Equidistant optimization of elliptical SRF standing wave cavities

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Abstract

A record accelerating rate was achieved earlier in standing wave (SW) SRF cavities when their shape was optimized for lower peak surface magnetic field sacrificing the peak surface electric field. **In view of new materials with higher limiting magnetic fields, expected for SRF cavities, in the first line the Nb_3Sn, the approach to optimization of cavity shape should be revised.** A method of equidistant optimization, offered earlier for traveling wave (TW) cavities is applied to SW cavities. It is shown here that without limitation by magnetic field, the maximal accelerating rate is defined not only by limitations of the electric field but to a significant degree by the cavity shape. For example, for a cavity with the aperture radius R_a = 35 mm the minimal ratio of the peak surface electric field to the accelerating rate is about E_{pk}/E_{acc} = 1.54. So, **with the maximal surface field experimentally achieved E_{pk} = 125 MV/m, the maximal achievable accelerating rate is about 80 MeV/m even if there are no restrictions by magnetic field.** Optimized cavity shapes with and without limitations by magnetic field are presented. Another opportunity – optimization for a low magnetic field, is opening for the same material, Nb_3Sn, with the purpose to have a high quality factor and increased accelerating rate that can be used for industrial linacs.

Geometry

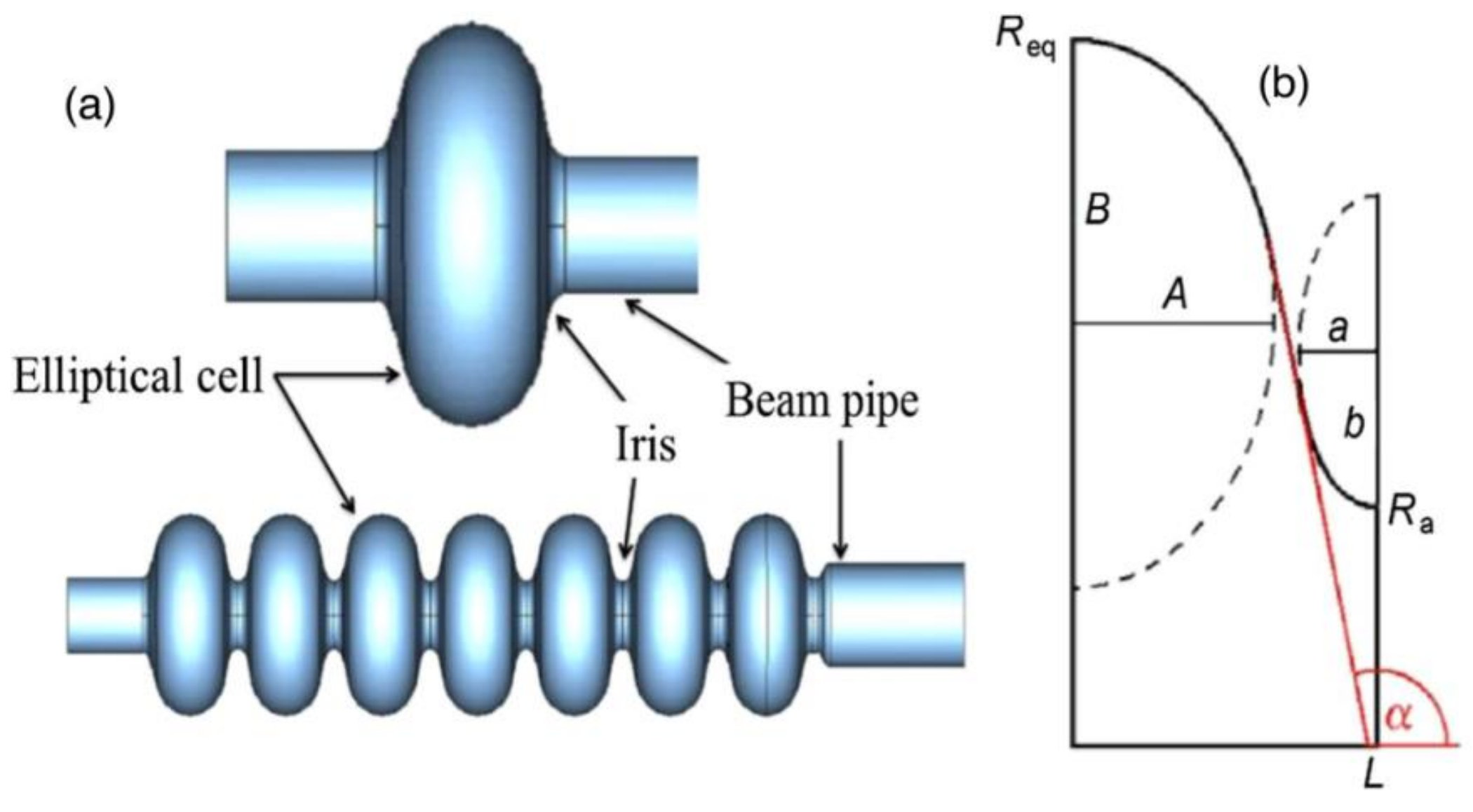


Fig. 1. (a) Single-cell and multicell elliptical cavities; (b) geometry of the half-cell.

Contemporary superconducting rf cavities for high energy particle accelerators consist of a row of cells coupled together. The contour of a half-cell consists of two elliptic arcs and a straight segment tangential to both.

Equidistant approach for optimization

Optimization of an elliptical cavity is usually done as a search for minimum B_{pk}/E_{acc} when the value of E_{pk}/E_{acc} is given. It is also possible to minimize E_{pk}/E_{acc} for a given B_{pk}/E_{acc} but **the truth is that we need to reach as high as possible accelerating gradient E_{acc} before field emission or magnetic quench limit further increase of the accelerating gradient.** So, the ideal situation would be to reach both limits simultaneously using all the possibilities to increase E_{acc}. If we know the maximal achievable surface peak fields E^*_{pk} and B^*_{pk}, then the cavity having equal values of E_{pk}/E^*_{pk} and B_{pk}/B^*_{pk} will be at equal distances from either limit. Then the criterion of the shape optimization can be written as the minimum of the maximum of two values: E_{pk}/E^*_{pk} and B_{pk}/B^*_{pk}, or, shortly,

$$\mathbf{min\ max}\ (E_{pk}/E^*_{pk},\ B_{pk}/B^*_{pk}).$$

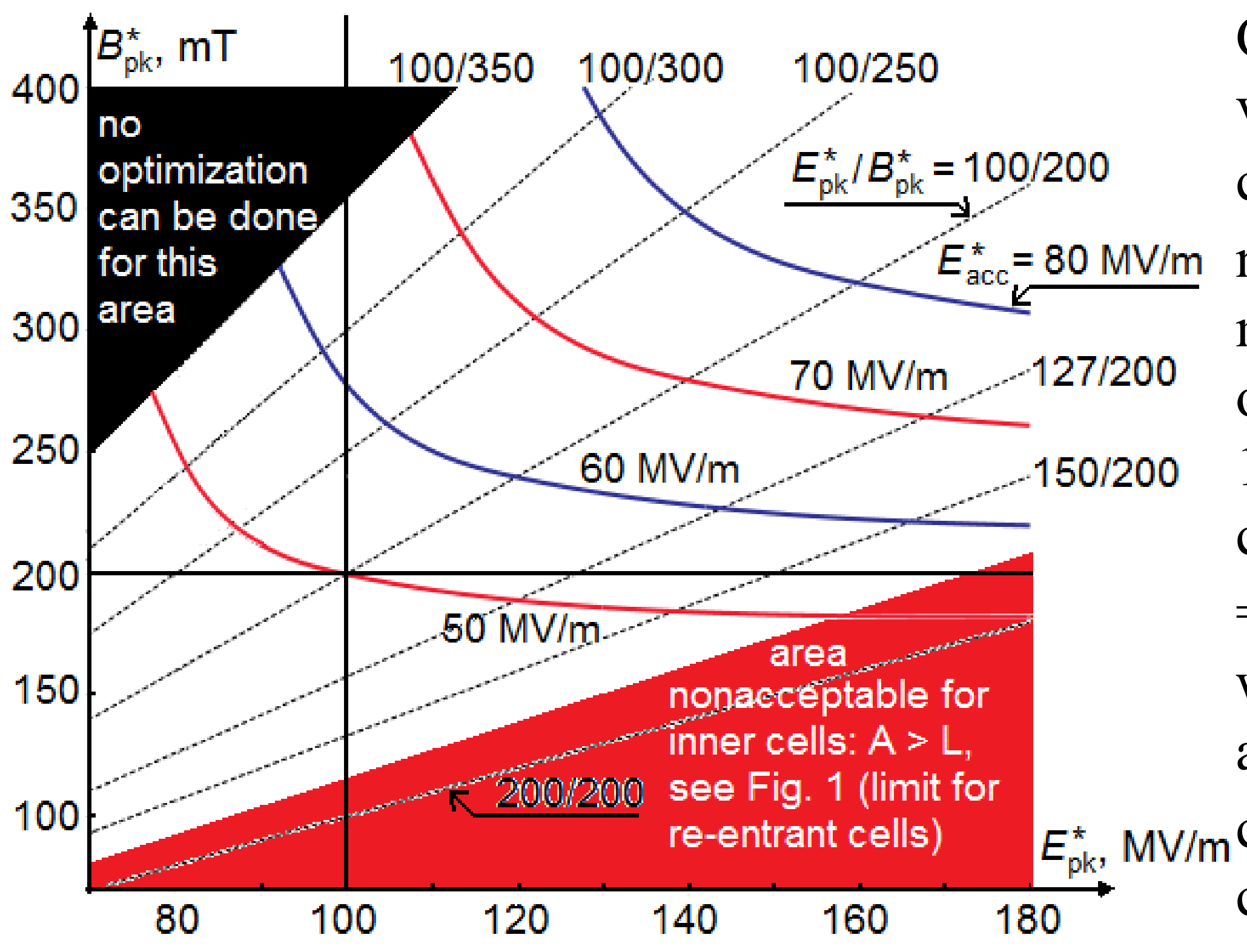
We named this approach *the equidistant optimization*.

The definition given above for the equidistant optimization can be rewritten in an equivalent form more convenient for calculations:

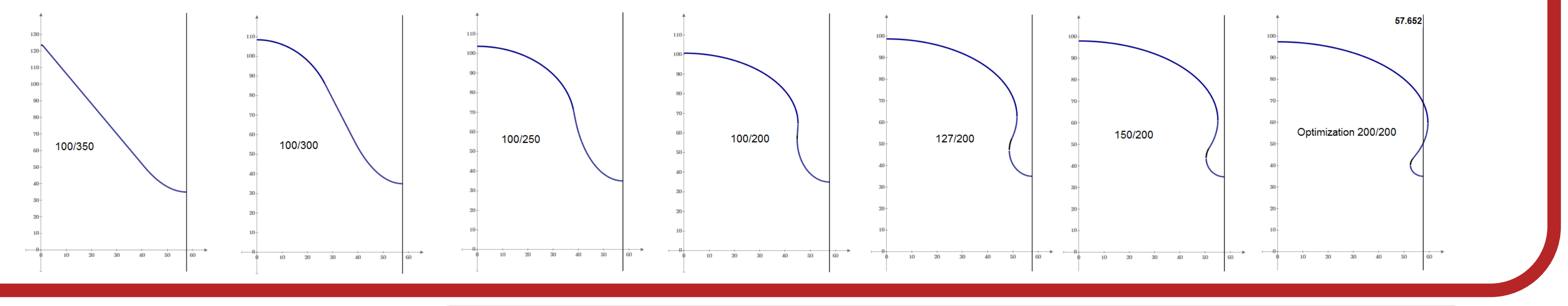
Goal = min E_{pk} if $E_{pk}/B_{pk} > E^*_{pk}/B^*_{pk}$ or Goal = min B_{pk} if $E_{pk}/B_{pk} < E^*_{pk}/B^*_{pk}$,

where the Goal is a combination of the geometrical parameters *A, B, a*, and *b*, giving the desired minimum.

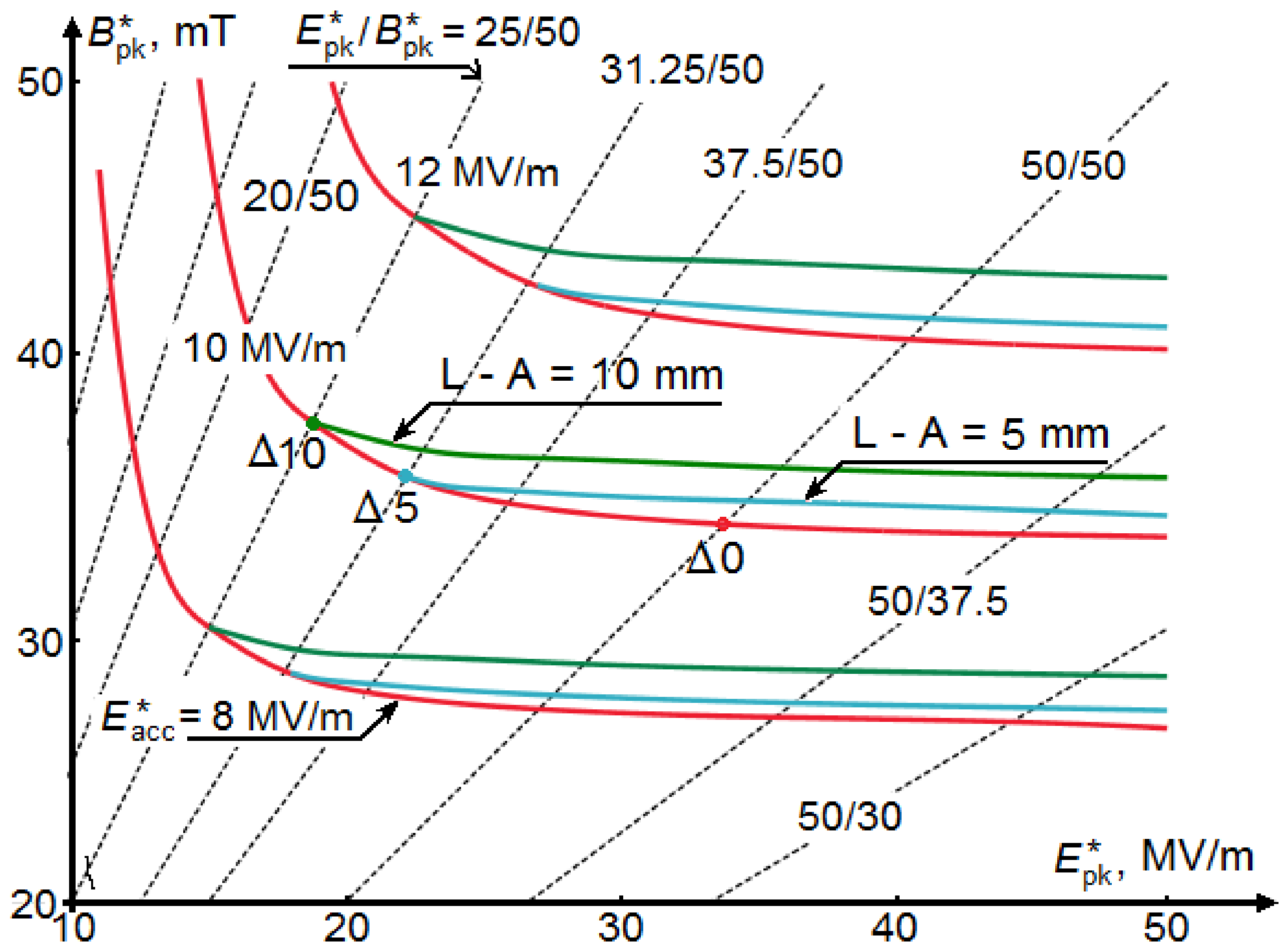
Results of optimization for inner cells with Ra = 35 mm



Calculations show that the minimal value of E_{pk}/E_{acc} is 1.536 that corresponds to E^*_{pk}/B^*_{pk} = 100/350. So, no benefit from increasing the limiting magnetic field above 350 mT can be obtained if the limiting electric field is 100 MV/m. The maximum can be counted on is about 65 MV/m. For E^*_{pk} = 125 MV/m that can be achieved now with very thorough surface preparation and with B^*_{pk} = 400 mT that hopefully can be obtained with a new material, we can recon not more than on 80 MV/m.



Low magnetic field cavities for industrial linacs, Ra = 30 mm



This Figure shows that the acceleration rate of 10 MV/m can be achieved at B_{pk} = 35 mT. This is about 15 % less than in the case of the TESLA cavity shape, or 30 % less in losses and makes cryocooling more practical.

Now an industrial linac is under consideration, which is based on Nb_3Sn-coated ILC-type 1.3 GHz acceleration cavity. High Q_0 at 4.4 K allows conduction cooling and cryocooler instead of He bath and refrigerator, which is extremely attractive for linacs operating in industrial environment. However, a cryocooler may remove ~2.4 W, and it is not reasonable to increase the gradient beyond ~8 MeV/m using 2-3 cryocoolers, because of Q_0 drop. Further increase of the gradient is not reasonable, the loss, say at 10 MeV/m reaches 12 W/m and the number of cryocoolers is impractical. The reason, however, is that ILC structure is optimized for HIGH GRADIENT, not HIGH Q_0: E_{pk}/E_{acc} = 2 and B_{pk}/E_{acc} = 4.26 mT/(MeV/m). For E_{acc} ~ 10 MeV/m one has a surface electric field of 20 MV/m, it is too low compared to the FE onset. On the other hand, B_{pk} is too high providing significant drop in Q_0. It is possible for a production version to reoptimize the linac completely changing balance between E_{pk} and B_{pk} to smaller values of B_{pk}.

Equidistant optimization of a single-cell cavity

A single-cell cavity with dimensions of an inner cell of a multicell cavity will have values E_{pk}/E_{acc} and B_{pk}/E_{acc} different from those of the inner cell because of different boundary conditions. Now, the length of the cell L becomes an independent geometric parameter for optimization along with *A, B, a,* and *b*.

In the definition of E_{acc} only ΔU, energy gain in volts is important. So, we should normalize this value of ΔU on the same value L_0 for any geometric parameters: *A, B, a*, and *b*, including the distance between the ends of the smaller ellipse $2L$, and find the maximal $E_{acc} = \Delta U/L_0$ giving maximal ΔU.

A particle moving close to the speed of light, will be accelerated only on a length equal to $\lambda/2$, even if it enters the cavity at a non-optimal phase. This is another reason why E_{acc} should be defined as $E_{acc} = \Delta U/(\lambda/2) = \Delta U/(2L_0)$.

Table 2. Results of equidistant optimization “100/200” for the inner cell of a multicell cavity and for the single-cell cavity. Units for B_{pk}/E_{acc} are mT/(MV/m).

	A, mm	B, mm	a, mm	b, mm	L, mm	E_{pk}/E_{acc}	B_{pk}/E_{acc}	E_{acc}, MV/m
Inner cell	46.2	40.6	11.8	21.8	57.652	1.88	3.76	53.1
Single cell	25.57	30.17	6.74	18.86	30	1.59	3.18	62.9

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