with higher limiting magnetic fields, expected for SRF cavities, in the first line Nb3Sn, the approach to optimization of cavity shape should be revised. A method of equidistant optimization, offered earlier for traveling wave cavities is applied to SW cavities. It is shown here that without limitation by magnetic field, the maximal accelerating rate is defined to a significant degree by the cavity shape. For example, for a cavity with the aperture radius Ra = 35 mm the minimal ratio of the peak surface electric field to the accelerating rate is about Epk/Eacc = 1.54. So, with the maximal surface field experimentally achieved E_pk = 125 MV/m, the maximal achievable accelerating rate is about 80 MeV/m even if there are no restrictions by the magnetic field. Another opportunity ¿ optimization for a low magnetic field, is opening for the same material, Nb3Sn, with the purpose to have a high quality factor and increased accelerating rate that can be used for industrial linacs.

INTRODUCTION

A record accelerating rate was achieved earlier with elliptical SRF cavities [1] when their shape was optimized [2] so that the peak surface magnetic field B_{pk} was decreased by 10%, and the peak surface electric field E_{pk} was increased by 20% compared to the TESLA cavity with the same accelerating field E_{acc} on the cavity axis. This change of shape was done due to understanding that the superheating field B_{sh} (250 mT for Niobium [3], the factual limit is about 210 mT) is the fundamental limit to acceleration in SRF cavities and decreasing the value B_{pk}/E_{acc} we can increase the accelerating rate E_{acc}. New materials, and first of all Nb_3Sn, are promised that they could be run at twice the magnetic field of Nb [3]. Does this mean that the accelerating rate can be twice the rate achieved in the Nb cavities? It seems that the surface electric field can become the next limit to the highest achievable accelerating rate. The surface electric field up to 12 MV/m has been demonstrated [1, 4, 5] in single-cell cavities. So, if this field is a limit, we need to decrease E_{pk}/E_{acc} to achieve maximal acceleration rate. For better results, as can be supposed, we should stay at equal distances from both limits. A method of equidistant optimization was offered earlier for TW cavities [6]. Now we apply this method to the SW elliptic cavities which are better studied than the TW SRF cavities and are easier in production than the TW ones.

GEOMETRY OF AN ELLIPTIC CAVITY

For easier explanation of the following, let us remind the geometry of the cavities under consideration.

Contemporary superconducting rf cavities for high energy particle accelerators consist of a row of cells coupled together as shown in Fig. 1. The contour of a half-cell consists of two elliptic arcs and a straight segment tangential to both. The contour can be described by several geometrical parameters shown in Fig. 1(b). Three of these parameters, length of the half-cell L, aperture R_a, and equatorial radius R_{eq} are defined by physical requirements: $L = \lambda/4$ (for π-mode), where λ is the RF wave length; the aperture is defined by requirements for coupling between cells and by the level of wakefields that can be allowed for a given

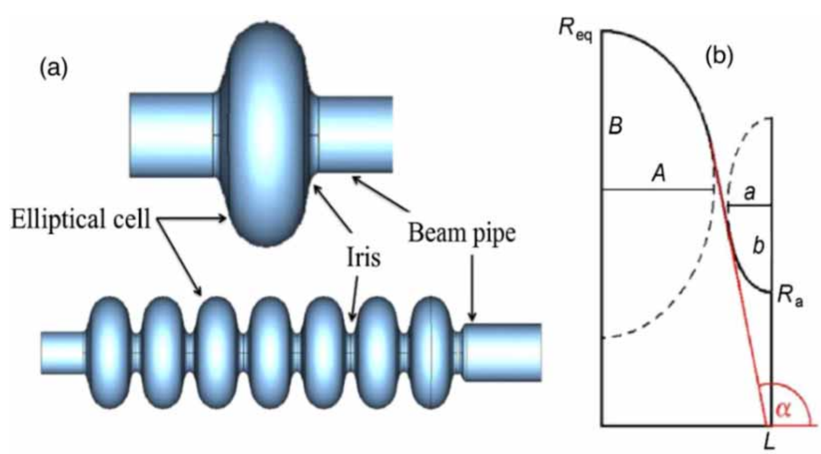

Figure 1: (a) Single-cell and multicell elliptical cavities; (b) geometry of the half-cell.

*vs65@cornell.edu

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accelerator; and the equatorial radius R_{eq} is used for tuning the cavity to a given frequency. The remaining four parameters (*A, B, a, b*) can fully describe the geometry. Here *A, B* and *a, b* are the half-axes of the equatorial and iris constitutive ellipses, respectively. The best combination of four parameters is the goal for the cavity shape optimization. The angle of the wall inclination between the axis of rotation and the straight segment of the wall is designated as α. The cavities with $\alpha < 90°$ are known as the reentrant cavities.

EQUIDISTANT APPROACH FOR OPTIMIZATON

Optimization of an elliptical cavity is usually done as a search for minimum B_{pk}/E_{acc} when the value of E_{pk}/E_{acc} is given. It is also possible to minimize E_{pk}/E_{acc} for a given B_{pk}/E_{acc} but the truth is that we need to reach as high as possible accelerating gradient E_{acc} before field emission or magnetic quench limit further increase of the accelerating gradient. So, the ideal situation would be to reach both limits simultaneously using all the possibilities to increase E_{acc}. If we know the maximal achievable surface peak fields E^*_{pk} and B^*_{pk}, then the cavity having equal values of E_{pk}/E^*_{pk} and B_{pk}/B^*_{pk} will be at equal distances from either limit. Then the criterion of the shape optimization can be written as a minimum of the maximum of two values: E_{pk}/E^*_{pk} and B_{pk}/B^*_{pk}, or, shortly, min max (E_{pk}/E^*_{pk}, B_{pk}/B^*_{pk}). We named this approach *the equidistant optimization*.

In the optimization, absolute values of E_{pk} and B_{pk} do not matter, only their ratio is important. This ratio depends on the geometry only. Values under the sign of minmax (see above) become equal in the result because E_{pk} and B_{pk} change reversely: when one of them increases, the other decreases, and vice versa.

The definition given above for the equidistant optimization can be rewritten in an equivalent form more convenient for calculations:

$$\text{Goal} = \min E_{pk} \text{ if } E_{pk}/B_{pk} > E^*_{pk}/B^*_{pk}$$
$$\text{or}$$
$$\text{Goal} = \min B_{pk} \text{ if } E_{pk}/B_{pk} < E^*_{pk}/B^*_{pk}, \quad (1)$$

where the Goal is a combination of the geometrical parameters *A, B, a*, and *b*, giving the desired minimum. The practice showed that the Goals defined by the first or the second way differ less than 0.01% if accuracy of the geometrical parameters is 0.01 mm.

So, the cavity shape optimized for given values of E^*_{pk} and B^*_{pk} depends only on their ratio E^*_{pk}/B^*_{pk}; optimization, for example, for E^*_{pk} = 120 MV/m and B^*_{pk} = 240 mT will be the same as optimization for E^*_{pk} = 100 MV/m and B^*_{pk} = 200 mT. Let us call this optimization "optimization 100/200" just as a remind that this ratio is for limiting surface fields of 100 MV/m and 200 mT.

Optimization for minimum B_{pk}/E_{acc} when the value of E_{pk}/E_{acc} is given, can be revised in the light of the method proposed here. For example, well optimized for a given aperture, the TESLA cavity has E_{pk}/E_{acc} = 2 and B_{pk}/E_{acc} = 4.2 mT/(MV/m). If we assume that both limits, E_{pk} and B_{pk}, are achieved simultaneously in this optimization, then $E_{pk}/B_{pk} = E^*_{pk}/B^*_{pk}$ = 2/4.2 (MV/m)/mT = 100/210 (MV/m)/mT. This means that this cavity can be treated as a cavity optimized for E^*_{pk} = 100 MV/m and B^*_{pk} = 210 mT, or, proportionally, for example, for E^*_{pk} = 80 MV/m and B^*_{pk} = 168 mT. The TESLA cavity cannot reach E_{acc} > 50 MV/m because in this case B^*_{pk} should be higher than 210 mT.

A possible future progress in the increase of achievable fields can change this proportion, and we have already this proportion changed [1]: a gradient of 59 MV/m was achieved in a single-cell reentrant cavity that corresponds to a peak surface electric field of 125 MV/m and a peak magnetic field of 206.5 mT. These values really look like maximal fields for existing cavities. The gradient was limited by a hard quench though the exponentially growing field emission when E_{pk} > 100 MV/m [4] shows that we are also close to the limit of the electric field. We can make another optimization with these parameters 125/206.5 ≈ 120/200, "optimization 120/200." If we knew what fields are maximum achievable, we will find the optimal shape from the first try. If we can afford the magnetic limiting field higher than 210 mT, but the limit in the electric field is still on the level of 125 MV/m, then for the accelerating rate higher than 62.5 MV/m we should have $E_{pk}/E_{acc} < 2$.

The difference between these two methods is in the fact that we do not know *a priori* what value of B_{pk}/E_{acc} we will have for a given value of E_{pk}/E_{acc} in the old method, but in the new method, we can choose the ratio between the extremal fields based on experiment, and then perform the optimization.

Values of E_{pk}/E_{acc} and B_{pk}/E_{acc} will be obtained as a result of optimization when limiting fields are given. The procedure of optimization for min max (E_{pk}/E^*_{pk}, B_{pk}/B^*_{pk}) consists in a systematical change of the elliptical half axes *A, B, a*, and *b* (Fig. 1) decreasing maximal value in parentheses, as a result both ratios become equal. This optimization method warrants a more complete study in comparison with conventional method for SW cavity optimization.

RESULTS OF OPTIMIZATION FOR INNER CELLS

Optimization was done for the inner cell of a multicell cavity with the aperture radius R_a = 35 mm. The end cells and the single-cell cavities should be optimized separately because of different boundary conditions at their ends. Besides, the end cells can be connected with beampipes of different configurations (Fig. 1). We will not analyze these cases. The purpose of this paper is to show the main features of the equidistant optimization for different limiting fields.

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Results of optimization – maximal achievable accelerating rates for different values of E^*_{pk}/B^*_{pk} - are presented in Fig. 2.

First question that can be asked is the following: what maximal acceleration can be achieved if there is no limitation by magnetic field? Calculations show that the minimal value of E_{pk}/E_{acc} is 1.536 that corresponds to E^*_{pk}/B^*_{pk} = 100/350. So, no benefit from increasing the limiting magnetic field above 350 mT can be obtained if the limiting electric field is 100 MV/m. The maximum can be counted on is about 65 MV/m. For E^*_{pk} = 125 MV/m that can be achieved now with very thorough surface preparation and with B^*_{pk} = 400 mT that hopefully can be obtained with a new material, we can recon not more than on 80 MV/m.

We examine here the elliptic cavities. However, the optimal cavity for the minimal E_{pk}/E_{acc} = 1.536 degenerates to a cavity with zero half-axes of the upper ellipse: A = 0, B = 0.

Dimensions and field ratios for different values of E^*_{pk}/B^*_{pk} are presented in Table 1. The areas corresponding the colored areas in Fig. 2 are also shown in color.

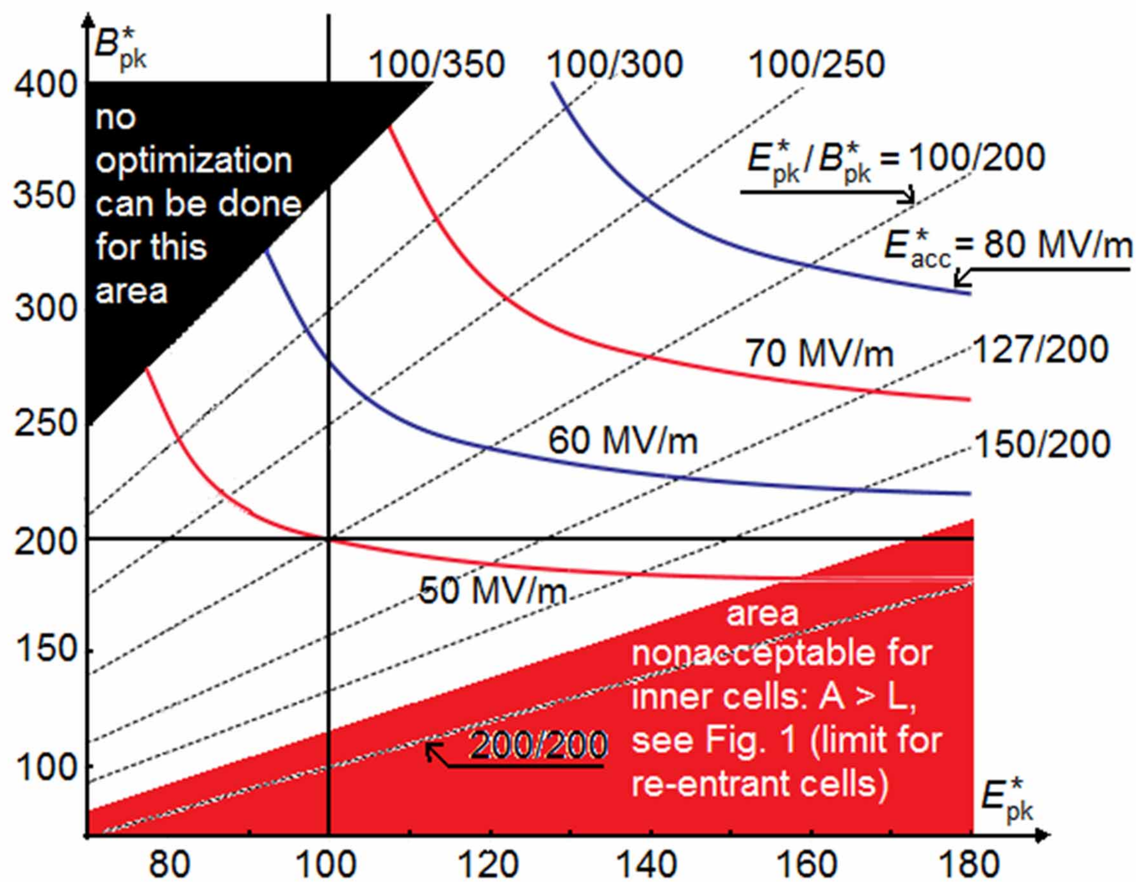

Figure 2: Maximal achievable accelerating rates for different limiting electric and magnetic fields. R_a = 35 mm.

Table 1: Result of equidistant optimization of inner cell of a multicell cavity, aperture radius is 35 mm, frequency is 1300 MHz. Units for B_{pk}/E_{acc} are mT/(MV/m).

E^*_{pk}/B^*_{pk},	100/350	100/300	100/250	100/200	127/200	150/200	172/200	200/200
E_{pk}/E_{acc}	1.536	1.62	1.745	1.998	2.399	2.767		3.605
B_{pk}/E_{acc}	5.43	4.86	4.36	4.00	3.78	3.69		3.60
A, mm	0	31.2	38.5	45.2	51.63	55.06	57.652	59.5
B, mm	0	45.4	38.3	35.9	36.10	36.78		37.65
a, mm	500	35.8	20.1	12.8	9.06	7.23		5.06
b, mm	2726	141.6	48.1	21.8	12.04	8.65		5.15
R_{eq}, mm	124.5	108.417	103.689	100.742	98.707	98.012		97.369
α, deg.	118.8	111.8	98.9	86.8	69.0	61.7		57.5

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HIGH-Q CAVITIES FOR INDUSTRIAL LINACS

Now an industrial linac is under consideration, which is based on Nb_3Sn-coated ILC-type 1.3 GHz acceleration cavity [7]. High Q_0 at 4.4 K allows conduction cooling and cryocooler instead of He bath and refrigerator, which is extremely attractive for linacs operating in industrial environment. However, a cryocooler may remove ~2.4 W, and it is not reasonable to increase the gradient beyond ~8 MeV/m using 2-3 cryocoolers, because of Q_0 drop, see Fig. 11 in [8]. Further increase of the gradient is not reasonable, the loss, say at 10 MeV/m reaches 12 W/m and the number of cryocoolers is impractical. The reason, however, is that ILC structure is optimized for HIGH GRADIENT, not HIGH Q_0: E_{pk}/E_{acc} = 2 and B_{pk}/E_{acc} = 4.26 mT/(MeV/m). For E_{acc} ~ 10 MeV/m one has a surface electric field of 20 MV/m, it is too low compared to the FE onset. On the other hand, B_{pk} is too high providing significant drop in Q_0. It is possible for a production version to reoptimize the linac completely changing balance between E_{pk} and B_{pk} to smaller values of B_{pk}.

For reentrant cavities, the distance L - A is the distance between inner surfaces of neighbor cells and should be big enough to account the thickness of material and the gap needed to weld the cells together.

Optimization for E^*_{pk}/B^*_{pk} > 0.5 (MV/m)/mT will lead to an increase of the half-axis A (Fig. 1) so that starting from the point marked as Δ0, see Fig. 3, the difference $\Delta = L -$ A can become less than zero when E^*_{pk} is increasing. However, we can make the optimization not increasing A above a given value. This is done in Fig. 3. The lower curve in each group of 3 curves presents the extreme case when the Δ is limited by zero for $E^*_{pk}/B^*_{pk} > 50/50$ and is shown as a reference, the next one marked as Δ5 corresponds to a limitation of 5 mm, and Δ10 is for 10 mm. The points Δ0 and so on are shown on the middle group of curves but are related to all groups because the shape of cells is the same on each dashed line.

Figure 3 shows that the acceleration rate of 10 MV/m can be achieved at $B_{pk} = 35$ mT. This is about 15% less than in the case of the TESLA cavity shape and makes cryocooling more practical.

Results of optimization for low magnetic field cavities are presented in Table 2. In the last line of the Table 2 are values of the product $G \cdot R_{sh}/Q$, where G is the geometry factor, and R_{sh}/Q is the geometric shunt impedance. This product defines losses in the cavity, e.g., for the TESLA cavity $R_{sh}/Q = 15400$ Ohm2. One can see that losses in the optimized cavities can be up to 30% less than in the TESLA cavity with the same accelerating rate due to lower surface magnetic field.

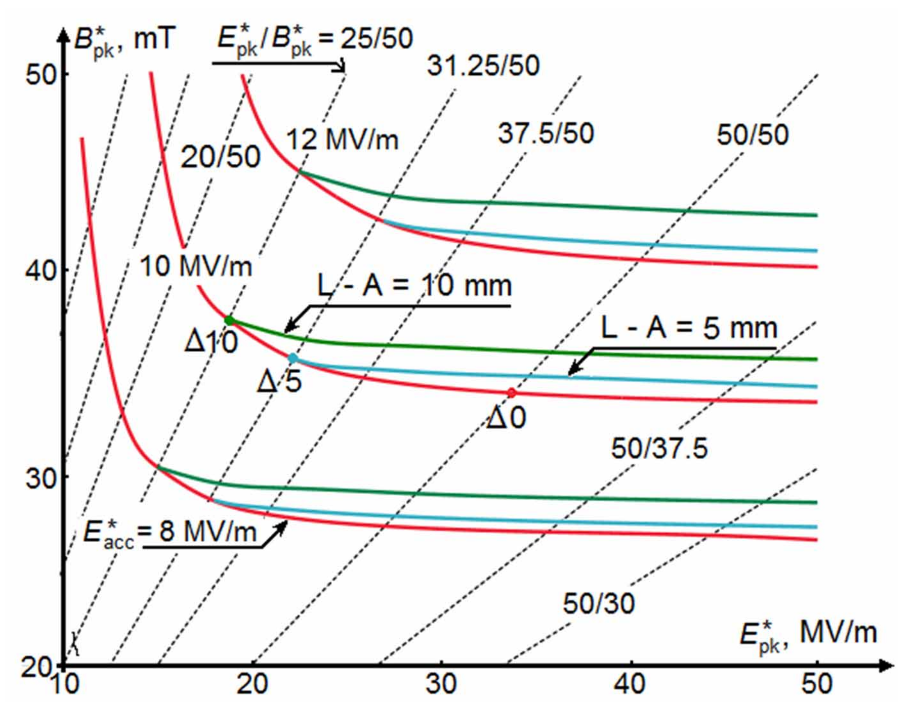

Figure 3: Equidistant optimization for inner cells of a multicell cavity when the increase of A is limited by certain values. Aperture radius R_a = 30 mm.

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Table 2: Geometrical and electromagnetic parameters for Fig. 3. Units for B_{pk}/E_{acc} are mT/(MV/m). If there are two lines in a Table cell, they belong to different A's (i.e., to different gaps $\Delta = L - A$): the first set is for the upper lines (Δ = 5 mm), the second set is for the second lines (Δ = 10 mm).

E^*_{pk}/B^*_{pk}, (MV/m)/mT	20/50 (0.4)	25/50 (0.5)	31.25/50 (0.625)	37.5/50 (0.75)	50/50 (1)	50/30 (1.667)
E_{pk}/E_{acc}	1.64	1.88	2.22 2.28	2.62 2.72	3.45 3.59	5.64 5.88
B_{pk}/E_{acc}	4.11	3.76	3.56 3.65	3.50 3.62	3.45 3.59	3.39 3.54
A, mm	39.45	46.25	52.04 47.65	52.65 47.65	52.65 47.65	52.65 47.65
B, mm	38.99	38.63	38.52 38.45	40.35 40.69	41.86 44.38	43.17 45.2
a, mm	19.55	11.9	8.61 10.46	9.31 11.20	10.35 13.64	11.34 14.7
b, mm	57.18	22.45	12.74 10.55	8.91 7.30	5.90 5.73	2.89 2.7
α, deg.	98.5	83.6	69.4 88.6	65.6 86.5	61.6 72.2	59.2 67.3
$G \cdot R_{sh}/Q$, Ohm^2	16230	18000	19160 18770	19680 19080	20220 19540	20860 20150

PROCEDURE OF OPTIMIZATION

2D computer code SuperLANS [9] and the TunedCell envelope code [10] have the accuracy needed for our optimization. These codes tune the cell with a given geometry to a needed frequency, changing its equatorial radius.

The goal function (1) belongs to a class of so-called ravine functions, these functions look like a surface in the ravine: it rapidly grows on the steep shores and slowly declines along the waterway. This property makes possible to change the geometric parameters in a broad range, but only if they belong to a correct direction, "along the waterway". This property is important to find a shape free of multipactor, not compromising much the goal function, moving along the "bottom" of the ravine.

Gradient descent becomes difficult for this class of functions. We used the "brute force", or "grid search" approach to search for a minimum of the goal function on a 4D grid. The step of the grid can be decreased until we reach the required accuracy. More details about the used procedure of optimization can be found in Ref. [11].

CONCLUSION

A new method – equidistant optimization - is implemented for standing wave cavities. Two cases are considered: inner cells of a cavity with high accelerating rate, and with low magnetic field suitable for industrial linacs. It is shown that for expected new materials with high critical magnetic field the limitation in accelerating rate comes to the surface electric field. For the cells with a low magnetic field, the found best shapes can decrease losses up to 30% compared to commonly used cavities.

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