

ABSTRACT

We report calculations of a dc superheating field H_{sh} in superconductors with nanostructured surfaces. Numerical simulations of the Ginzburg-Landau (GL) equations were performed for a superconductor with an inhomogeneous impurity concentration, a thin superconducting layer on top of another superconductor, and S-I-S multilayers were performed. The superheating field was calculated taking into account the instability of the Meissner state with a nonzero wavelength along the surface, which is essential for realistic values of the GL parameter κ . Simulations were done for the materials parameters of Nb and Nb₃Sn at different values of κ and the mean free paths. We show that the impurity concentration profile at the surface and thicknesses of S-I-S multilayers can be optimized to reach H_{sh} exceeding the bulk superheating fields of both Nb and Nb₃Sn. For example, a S-I-S structure with 90 nm thick Nb₃Sn layer on Nb can boost the superheating field up to ~500 mT, while protecting the SRF cavity from dendritic thermomagnetic avalanches caused by local penetration of vortices.

INTRODUCTION

- The superheating field is the maximum magnetic field H that can be screened by a superconductor in the vortex-free Meissner state [1].
- Recent advancements in Nb cavities have pushed H_{sh} to its theoretical limit, prompting the exploration of surface nano-structuring techniques to further increase H_{sh} [2,3,4].
- The goal of this study is to investigate the influence of the Ginzburg-Landau parameter κ on H_{sh} for superconductors with a nanostructured surface.

NUMERICAL CALCULATION OF SUPERHEATING FIELD

Breakdown of the Meissner state at H_{sh} occurs via a periodic modulation of the order parameter with a wavelength $\sim(\xi^3\lambda)^{1/4}$.

GL equations:

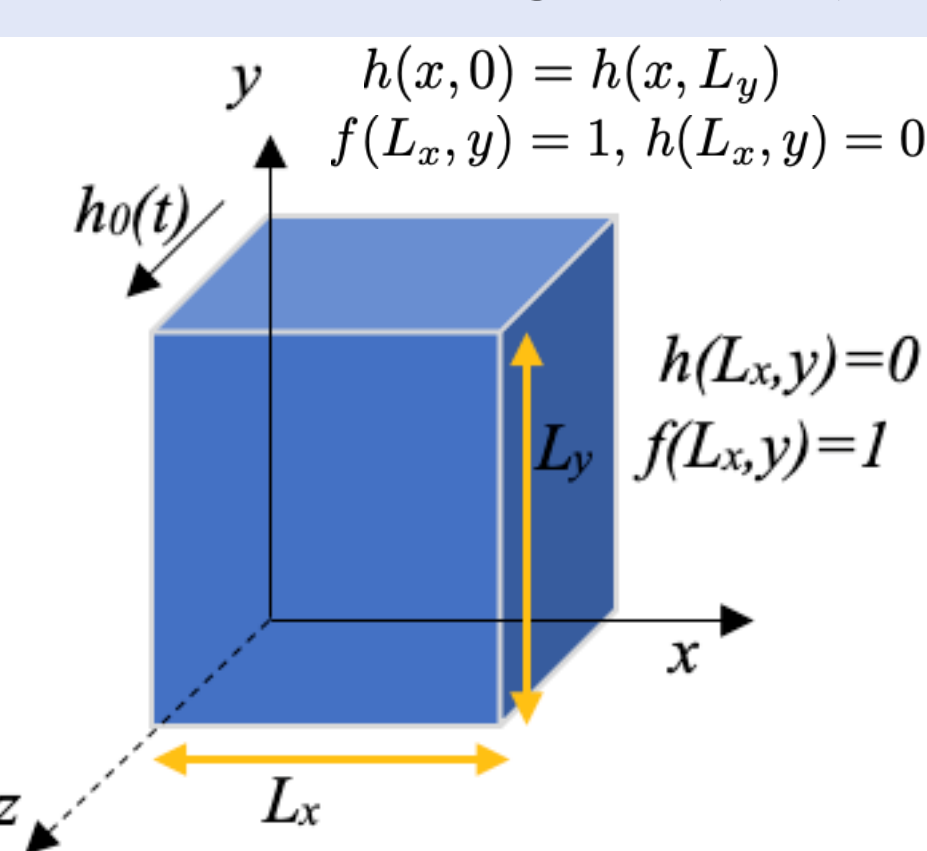
$$\dot{f} - f + f^3 - \nabla^2 f + \frac{\kappa^2}{f^3} [(\partial_x h)^2 + (\partial_y h)^2] = 0$$

$$\nabla \cdot \left(\frac{\nabla h}{f^2} \right) = \frac{h}{\kappa^2}$$

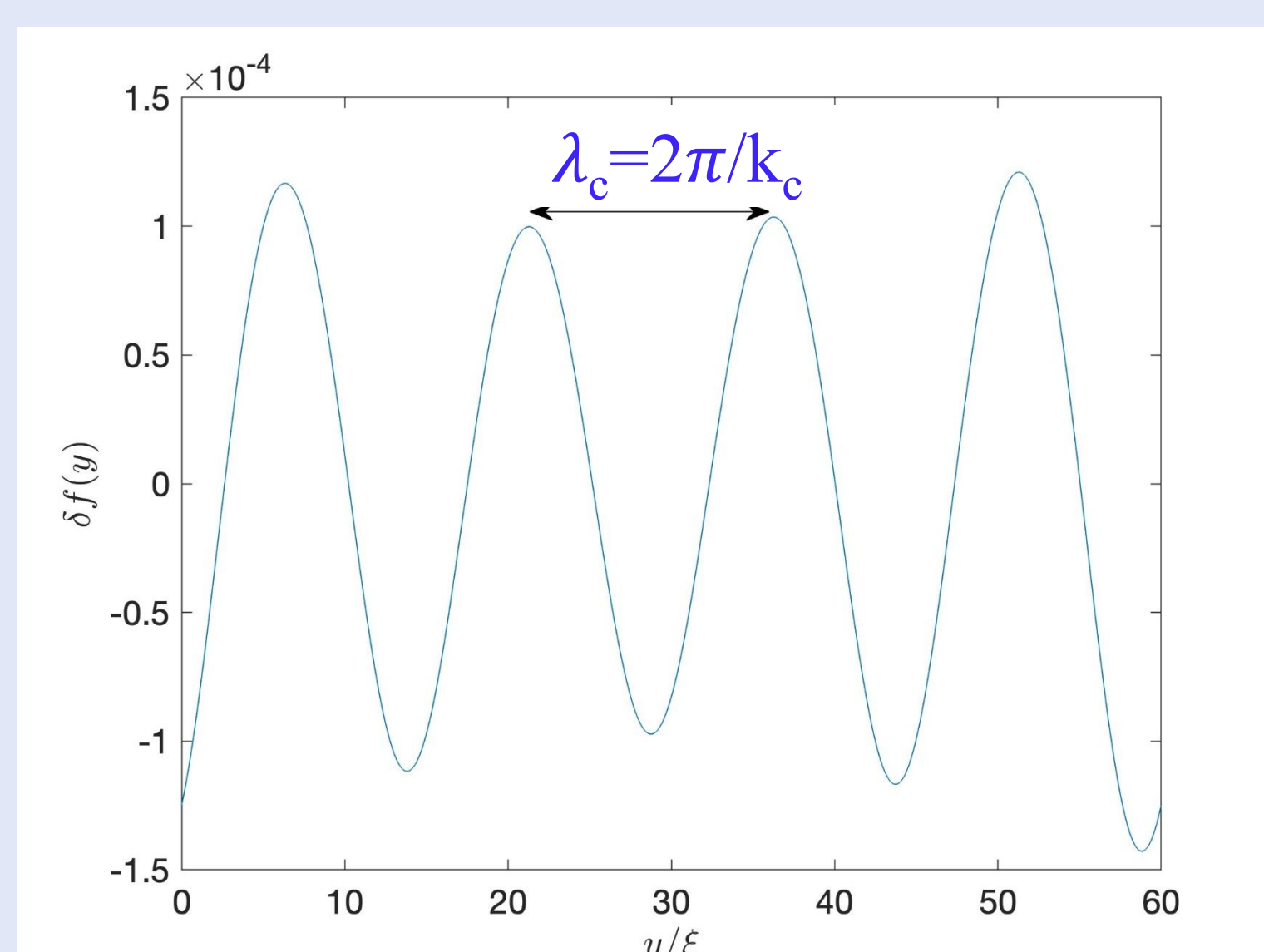
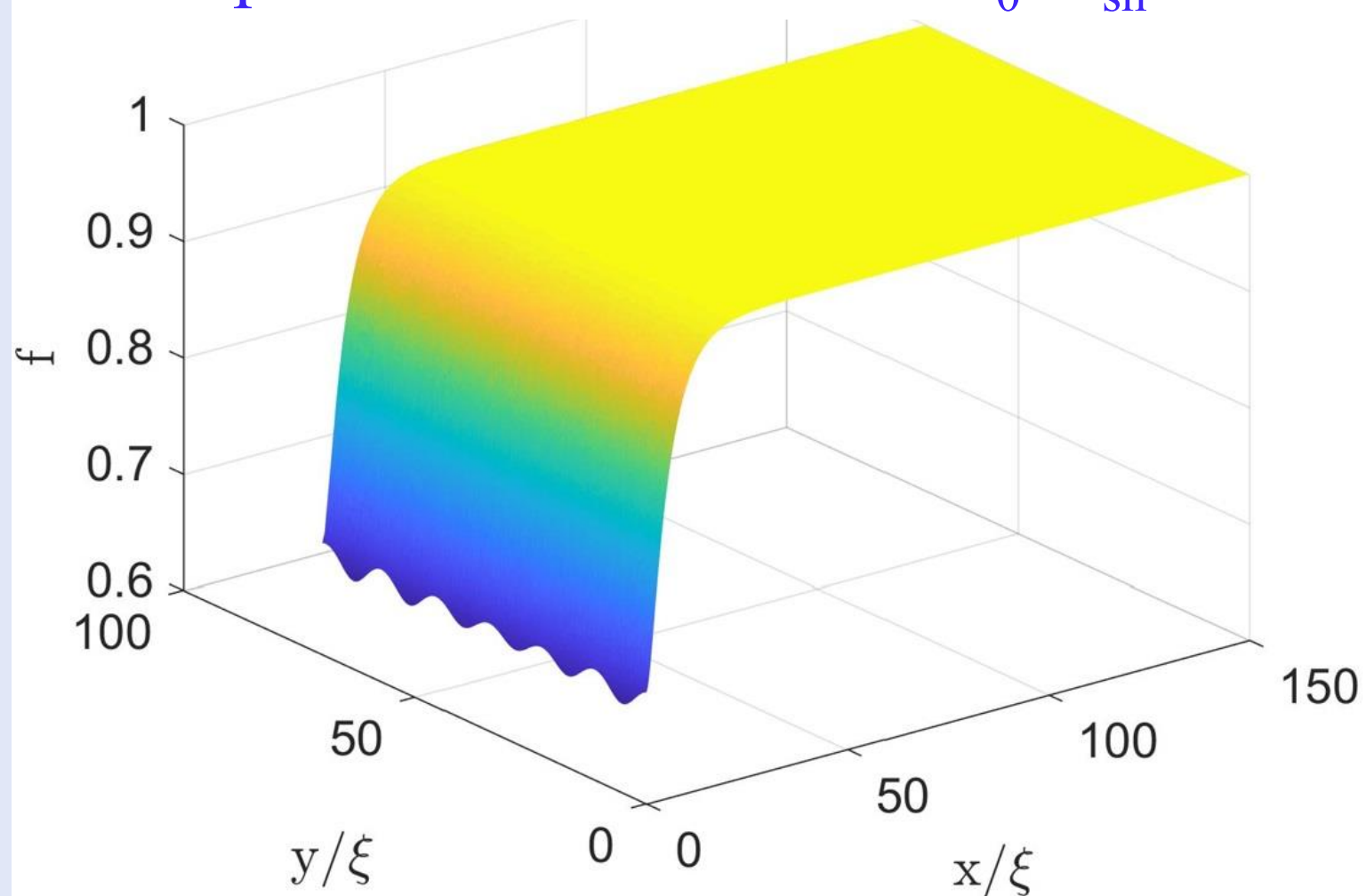
$$f = \Delta/\Delta_0, \quad h = H/\sqrt{2}H_c$$

$$h_0(t) = \theta(t - t_0)(0.01t_0 + 0.00005(t - t_0)) + 0.01t\theta(t_0 - t)$$

$\Delta(x,y,z)$ - amplitude of the complex order parameter $\psi = \Delta e^{i\theta}$, Δ_0 is equilibrium order parameter in the bulk, All lengths are in the unit of ξ , k_c -critical momentum, $\theta(t)$ - Heaviside step function, t_0 was chosen such that $h_0(t_0) < h_{sh}$.



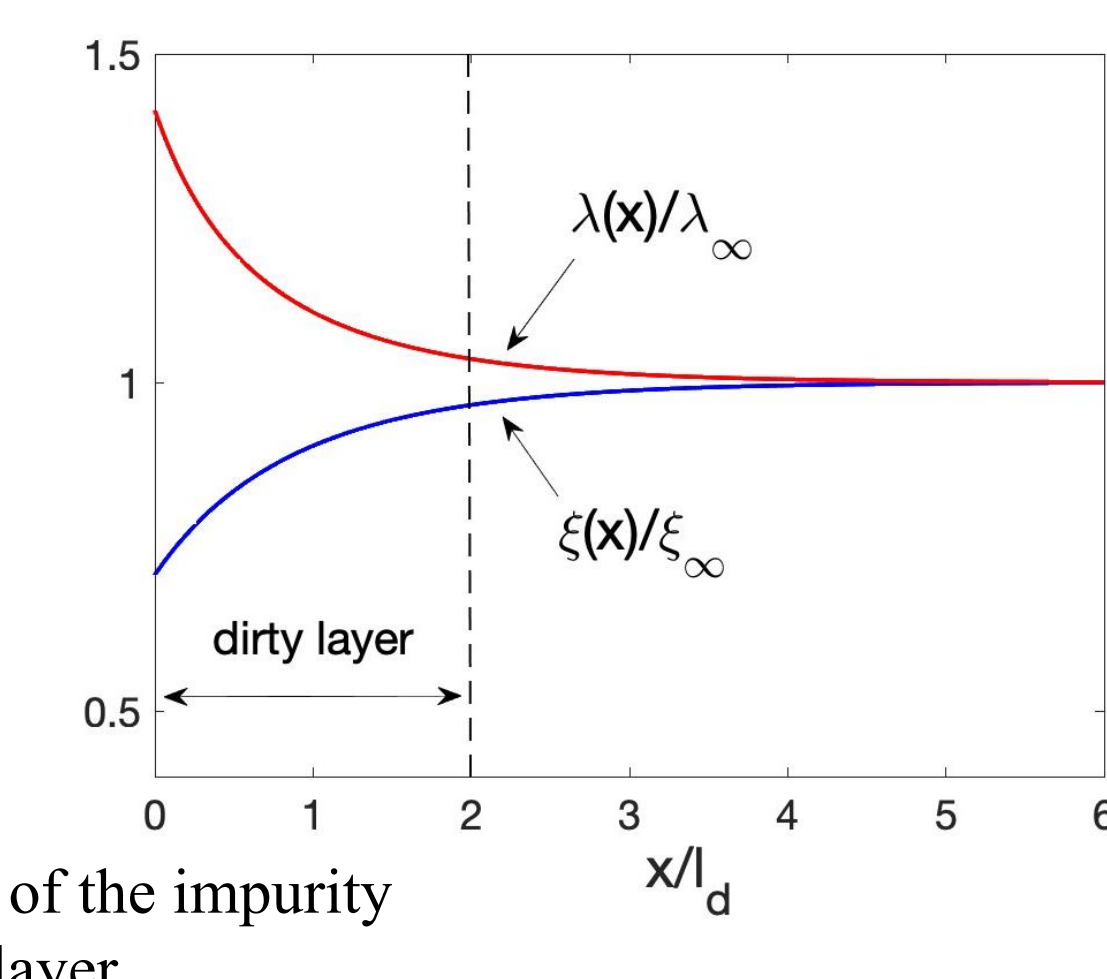
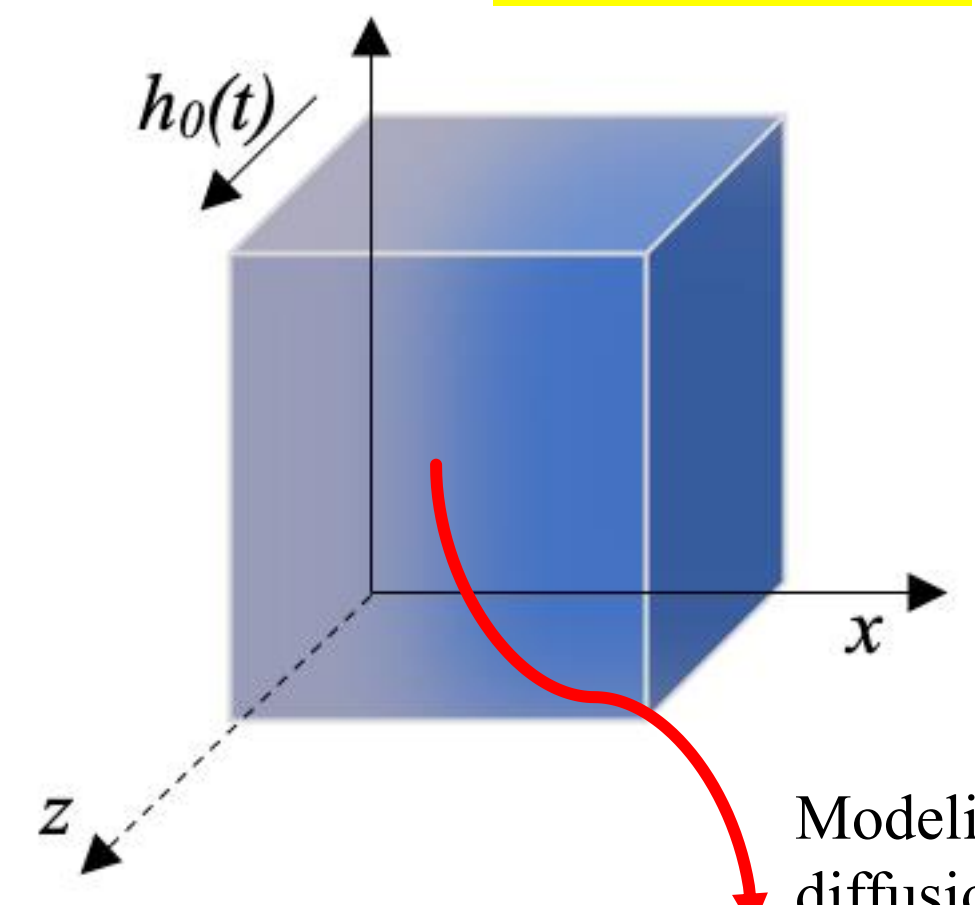
Order parameter calculated at $h_0=h_{sh}+0$



A snapshot of $\Delta f(y,0)$ at $h_0=h_{sh}+0$.

SUPERCONDUCTOR WITH AN IMPURITY DIFFUSION LAYER

S-S structure



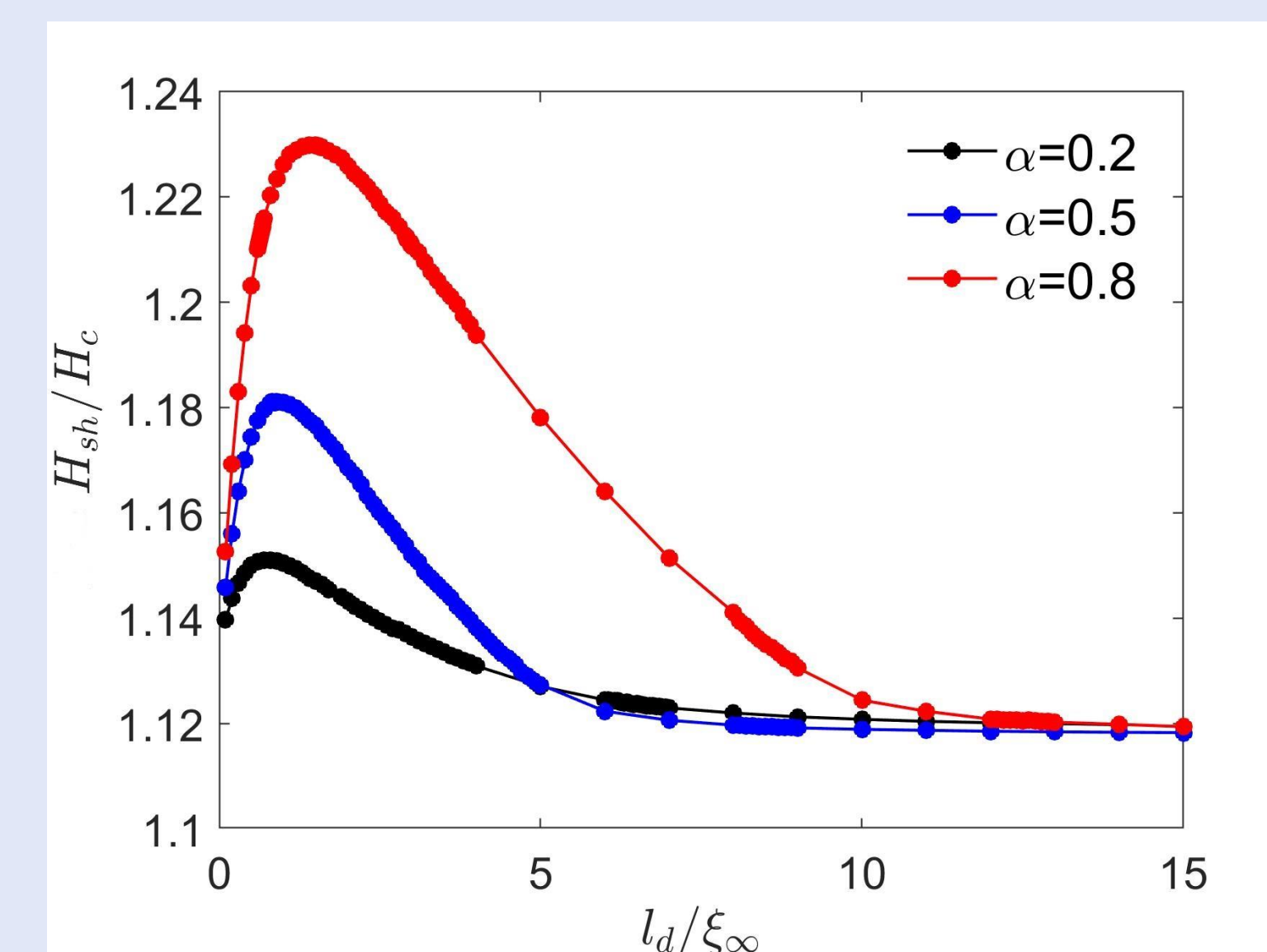
- The parameter α models the reduction of $\xi(0,y,z)$ and enhancement of $\lambda(0,y,z)$
- l_d is a thickness of the diffusion layer

$$S_\gamma = \xi^2(x)/\xi_\infty^2 = \lambda_\infty^2/\lambda^2(x) = 1 - \alpha \exp(-x/l_d)$$

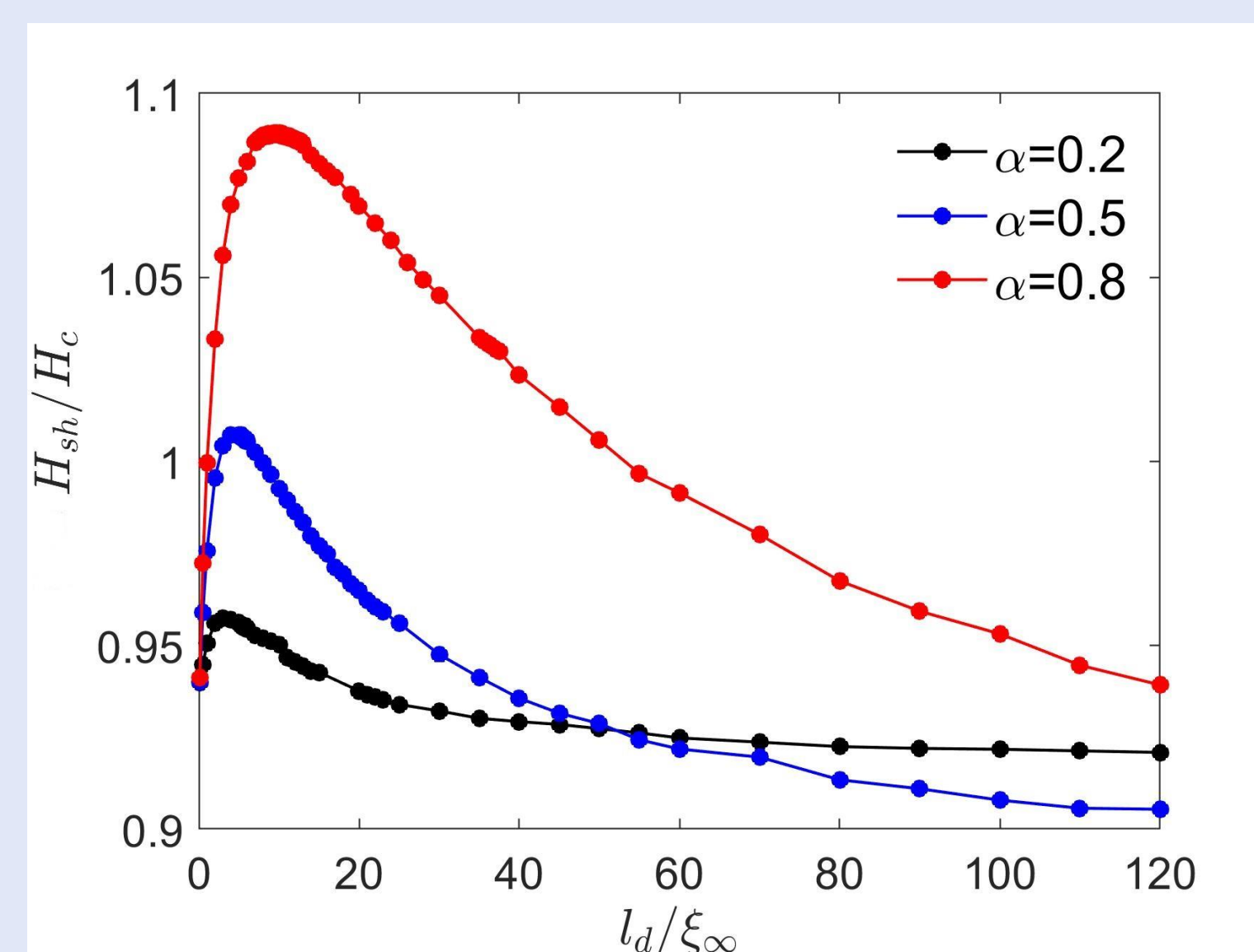
GL equations:

$$\dot{f} = f - f^3 + \nabla \cdot (S_\gamma \nabla f) - \frac{\kappa^2}{S_\gamma f^3} [(\partial_x h)^2 + (\partial_y h)^2]$$

$$\nabla \cdot \left(\frac{\nabla h}{f^2} \right) = \frac{S_\gamma}{\kappa^2} h + \frac{1}{S_\gamma f^2} (\partial_x S_\gamma \cdot \partial_x h)$$

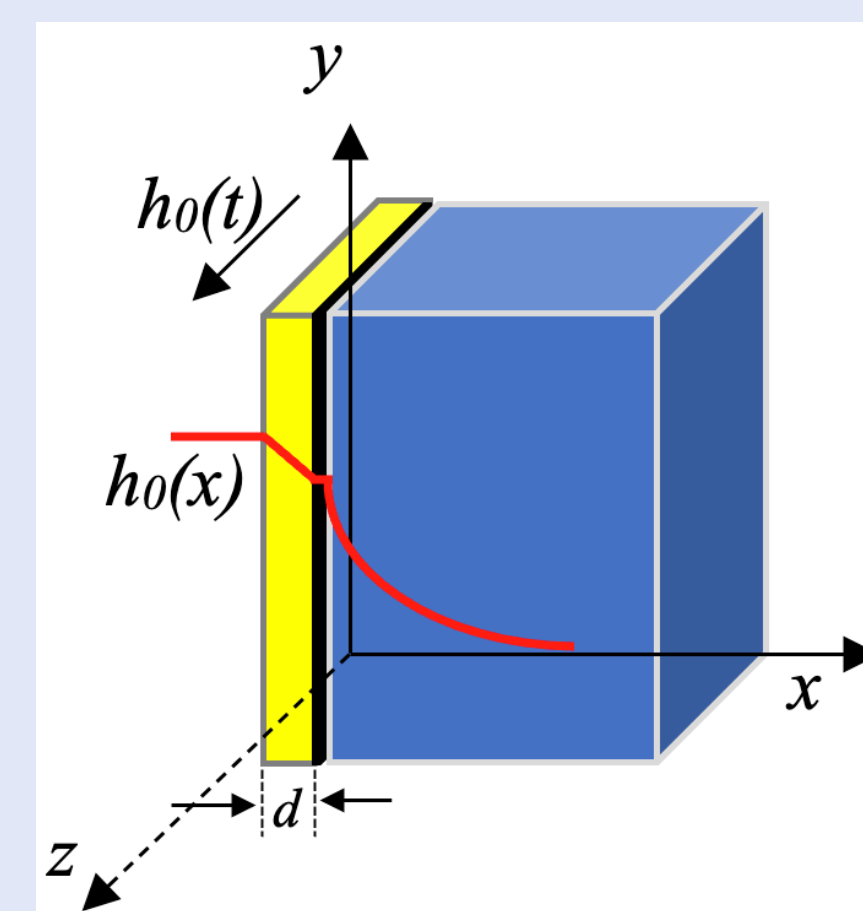


H_{sh} vs l_d for different α at $\kappa=2$.



H_{sh} vs l_d for different α at $\kappa=10$.

S-I-S STRUCTURES



Geometries:

- Nb₃Sn-I-Nb₃Sn
- Dirty Nb₃Sn-I-Nb₃Sn
- Nb₃Sn-I-Nb

Assumption:

The I-layer is thick enough to fully suppress the Josephson coupling between the S-overlayer and the bulk.

GL equations (Film):

Same materials present both sides

$$\dot{f}_1 = f_1 - f_1^3 + \frac{\xi_2^2}{\xi_1^2} \nabla^2 f_1 - \frac{\xi_2^2 \kappa_1^2}{\xi_1^2 f_1^3} [(\partial_x h_1)^2 + (\partial_y h_1)^2]$$

$$\nabla \cdot \left(\frac{\nabla h_1}{f_1^2} \right) = \frac{h_1 \xi_2^2}{\xi_1^2 \kappa_1^2}$$

GL equations (Film):

Different materials present both sides

$$\dot{f}_1 = \zeta f_1 - f_1^3 + s \nabla^2 f_1 - \frac{\tilde{\kappa}^2}{f_1^3} [(\partial_x h_1)^2 + (\partial_y h_1)^2]$$

$$\nabla \cdot \left(\frac{\nabla h_1}{f_1^2} \right) = \frac{\lambda_1^2 h_1}{\lambda_2^2 \zeta \kappa_2^2}$$

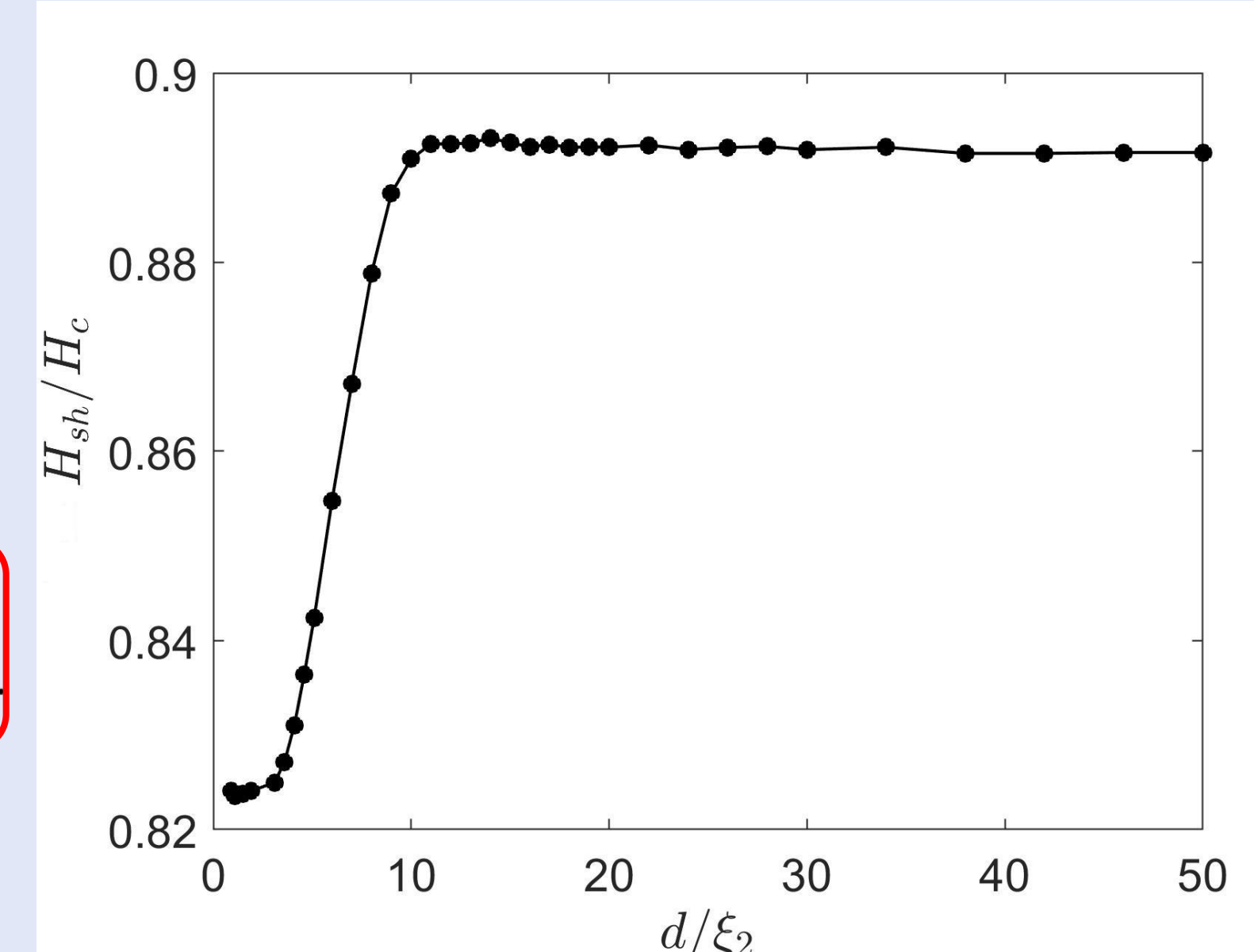
$$\zeta = \frac{1-T/T_{c1}}{1-T/T_{c2}}, \quad s = \frac{\xi_1^2}{\xi_2^2} \zeta, \quad \tilde{\kappa}^2 = \frac{\xi_1^2 \lambda_1^4}{\xi_2^2 \lambda_2^4} \kappa_2^2 \zeta^3$$

* The subscript 1 and 2 represent film and bulk parameters, respectively

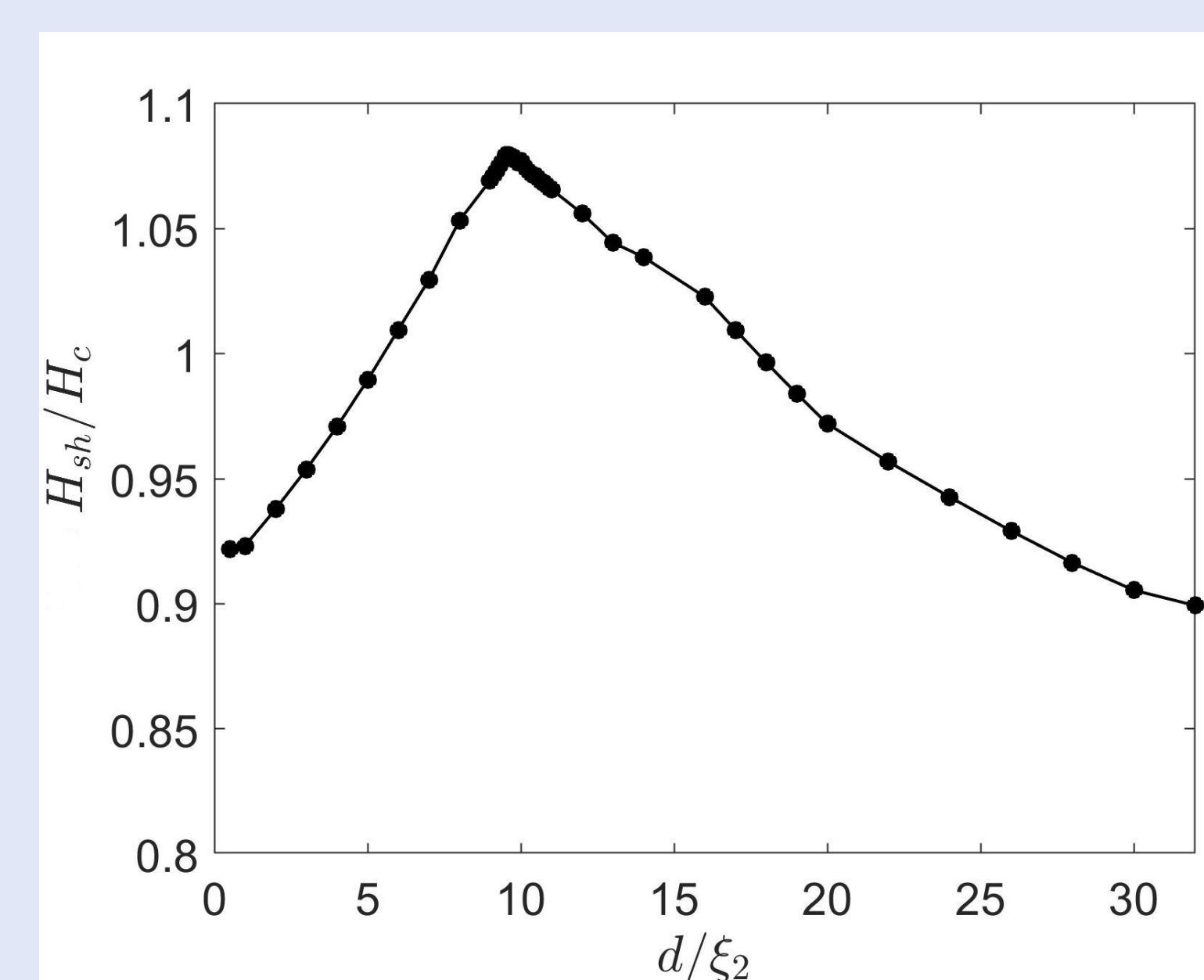
Additional boundary conditions:

$$h(d+0, y) = h(d-0, y)$$

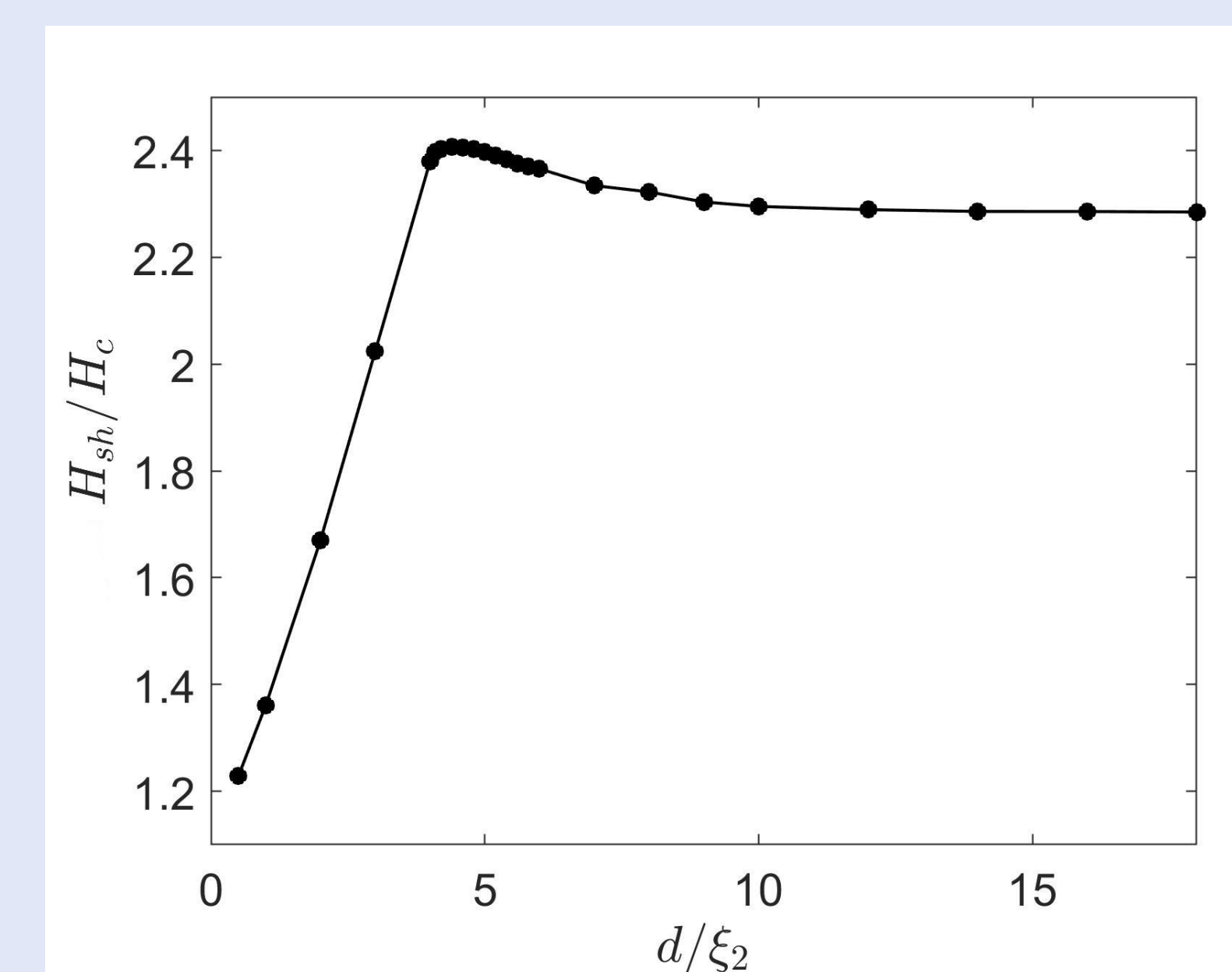
zero current $\partial_y h(d+0) = \partial_y h(d-0) = 0$ through the I layer.



H_{sh} vs. d for the case of Nb₃Sn-I-Nb₃Sn



H_{sh} vs. d for the case of dirty Nb₃Sn-I-Nb₃Sn



H_{sh} vs. d for the case of Nb₃Sn-I-Nb

SUMMARY

- The Ginzburg-Landau theory was employed to analyze the effect of impurity profiles and high T_c superconducting layers on the dc superheating field in S-S and S-I-S structures.
- Numerical simulations covered the entire range of $1 < \kappa < \infty$ and took into account the instability $H=H_{sh}$ at a finite wave number k_c , which is particularly relevant for Nb with $\kappa \sim 1$.
- Optimizing the diffusion length can enhance H_{sh} by 5-20% at $\kappa=10$ and 2-9% at $\kappa=2$.
- A dirty overlayer of Nb₃Sn deposited on a clean Nb₃Sn, can boost the superheating field by about 10%.
- A S-I-S structure consisting of a Nb₃Sn overlayer on Nb can increase the superheating field by approximately 2.2 times as compared to the bulk Nb.

[1] M. Tinkham, Introduction to superconductivity. (Dover Publ., Mineola, New York, 2004).
 [2] A. Gurevich, (2006). Enhancement of rf breakdown field of superconductors by multilayer coating. Appl. Phys. Lett, 88(1), 012511.
 [3] A. Gurevich, (2015). Maximum screening fields of superconducting multilayer structures. AIP Advances, 5(1), 017112.
 [4] D. B. Liarte, S. Posen, M. K. Transtrum, G. Catelani, M. Liepe, and J. P. Sethna, (2017). Theoretical estimates of maximum fields in superconducting resonant radio frequency cavities: stability theory, disorder, and laminates. Supercond. Sci. Technol, 30, 033002.