



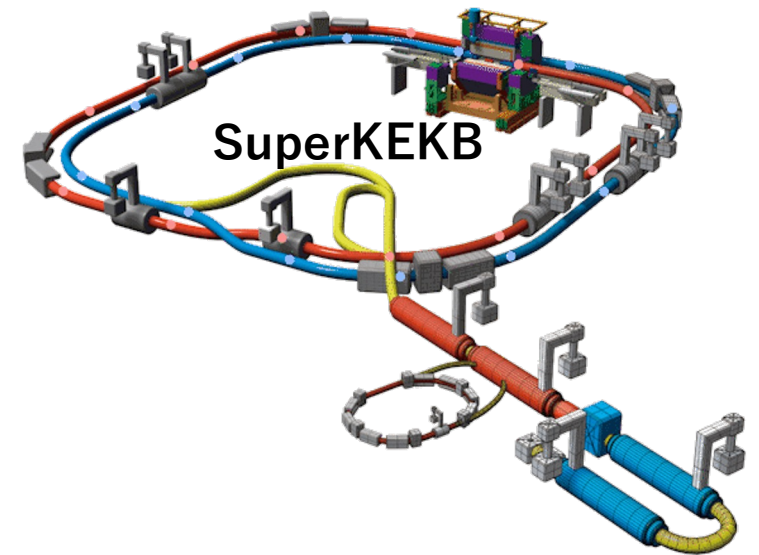
Cavity Beam Interaction /HOMs and Dampers

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High-current electron-positron ring collider

High Beam Current-related issues in RF system

In RF system of high beam current electron (positron) ring accelerator such as SuperKEKB, there are many challenging issues to realize the stable and high-performance beam operation.

- Large Beam Power Handling (Optimization for beam loading)
 - Optimum tuning, Optimum coupling
 - Phase difference among RF stations to share the beam power
- Instabilities due to accelerating mode
 - Coupled Bunch Instability (CBI) related to $\mu = -1, -2$ and -3 modes
 - New CBI damper system
 - Zero-mode related to Robinson stability
 - Direct RF feedback (DRFB)
 - Zero-mode damper (ZMD)
- Coupled Bunch Instability (CBI) due to Higher-Order modes (HOMs)
 - ARES and SCC are designed as HOM-damped structure with HOM absorbers.
 - Additionally, a bunch-by-bunch feedback system is effective.

Contents

- ◆ Optimization for Beam Loading
 - Impedance Characteristics of Cavity (Equivalent Circuit Model)
 - Frequency Spectrum of Beam
 - Optimum Tuning
 - Optimum Coupling

- ◆ Instabilities due to Accelerating Mode
 - Coupled Bunch Instability (CBI) related to $m=-1$, -2 and -3 modes
 - Static Robinson Instability (zero-mode)

- ◆ HOM
 - HOM damping in KEKB SCC
 - Large HOM power in KEKB and SuperKEKB

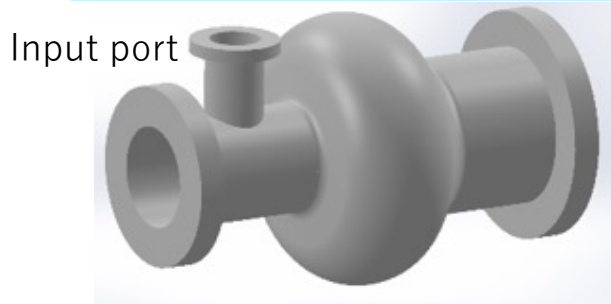
Definitions of Basic Terms in this Lecture

electron/positron storage ring

ex. of SuperKEKB



Superconducting Cavity



RF frequency f_{rf}

~ 508.9 MHz

Cavity resonant frequency f_0

Revolution frequency f_{rev}

~ 100 kHz

Revolution time T_0

~ 10 μ s

Harmonic number h

5120

Beam current (average) I_b

2.6 A/3.6 A

Particle speed v

$$\beta = \frac{v}{c} \approx 1$$

Circumference C

~ 3 km

Bunch charge q

e-/e+ 10/14 nC

Number of Bunch N_b

2500

Bunch space T_b

~ 4 ns

Frequency f

$$\omega = 2\pi f$$

Angular frequency ω

Wavelength λ

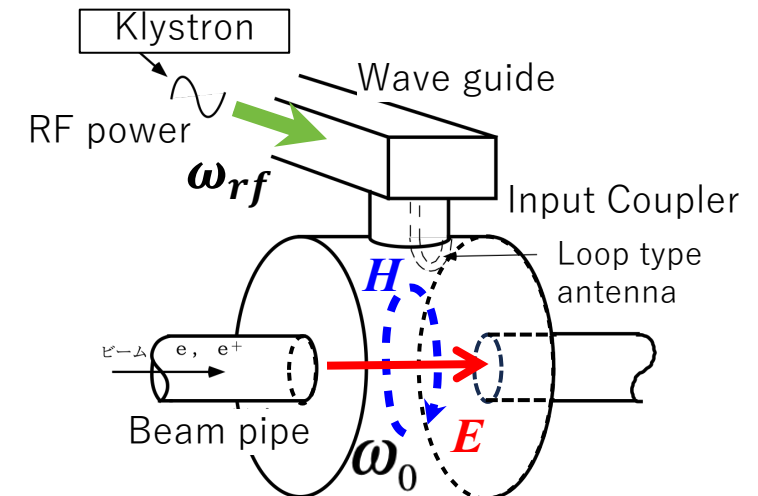
$$f = \frac{v(\approx c)}{\lambda}$$

Speed of light c

Imaginary number j

$$f_{rev} = v/C = 1/T_0$$

$$f_{rf} = h f_{rev} \quad I_b = f_{rev} q N_b$$



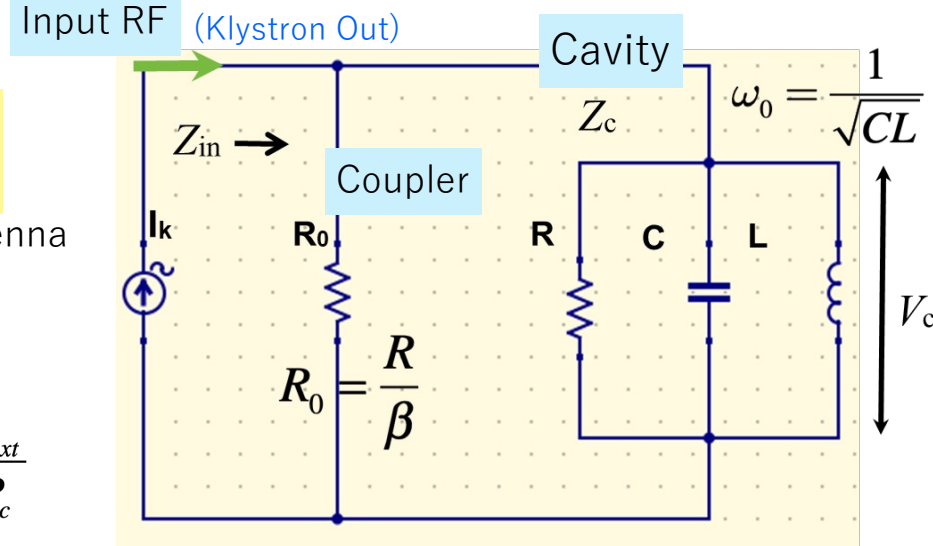
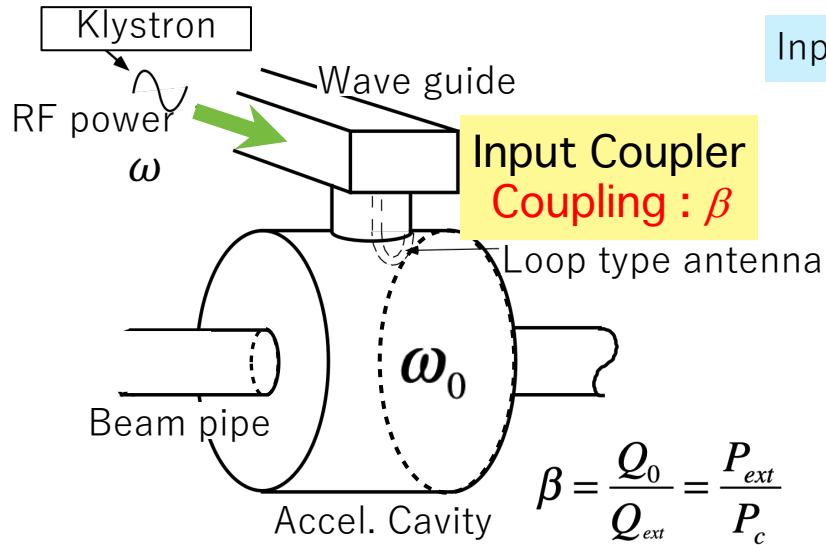
TM010 mode

Contents

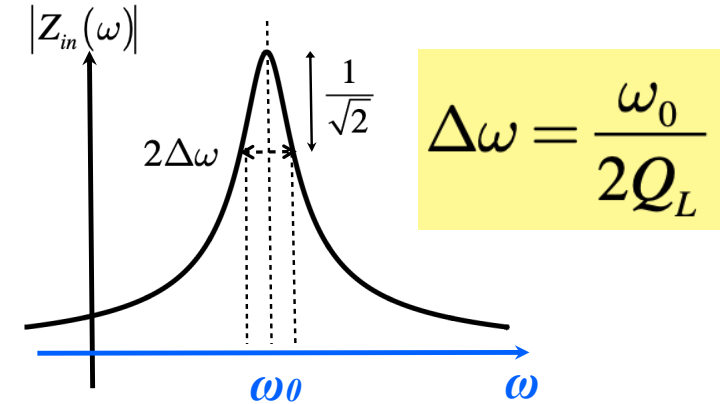
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Equivalent Circuit Model of Cavity

Frequency Dependence of Cavity Response (Impedance)



Q-value corresponds to peak width of impedance.

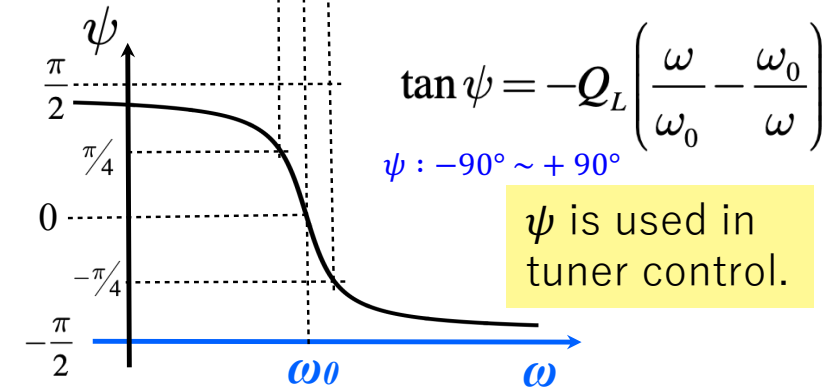


Input Impedance

$$Z_{in}(\omega) = V_c(\omega) / I_k(\omega)$$

$$Z_{in}(\omega) = \frac{R/Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

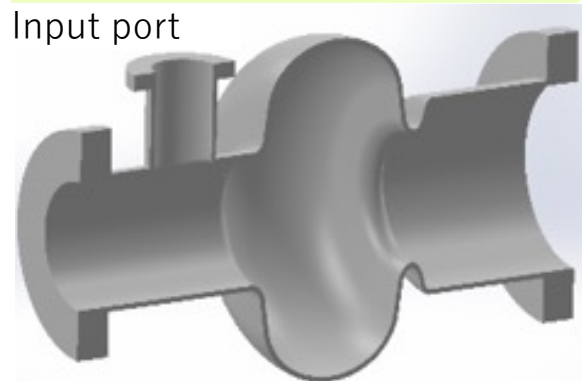
$$R = \frac{R_{sh}}{2} \quad Q_0 = \omega_0 RC$$



ψ is used in tuner control.

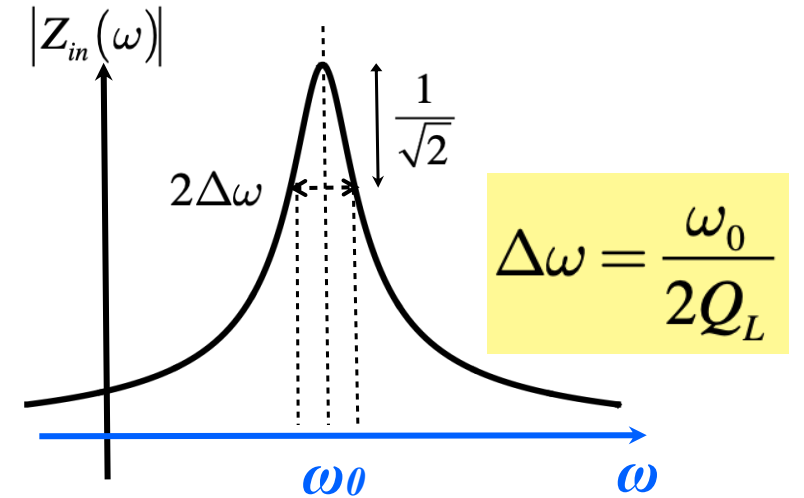
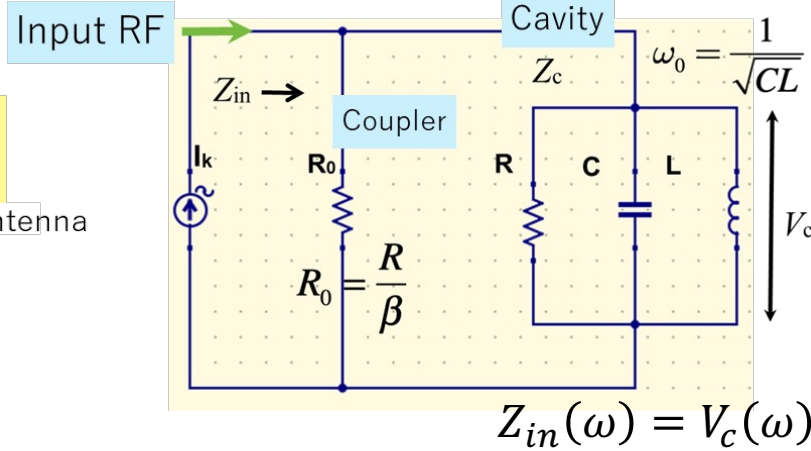
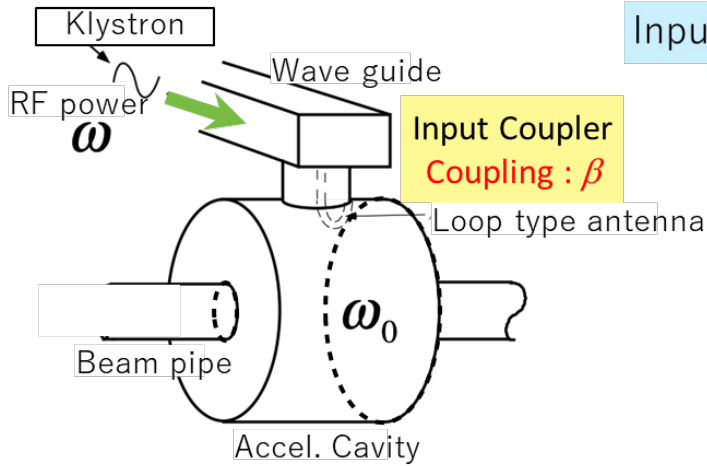
ψ : Phase difference between input RF and excited voltage ($\omega \neq \omega_0$)

Superconducting Cavity



Equivalent Circuit Model of Cavity

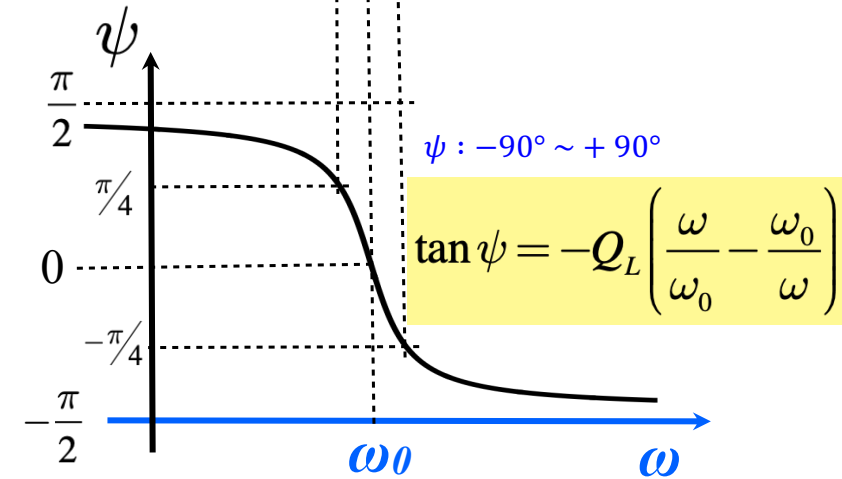
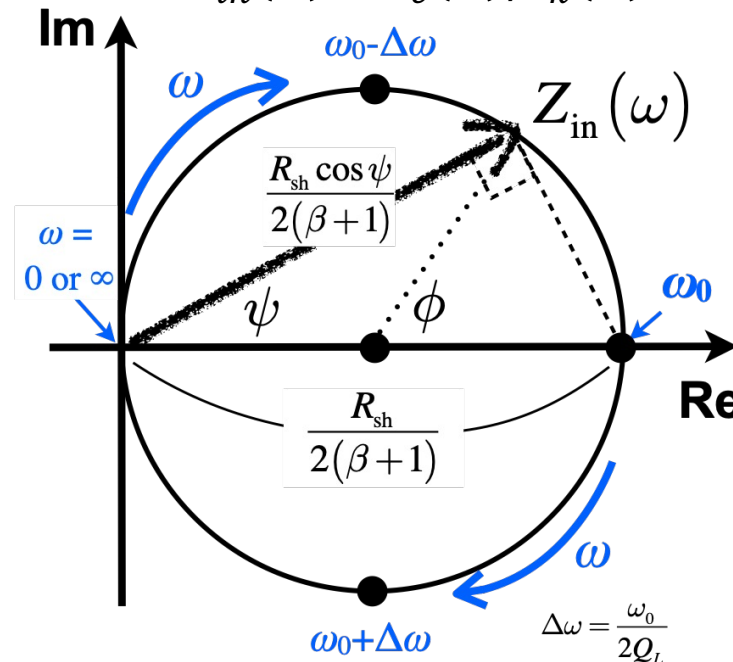
Frequency Dependence of Cavity Response (Impedance)



$$Z_{in}(\omega) = \frac{R/Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = \frac{R/(\beta+1)}{1 + jQ_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

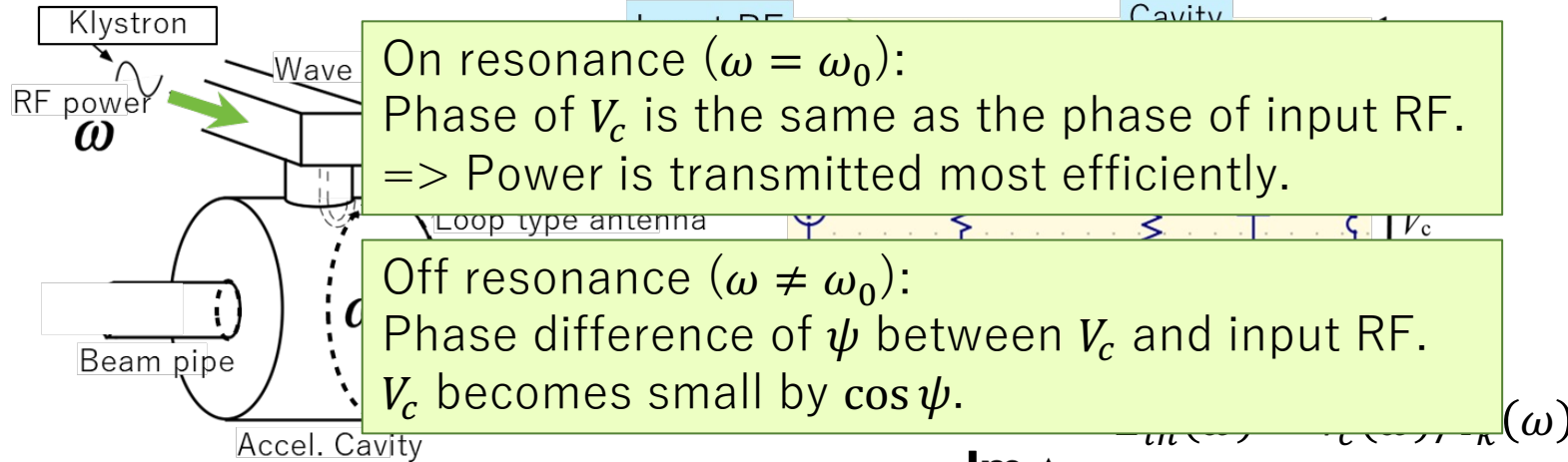
$$= \frac{R_{sh} \cos \psi}{2(\beta+1)} \cdot e^{j\psi} = \frac{R_{sh}}{4(\beta+1)} \cdot (1 + e^{j\phi})$$

$$\phi = 2\psi \quad R = \frac{R_{sh}}{2} \quad Q_0 = \omega_0 RC$$



Equivalent Circuit Model of Cavity

Frequency Dependence of Cavity Response (Impedance)



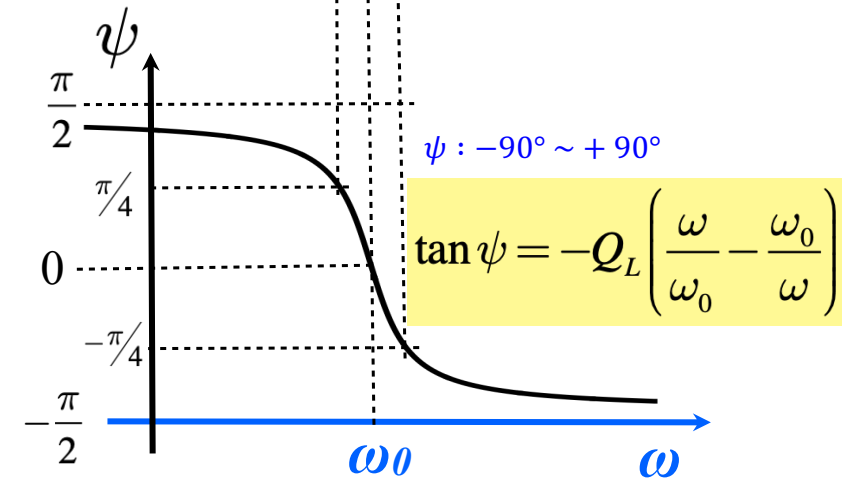
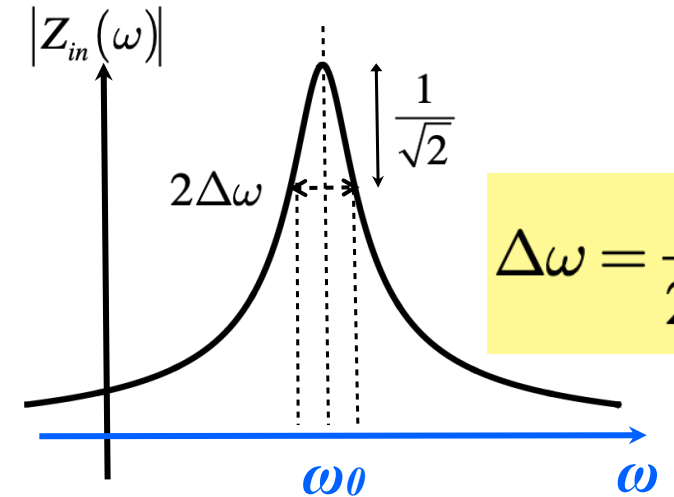
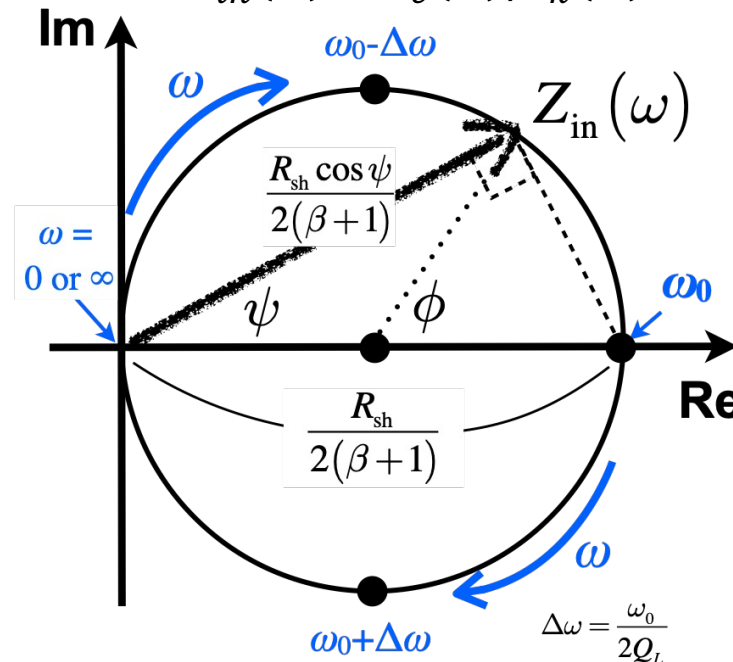
On resonance ($\omega = \omega_0$):
Phase of V_c is the same as the phase of input RF.
=> Power is transmitted most efficiently.

Off resonance ($\omega \neq \omega_0$):
Phase difference of ψ between V_c and input RF.
 V_c becomes small by $\cos \psi$.

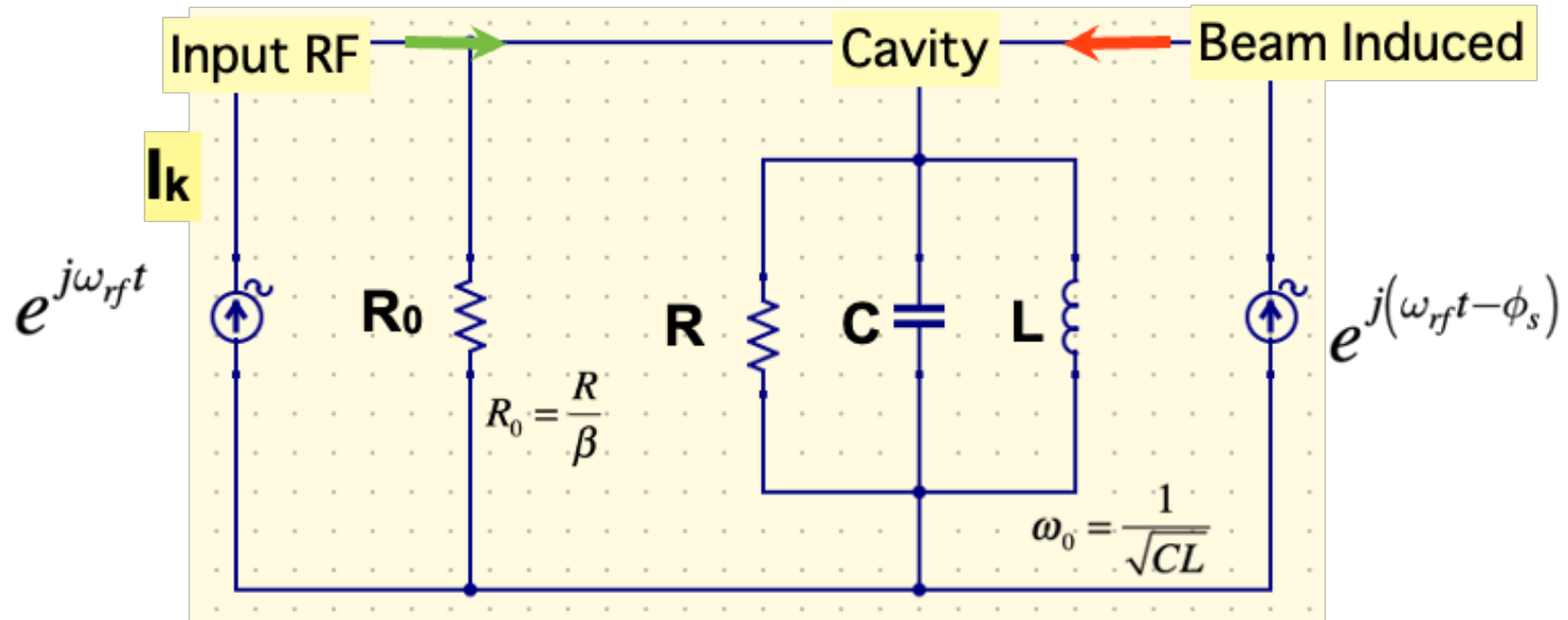
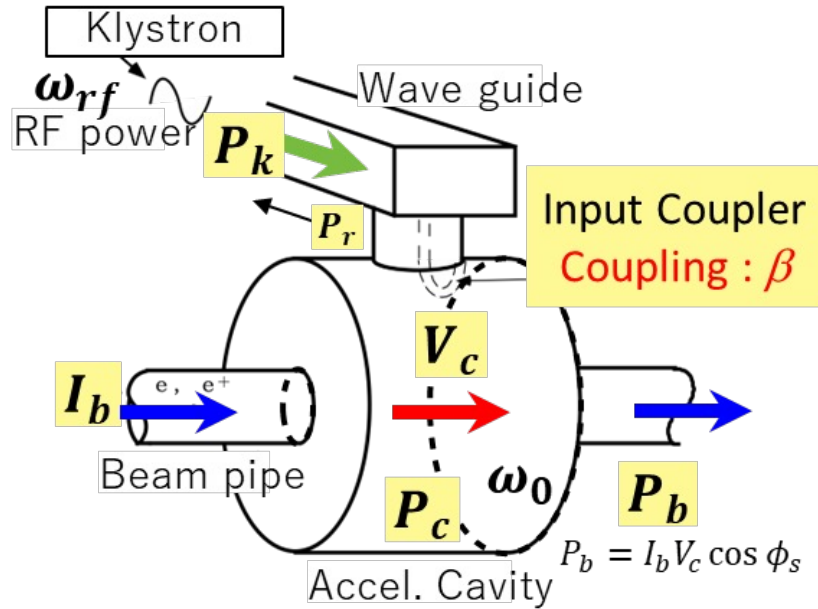
$$Z_{in}(\omega) = \frac{R/Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = \frac{R/(\beta+1)}{1 + jQ_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$= \frac{R_{sh} \cos \psi}{2(\beta+1)} \cdot e^{j\psi} = \frac{R_{sh}}{4(\beta+1)} \cdot (1 + e^{j\phi})$$

$$\phi = 2\psi \quad R = \frac{R_{sh}}{2} \quad Q_0 = \omega_0 RC$$



Consideration of Beam



Beam Loading

Additional Load to be compensated by the Input RF

$$Z_{in}(\omega) = \frac{R/Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = \frac{R/(\beta+1)}{1 + jQ_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

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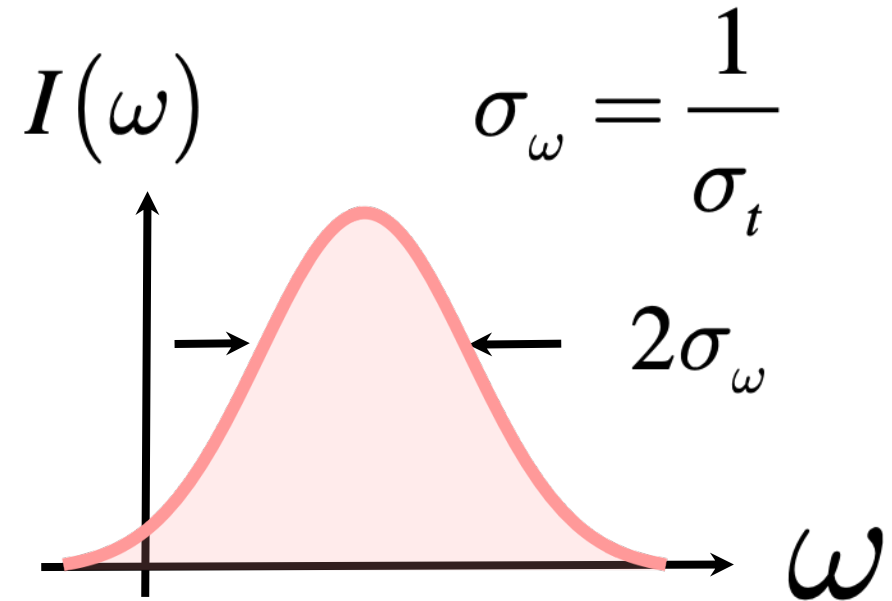
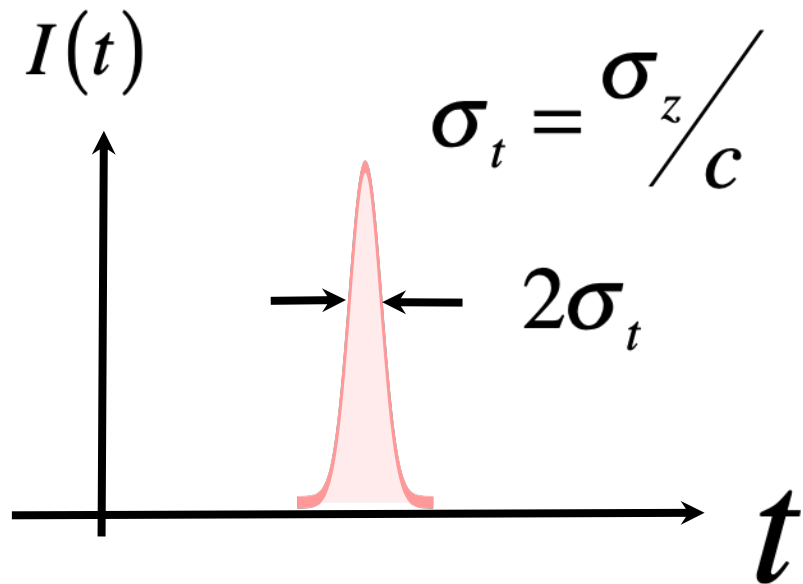
◆ HOM

- HOM damping in KEKB SCC
- Large HOM power in KEKB and SuperKEKB

Frequency Spectrum of Bunched Beam

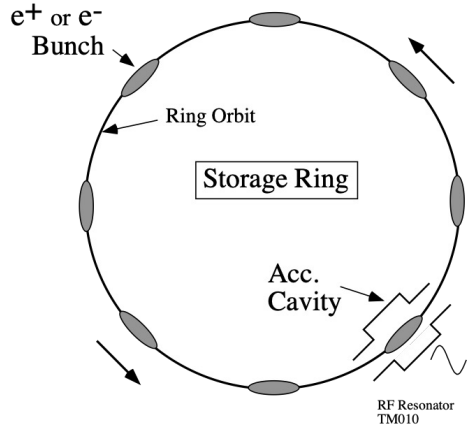
A gaussian bunch passing through the cavity only once

σ_z : bunch length



The width of the spectrum is depending on the bunch length.

Frequency Spectrum of Bunch Train



$$\omega_{rf} = m \frac{2\pi}{T_b}$$

I_b : Average Beam Current (DC component)

T_b : Interval time of bunch

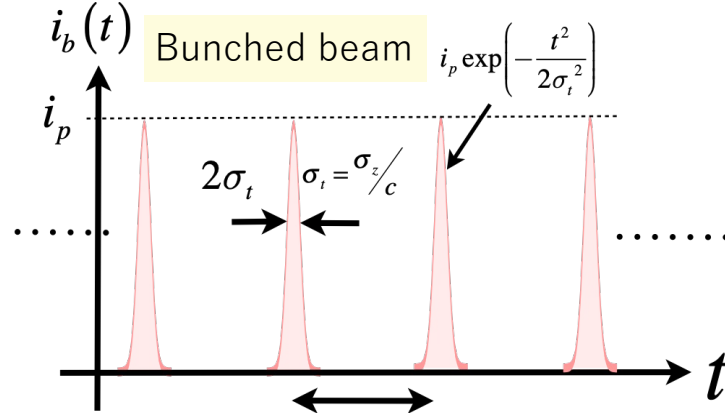
m : integer

When $m = 1$, $\frac{2\pi}{T_b} = \omega_{rf}$.

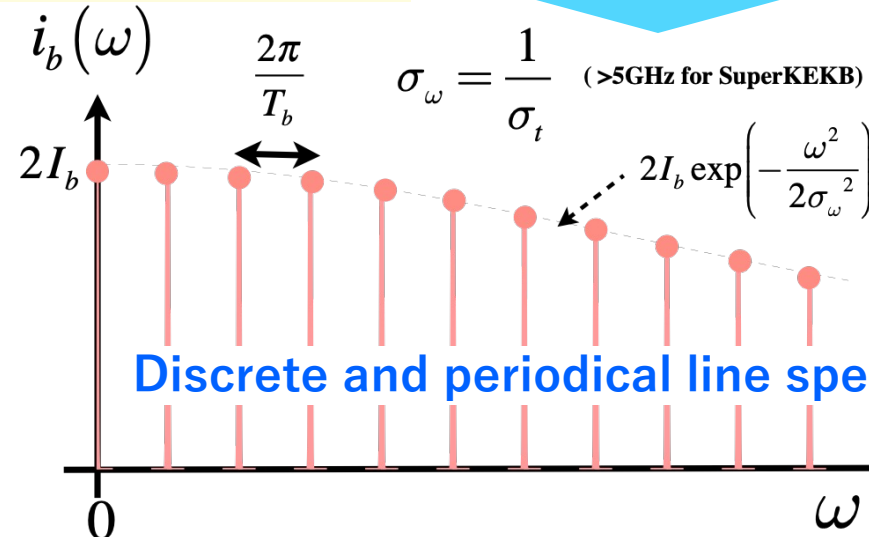
When all bucket filled by bunches, interval of line spectra becomes ω_{rf} .

Multi Gaussian bunches passing through multi turns

Bunch train



Frequency spectrum



Discrete and periodical line spectra

Ideal bunch train

- No difference between all bunches
- Gaussian shape
- Equal spaced bunches with interval of T_b

- Width of impedance of cavity is enough narrow. = high Q_L

➡ considering only near ω_{rf}

- $\sigma_\omega \gg \omega_{rf}$ ($\sigma_z \ll \lambda_{rf}$)

$$i_b(\omega_{rf}) \sim 2I_b \exp\left(-\frac{\omega_{rf}^2}{2\sigma_\omega^2}\right) \sim 2I_b$$

$$I(\omega_{rf}) \approx 2I_b$$

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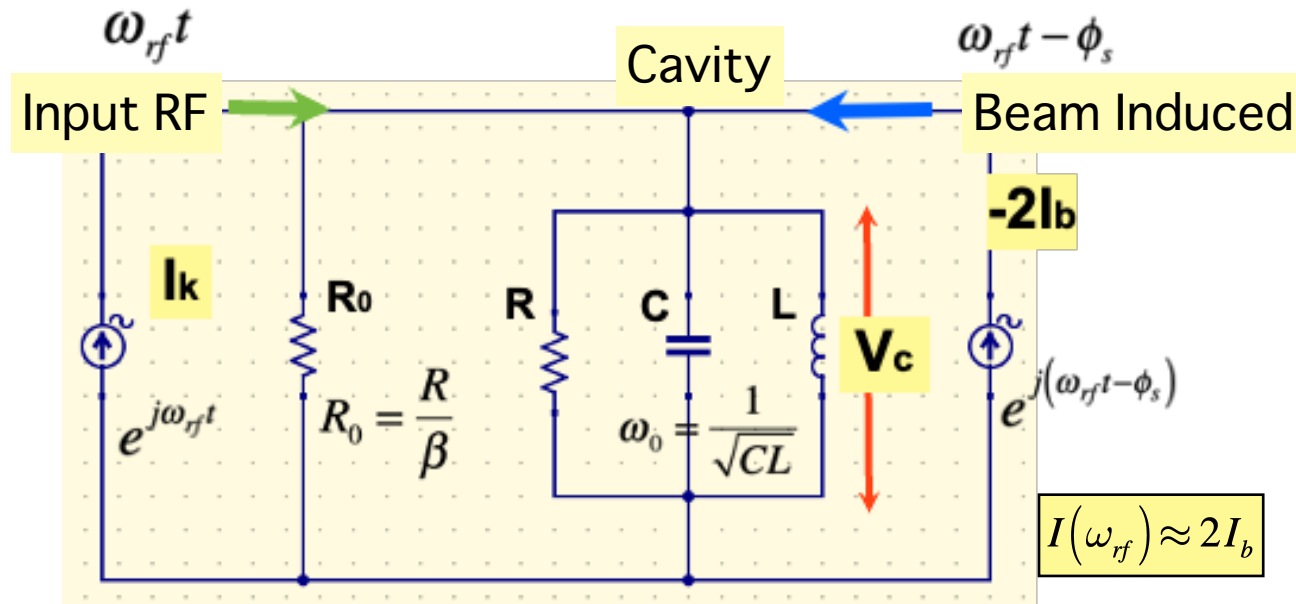
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Beam is accelerating at the synchronous phase.

Equivalent Circuit Model with Beam Acceleration



$$V_k(\omega) = Z_{in}(\omega) I_k(\omega)$$

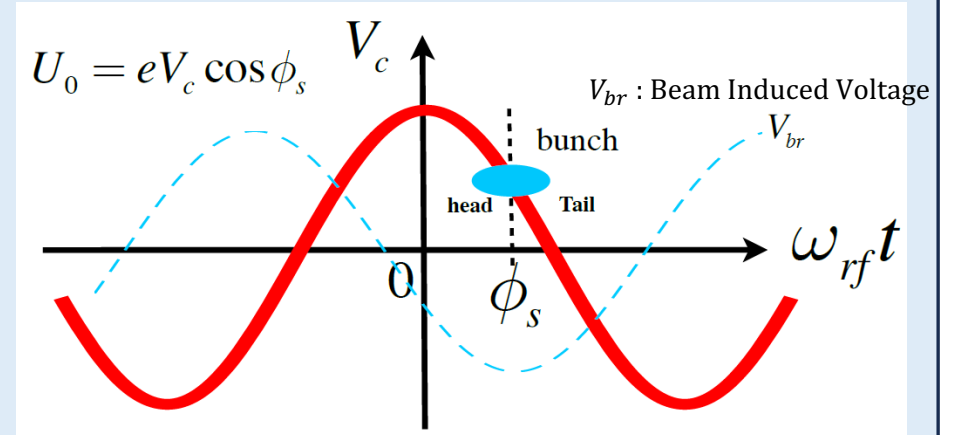
I_b : Average Current
(Decelerating direction)

$$V_b(\omega) = Z_{in}(\omega) I_b(\omega)$$

$$V_c = V_b + V_k \quad \text{in } \omega_{rf}$$

Cavity voltage \uparrow \uparrow \uparrow due to input power from Klystron
 Beam induced voltage

Principle of Phase Stability



Positive direction of V_c corresponds to beam acceleration.

In the electron storage ring, the beam is bunched at the synchronous phase ϕ_s to balance the radiation loss.

According to “**The Principle of Phase Stability**”, the restoring force works around the phase ϕ_s .
 Beam particles oscillate around ϕ_s .
 = Synchrotron oscillation

Cavity Voltage V_c in Beam Acceleration

$$\omega_0 = \omega_{rf}$$

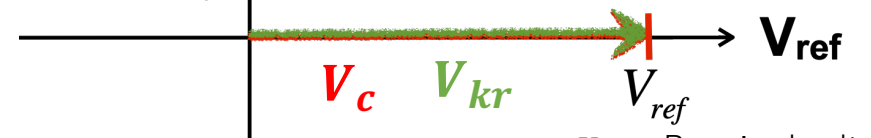
Without Beam

On - resonance

$$\omega_0 = \omega_{rf}$$

$$V_c = V_k = V_{kr} = \frac{R_{sh} I_k}{2(\beta + 1)} \equiv V_{ref}$$

$V_{kr} : V_k$ at $\omega_0 = \omega_{rf}$



$V_c \equiv V_{ref}$ without beam

With Beam

$$V_b = V_{br} = \frac{R_{sh} I_b}{\beta + 1}$$

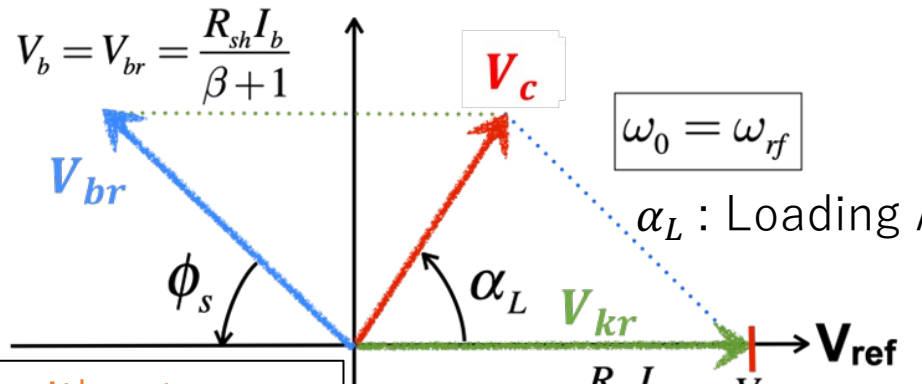
$$\omega_0 = \omega_{rf}$$

α_L : Loading Angle

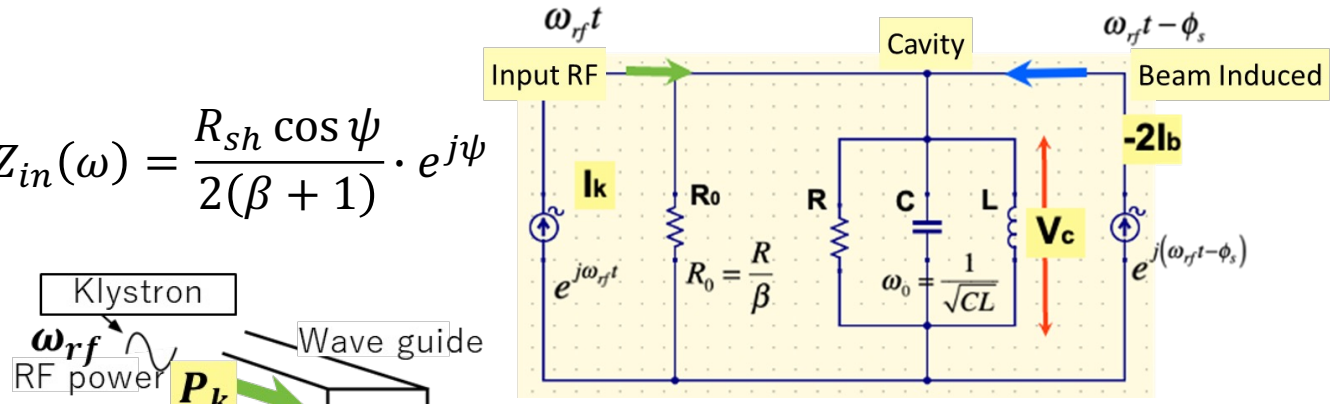
without I_k control (fixed)

$$V_k = V_{kr} = \frac{R_{sh} I_k}{2(\beta + 1)}$$

$V_{br} : V_b$ at $\omega_0 = \omega_{rf}$



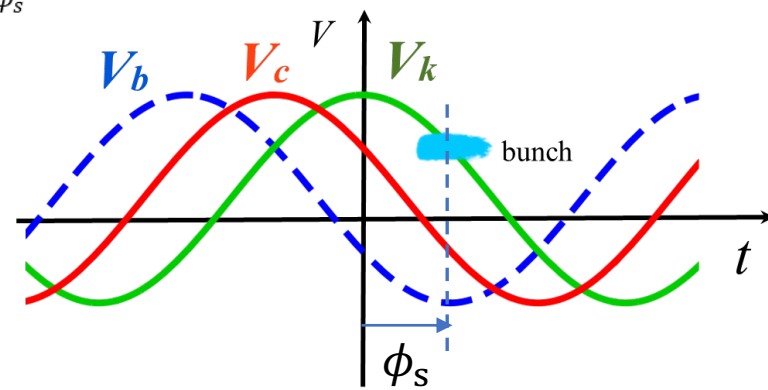
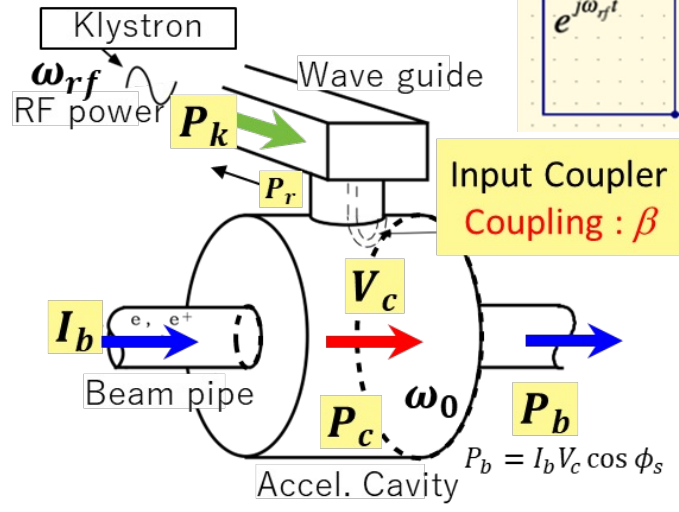
$$Z_{in}(\omega) = \frac{R_{sh} \cos \psi}{2(\beta + 1)} \cdot e^{j\psi}$$



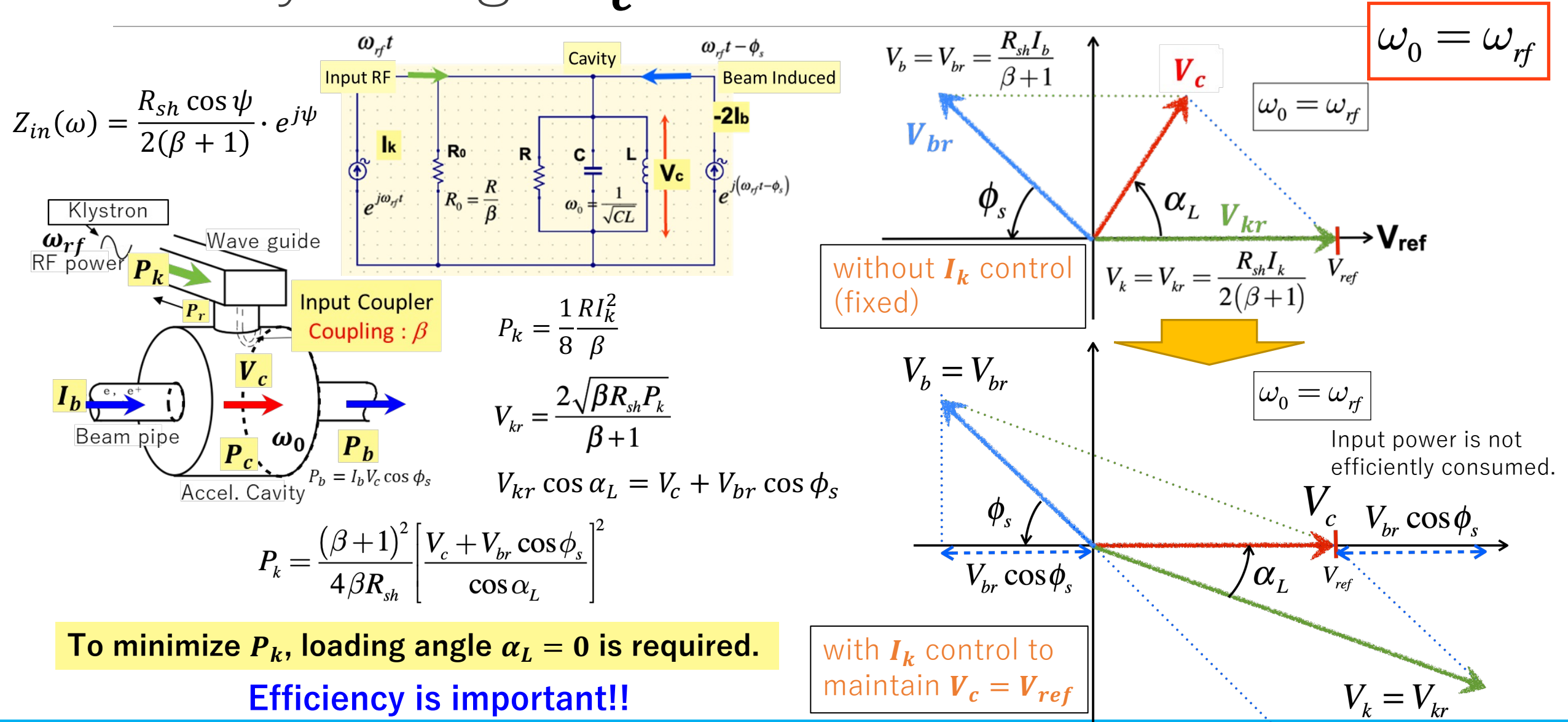
$$V_k(\omega) = Z_{in}(\omega) I_k(\omega)$$

$$V_b(\omega) = Z_{in}(\omega) I_b(\omega)$$

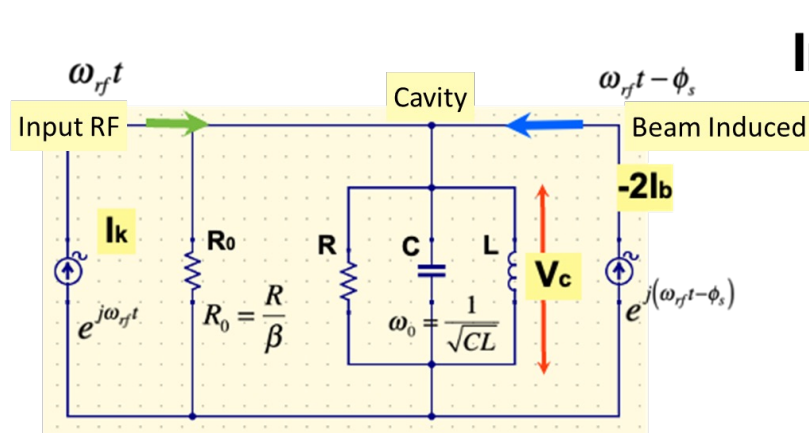
$$V_c = V_b + V_k$$



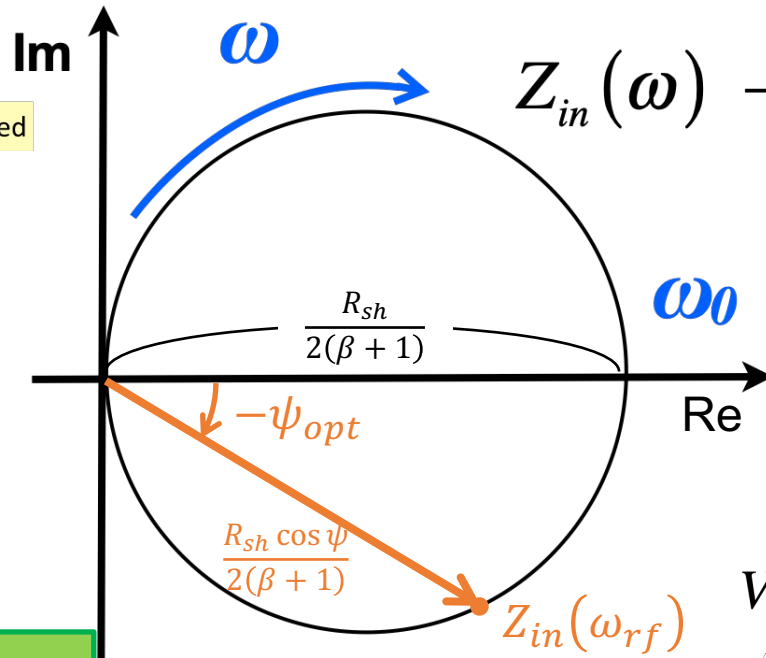
Cavity Voltage V_c in Beam Acceleration



Optimum Tuning



$$Z_{in}(\omega) = \frac{R_{sh} \cos \psi}{2(\beta + 1)} \cdot e^{j\psi}$$

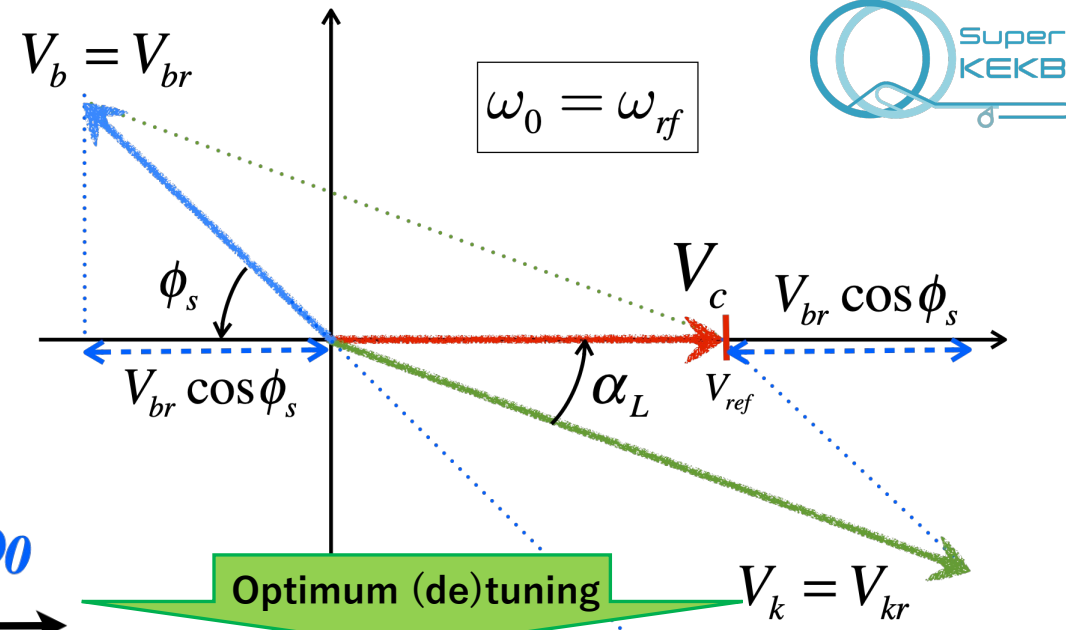


Optimum (de)tuning

Detuning the frequency of the cavity so that V_c and the input RF are in phase ($\alpha_L = 0$)

$$\omega_0 = \omega_{rf} + \Delta\omega_{opt} \quad \Delta\omega_{opt} = -\frac{\omega_{rf} I_b}{2V_c} \left(\frac{R_{sh}}{Q_0} \right) \sin \phi_s$$

Input power is minimized.

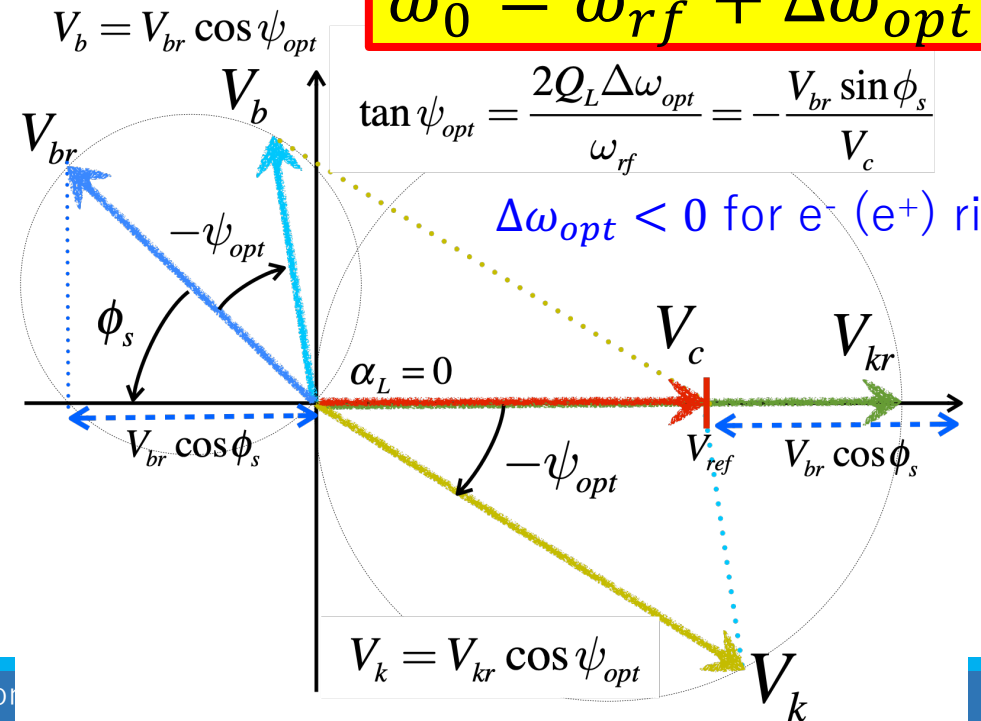


Optimum (de)tuning

$$\omega_0 = \omega_{rf} + \Delta\omega_{opt}$$

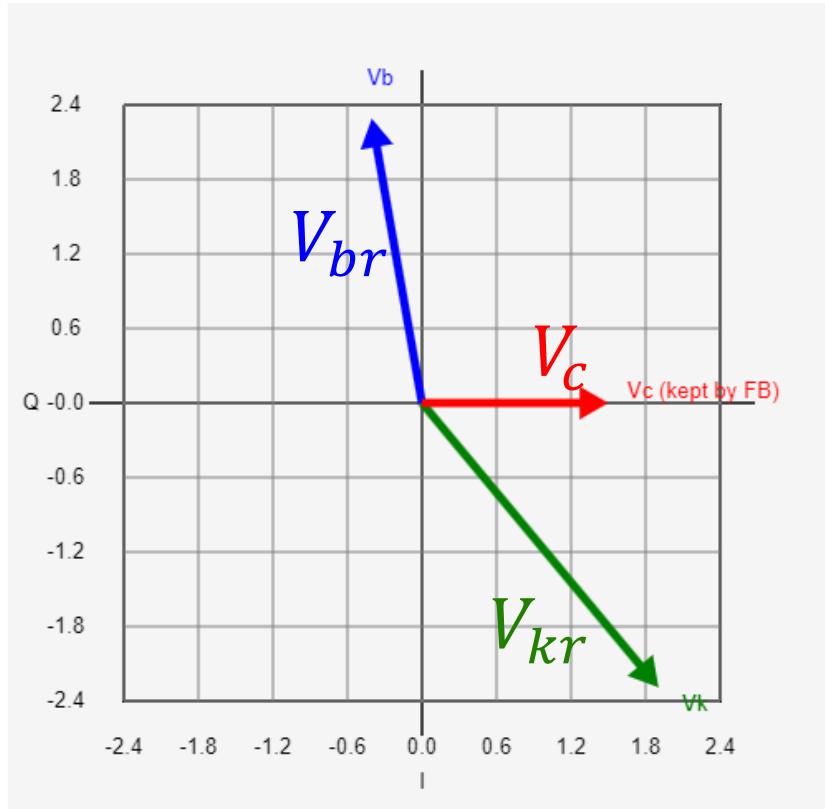
$$\tan \psi_{opt} = \frac{2Q_L \Delta\omega_{opt}}{\omega_{rf}} = -\frac{V_{br} \sin \phi_s}{V_c}$$

$\Delta\omega_{opt} < 0$ for e^- (e^+) ring

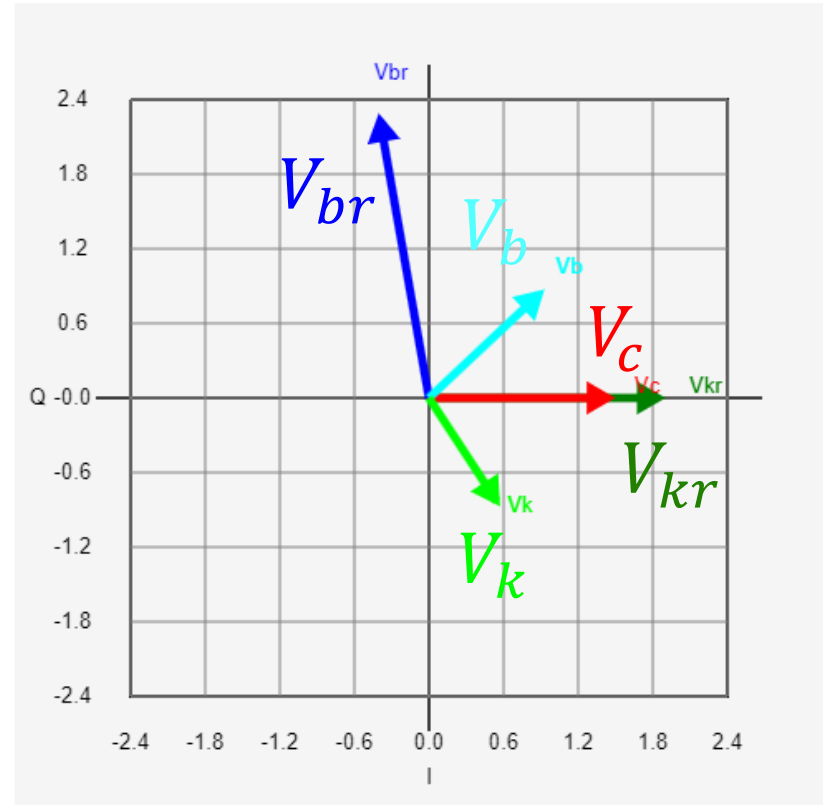


Efficiency by Optimum Tuning

Example of SuperKEKB, SC cavity operating parameters in $I_b=500\text{mA}$



without Optimum tuning



with Optimum tuning

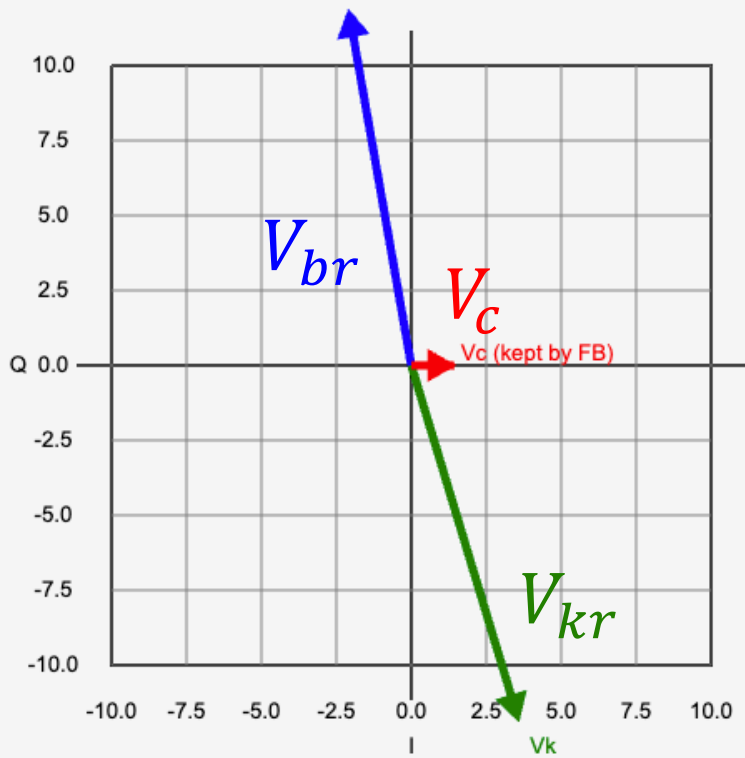
SCC operating parameters

Vref [MV]:	1.5
Φ_s [deg]:	80
R/Q [Ω]:	93
QL:	50000
f_rf [MHz]:	508.9

$$P_k \propto V_{kr}^2$$

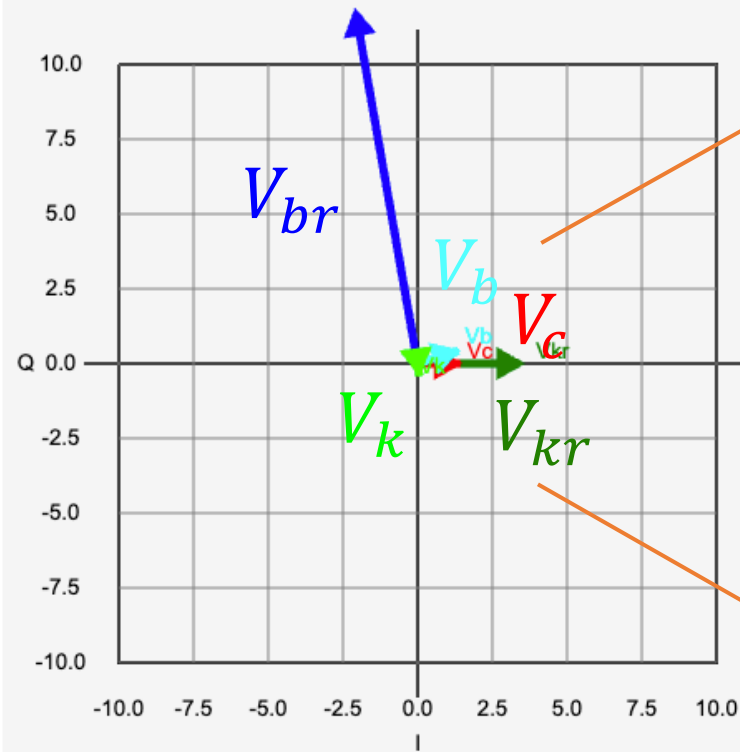
Efficiency by Optimum Tuning

Example of SuperKEKB, SCC operating parameters in $I_b=2.6$ A



Beam Current: 2600 mA

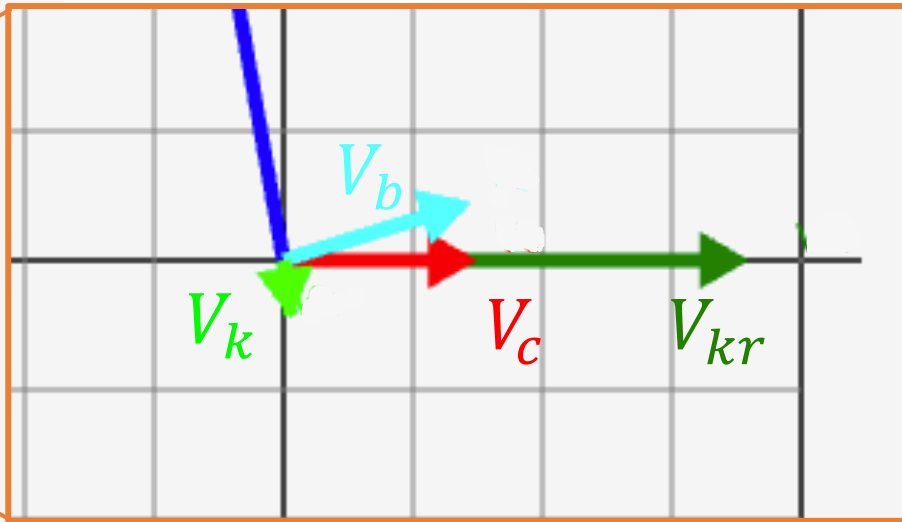
without Optimum tuning



Beam Current: 2600 mA

with Optimum tuning

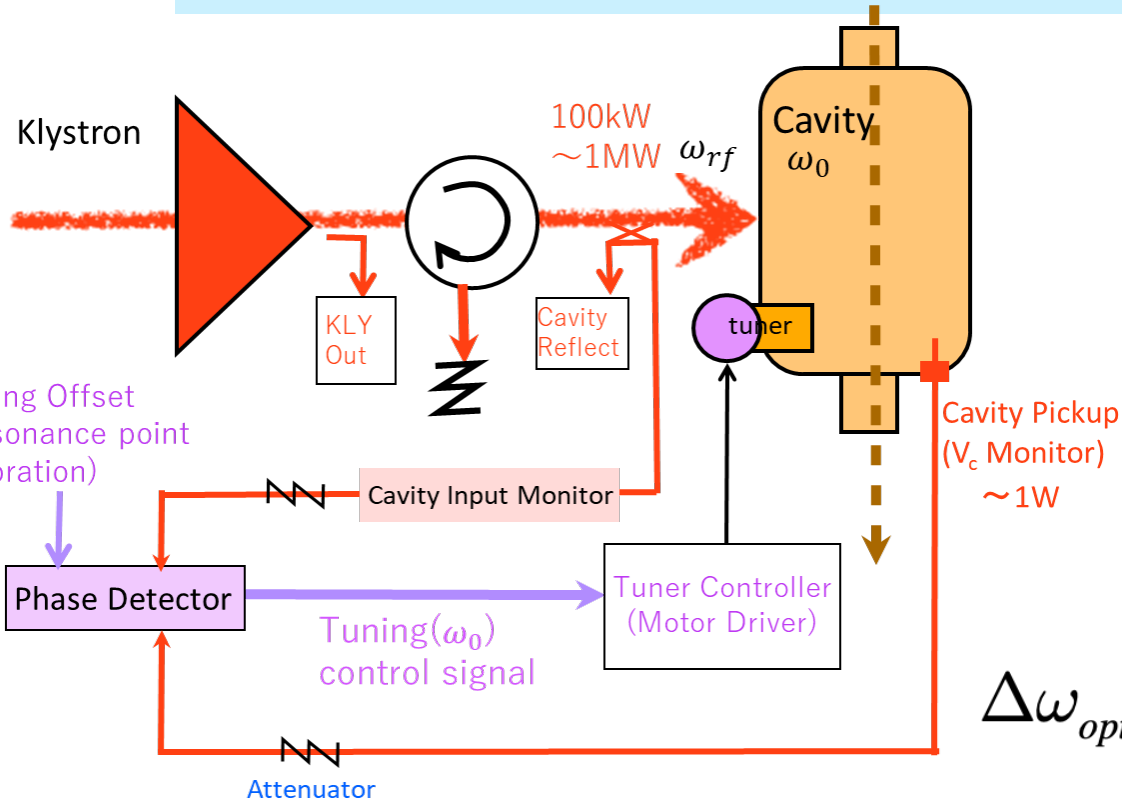
zoomed up



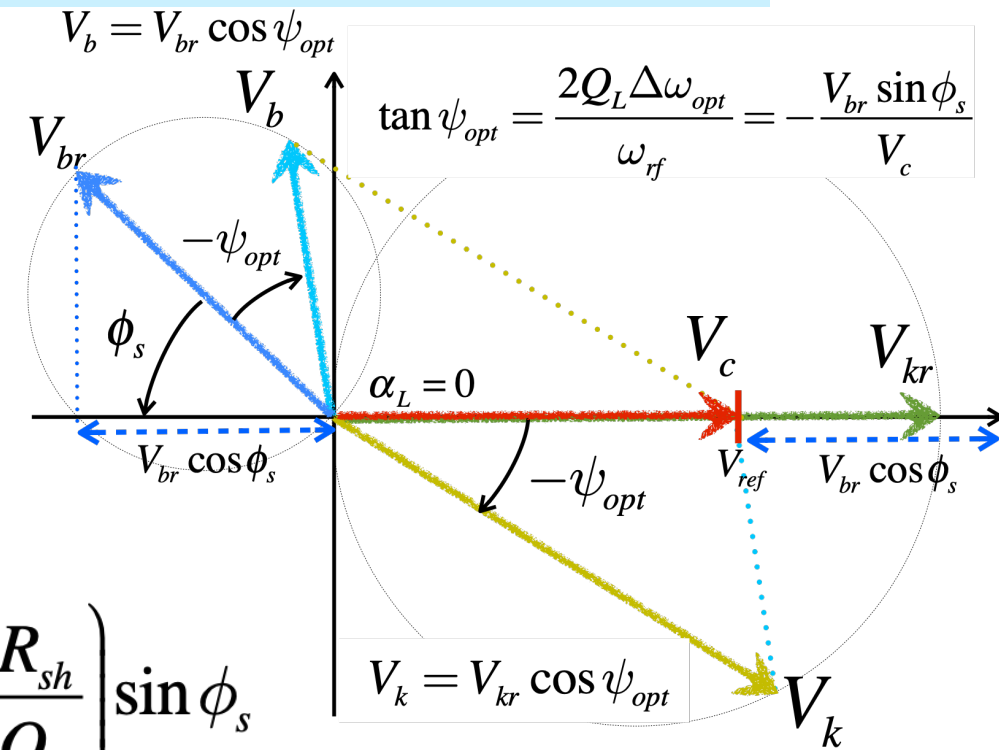
V_c is generated by V_b !!

Optimum Tuning by Auto-tuner Control

Auto-tuner control system keeps optimum tuning automatically.



$$\Delta\omega_{opt} = -\frac{\omega_{rf} I_b}{2V_c} \left(\frac{R_{sh}}{Q_0} \right) \sin\phi_s$$



Optimum tuning is also effective to suppress the coupled bunch instability ($\mu \geq 0$ modes).

In the high current ring, the optimum detuning is indispensable.

But the detuning frequency should be smaller than revolution frequency to avoid instability.

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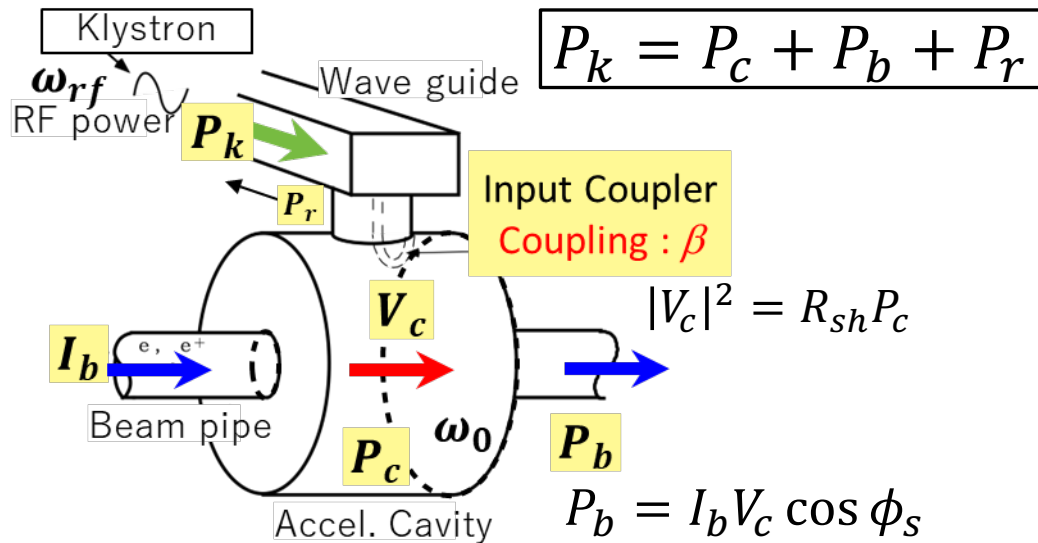
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Optimum Coupling (to be more efficiency)

Optimization of input coupling constant β : Reduce reflection power in beam acceleration



Coupling β	Reflectivity	No Beam
$\beta = \frac{Q_0}{Q_{ext}} = \frac{P_{ext}}{P_c}$	$\Gamma = \frac{\beta - 1}{\beta + 1}$	$\beta = 1$
		→ zero-reflection

Minimum power for beam (I_b) acceleration

P_c : Wall loss (for V_c excitation) $|V_c|^2 = R_{sh}P_c$
 P_b : Feeding power to Beam I_b $P_b = I_b V_c \cos \phi_s$

$P_k = P_c + P_b$ (expecting with $P_r = 0$)

β should be set to make matching for zero reflection with beam acceleration. (Dissipation is $P_c \rightarrow P_c + P_b$ in cavity.)

$\beta' = \frac{P_{ext}}{P_c + P_b} = \frac{\beta P_c}{P_c + P_b}$
 $\beta' = 1$ is matching condition with beam.

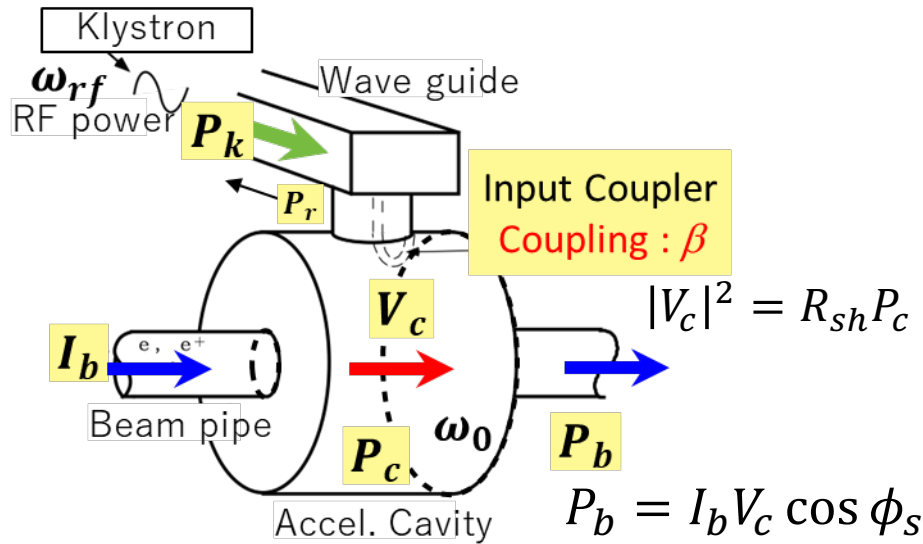
Optimum Coupling

$\beta_{opt} = 1 + \frac{P_b}{P_c}$

If the beam current increase more than I_b , P_b increase and $\beta' < 1$. (=looks like Under Coupling). It is better to design with a margin for P_b .

In general, β cannot be changed during the machine operation. It is important to choose the optimum coupling with margin in design and manufacturing.

Example of SuperKEKB HER ($I_b = 2.6 \text{ A}$)



Normal Conducting Cavity (ARES)

$$V_c = 0.5 \text{ MV}, P_c = 150 \text{ kW}, \phi_s = 65^\circ$$

$$P_b = 2.6 \text{ A} \times 0.5 \text{ MV} \times \cos 65^\circ \sim 550 \text{ kW}$$

$$\beta_{opt} = 1 + \frac{P_b}{P_c} = 1 + \frac{550 [\text{kW}]}{150 [\text{kW}]} \sim 4.7$$

Superconducting Cavity (SCC)

$$V_c = 1.5 \text{ MV}, P_c = 24 \text{ W}, \phi_s = 83^\circ$$

$$P_b = 2.6 \text{ A} \times 1.5 \text{ MV} \times \cos 83^\circ \sim 475 \text{ kW}$$

$$\beta_{opt} = 1 + \frac{P_b}{P_c} = 1 + \frac{475 [\text{kW}]}{24 [\text{W}]} \sim 2 \times 10^4$$

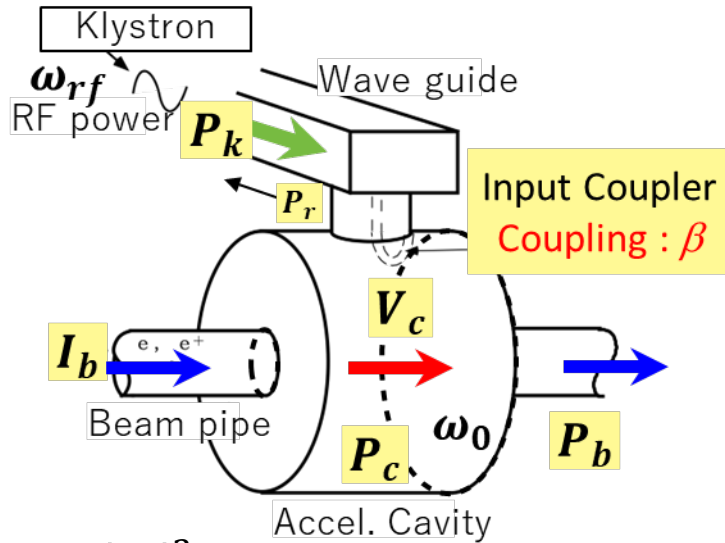
Q_L or Q_{ext} are used as the input coupling in SC cavity.

Optimum coupling ($P_k \approx P_b, Q_L \approx Q_{ext} \ll Q_0$)

$$Q_{ext.opt} \approx \frac{V_c}{\left(\frac{R_{sh}}{Q_0}\right) I_b \cos \phi_s} = \frac{1.5 \text{ MV}}{93 \Omega \times 2.6 \text{ A} \times \cos 83^\circ} \sim 5 \times 10^4$$

Cavity Parameters	ARES	SCC
R_{sh}/Q_0 [Ω]	15	93
Q_0	1.2×10^5	1×10^9
V_c [MV]	0.5	1.5
P_c [kW]	150	0.024
β	5	-
Q_L	2×10^4	5×10^4

Required Klystron Power for SuperKEKB



$$|V_c|^2 = R_{sh} P_c$$

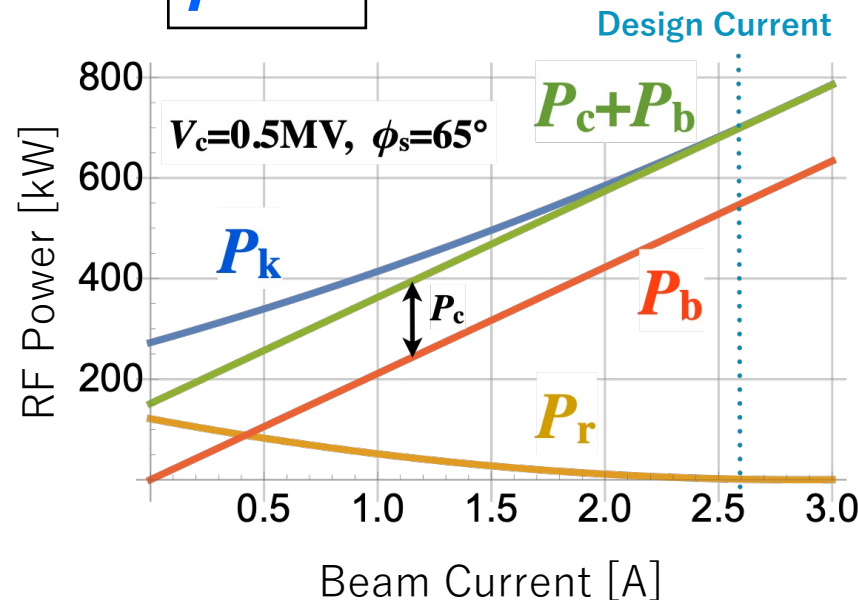
$$P_b = I_b V_c \cos \phi_s$$

P_c is kept constant by keeping V_c constant in FB control.

optimum tuning
optimum coupling

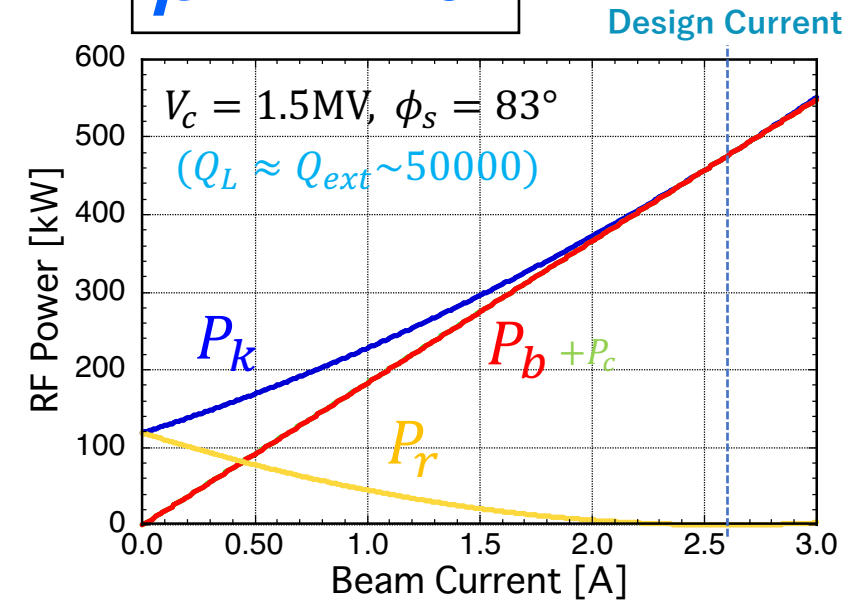
Normal Conducting Cavity (ARES)

$$\beta = 5$$



Superconducting Cavity (SCC)

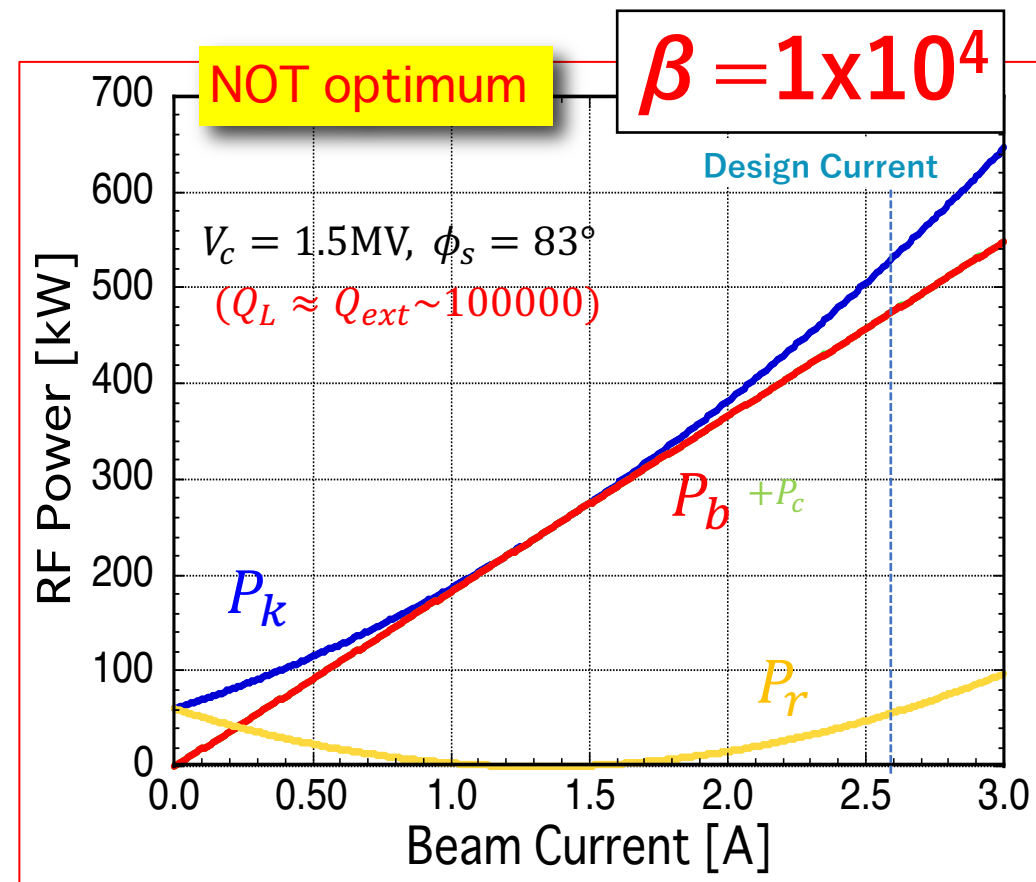
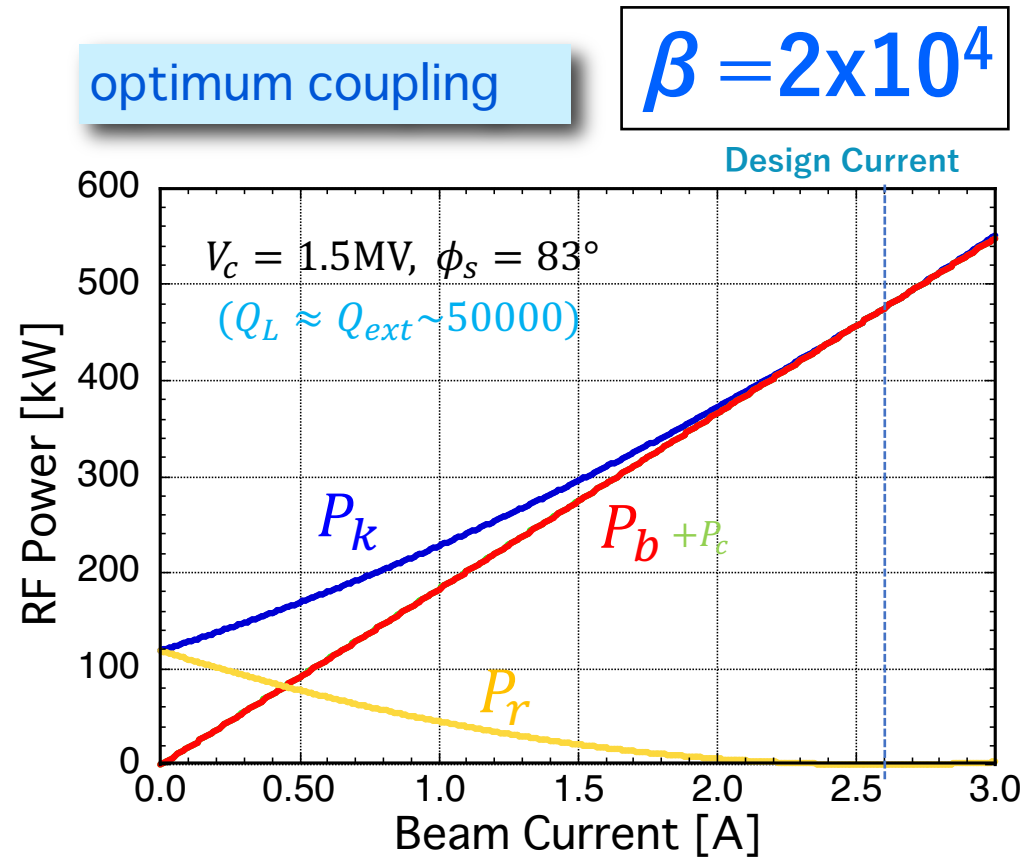
$$\beta = 2 \times 10^4$$



Required Klystron Power for SuperKEKB

Superconducting Cavity (SCC)

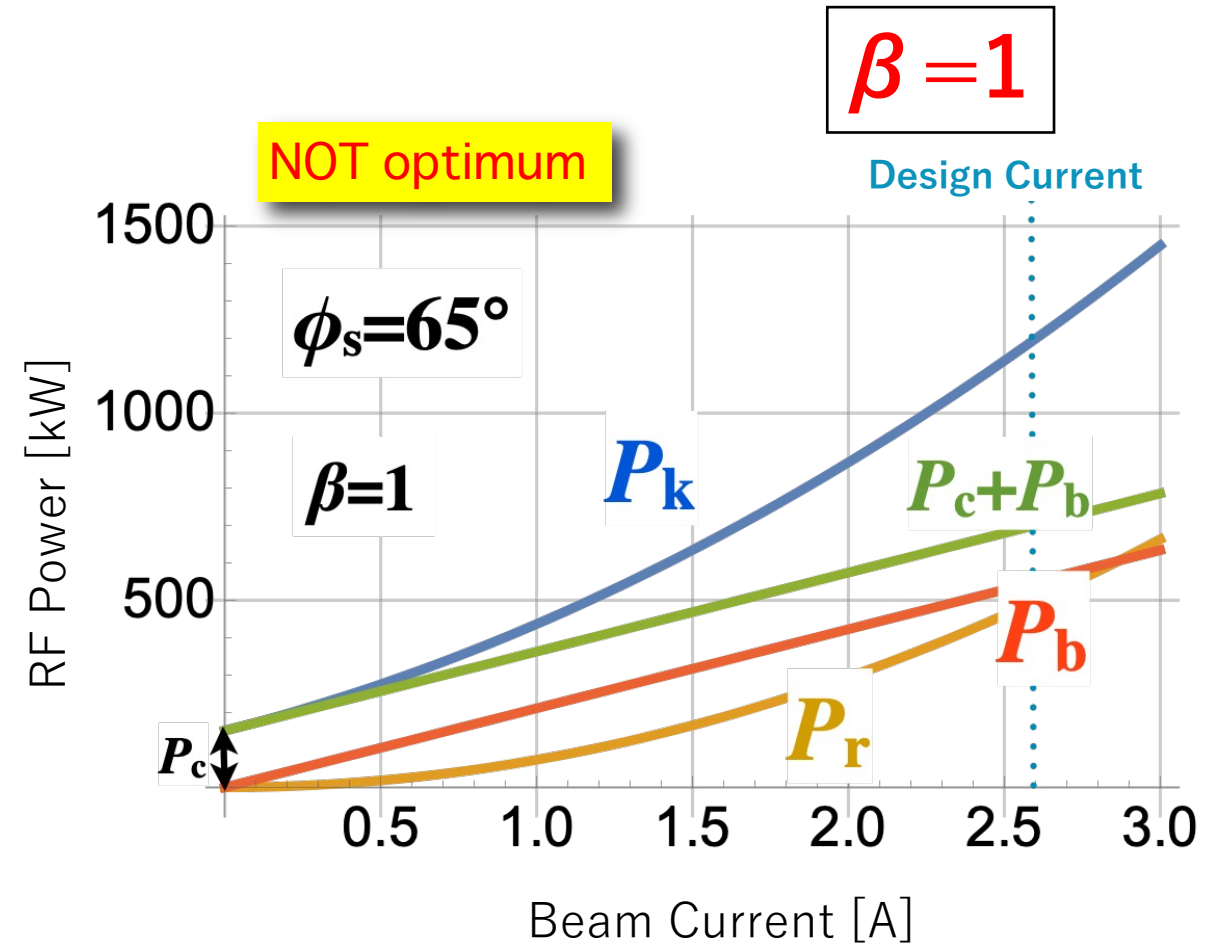
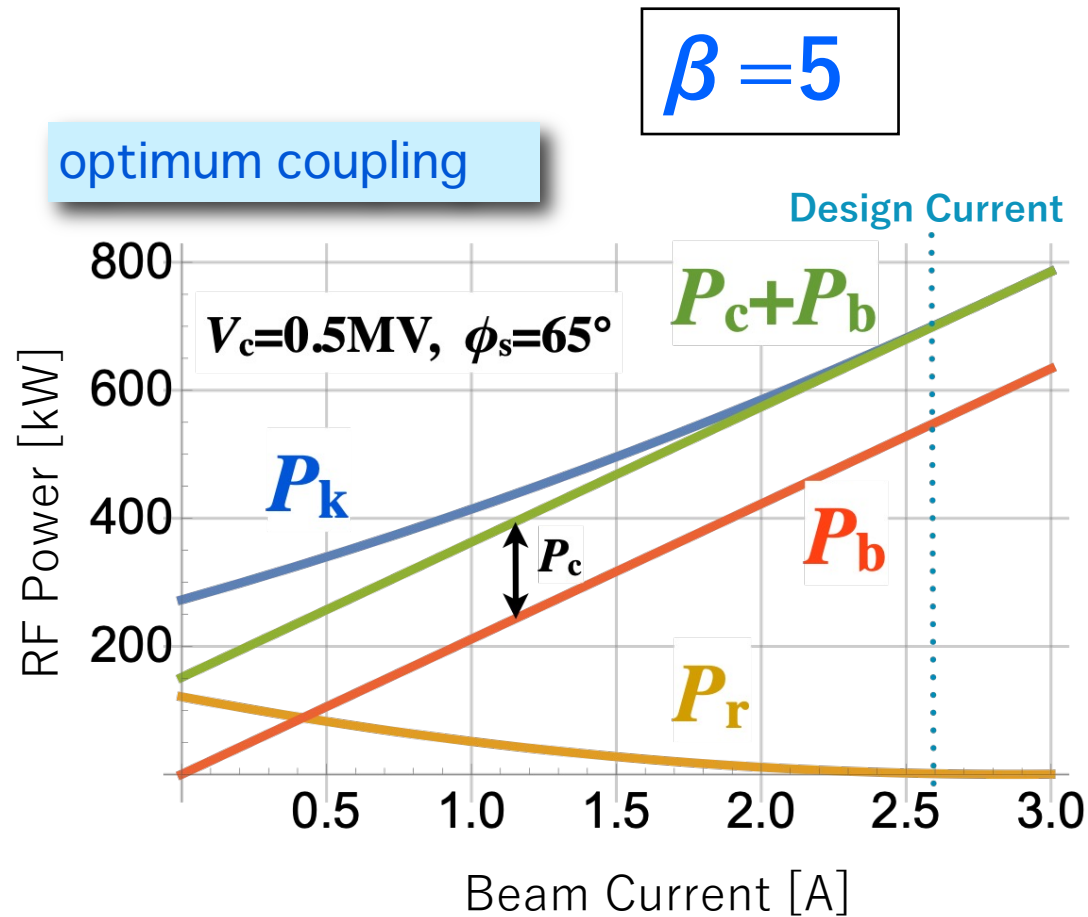
optimum tuning



Required Klystron Power for SuperKEKB

Normal Conducting Cavity (ARES)

optimum tuning

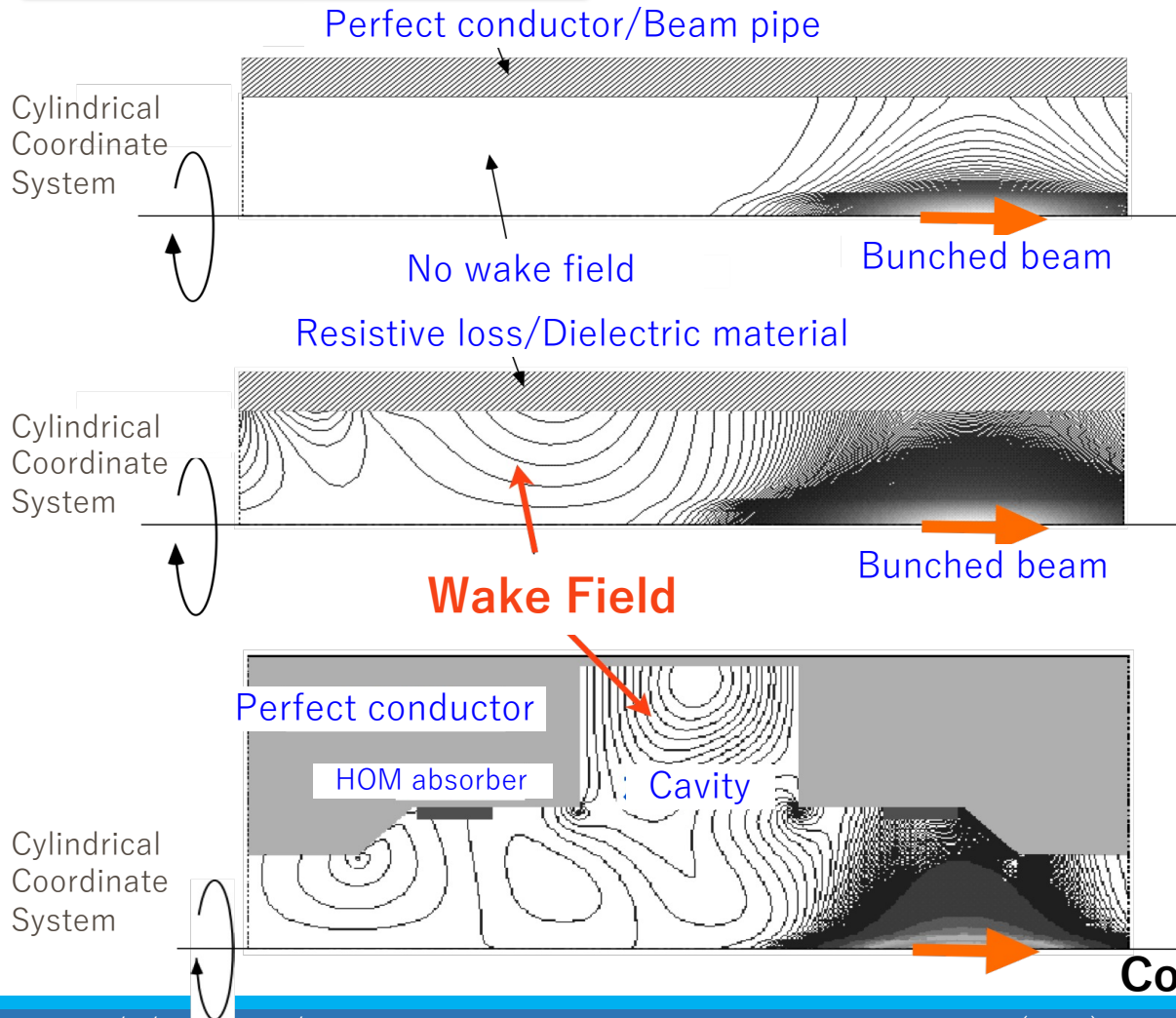


Contents

- ◆ Optimization for Beam Loading
 - Impedance Characteristics of Cavity (Equivalent Circuit Model)
 - Frequency Spectrum of Beam
 - Optimum Tuning
 - Optimum Coupling
- ◆ Instabilities due to Accelerating Mode
 - Coupled Bunch Instability (CBI) related to $\mu = -1, -2$ and -3 modes
 - Static Robinson Instability (zero-mode)
- ◆ HOM
 - HOM damping in KEKB SCC
 - Large HOM power in KEKB and SuperKEKB

CBI excited by wake fields

Calculated by "MAFIA"



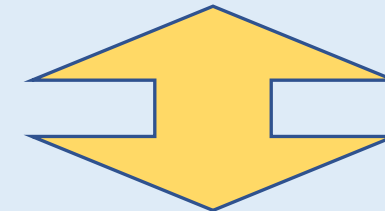
Wake field are induced by the interaction between the beam and the resistive loss and/or structure of the beam pipe.
(=impedance of the surrounding component).

Bunches in a ring are coupled via the wake field.

Oscillation of beam particles is increased.
Eventually, Beam is lost.

Coupled Bunch Instability (CBI)

Cavity is the essential component.

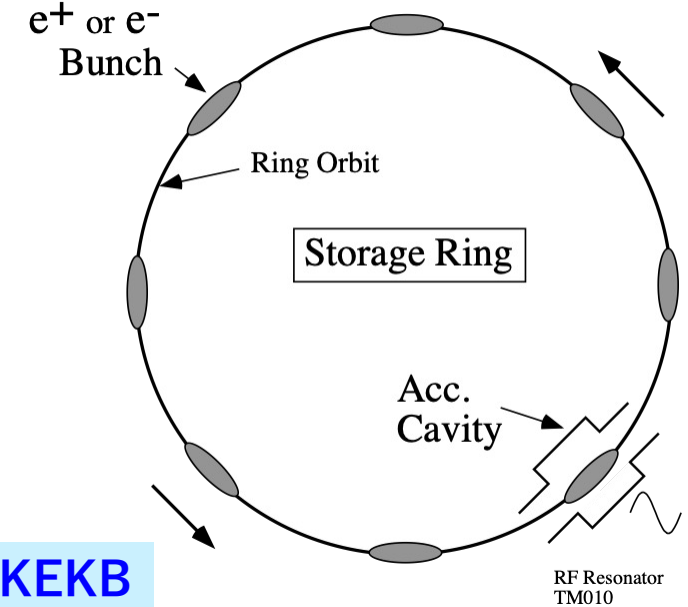
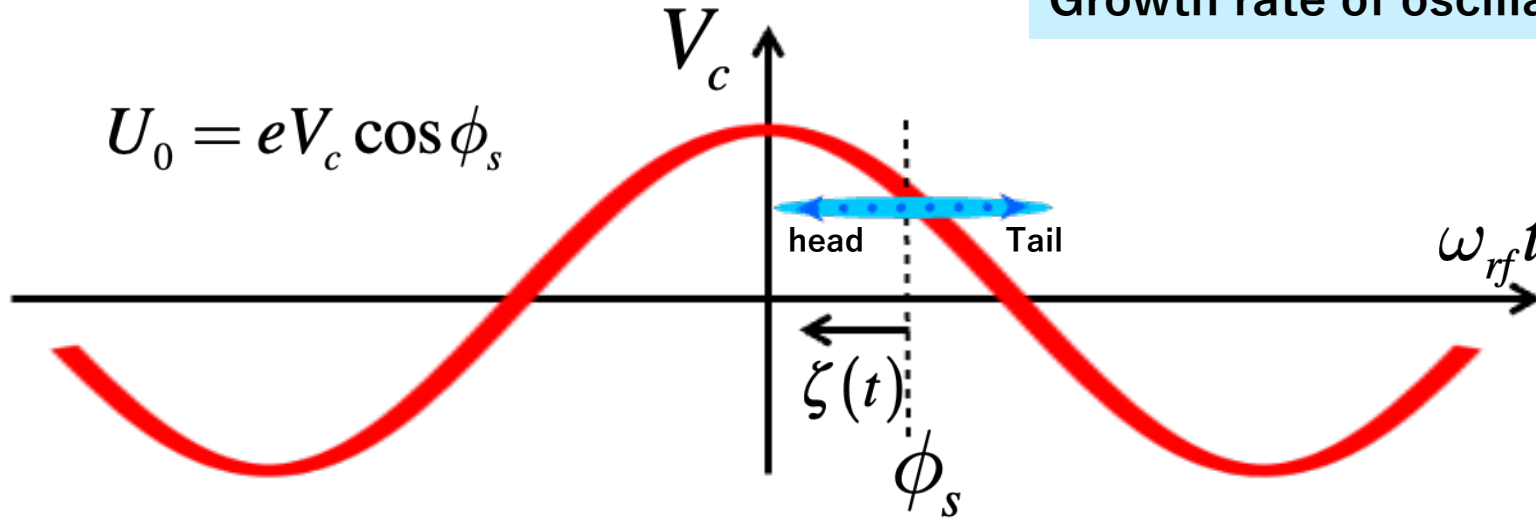


Cavity can be a major cause of CBI.

Consideration of only CBI caused by the acceleration mode

Evaluation of CBI - Instability occur or not. -

Growth rate of oscillation



Synchrotron Oscillation

: Beam particles oscillate around the synchronous phase ϕ_s .

$$\zeta(t) \propto e^{-i\Omega t} = e^{\alpha t} \cdot e^{-j\omega_s t} = e^{t/\tau} \cdot e^{-j\omega_s t}$$

$$\Omega = \omega_s + j\alpha$$

$$\alpha = \frac{1}{\tau} \quad \text{growth rate}$$

SuperKEKB

RF frequency : $f_{rf} \sim 509\text{MHz}$

Revolution frequency : $h = 5120$

$$f_{rev} = f_{rf} / h \sim 100\text{kHz} \quad T_{rev} \sim 10\mu\text{s}$$

Synchrotron frequency : $f_s \sim 2\text{kHz}$

$\alpha > 0$ oscillation increase exponentially

Instability is evaluated by the growth rate.

Coupling Impedance and Growth rate of CBI

Voltage $V(\omega)$ induced in structure by beam current $I(\omega)$,

Coupling Impedance

$$\mathbf{V}(\omega) = -\mathbf{Z}(\omega)\mathbf{I}(\omega)$$

Induced Voltage

Beam Current

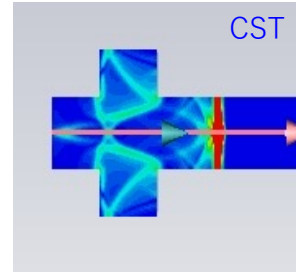
Longitudinal mode :

$$V_{\parallel}(\omega) = -Z_{\parallel}(\omega)I(\omega)$$

(accelerating mode)

Transvers mode:

$$\mathbf{V}_{\perp}(\omega) = -\mathbf{Z}_{\perp}(\omega)\mathbf{I}(\omega)$$



Instability occur in a structure (cavity)?



Evaluate “Coupling Impedance” with beam

Oscillation mode μ :

$$\zeta(t) \propto e^{\alpha_{\mu}t} = e^{t/\tau_{\mu}}$$

growth rate

$$\alpha_{\mu} = \frac{1}{\tau_{\mu}} \propto \text{Re}\{Z(\omega_{\mu})\}$$

ω_{μ} : angular frequency of mode μ

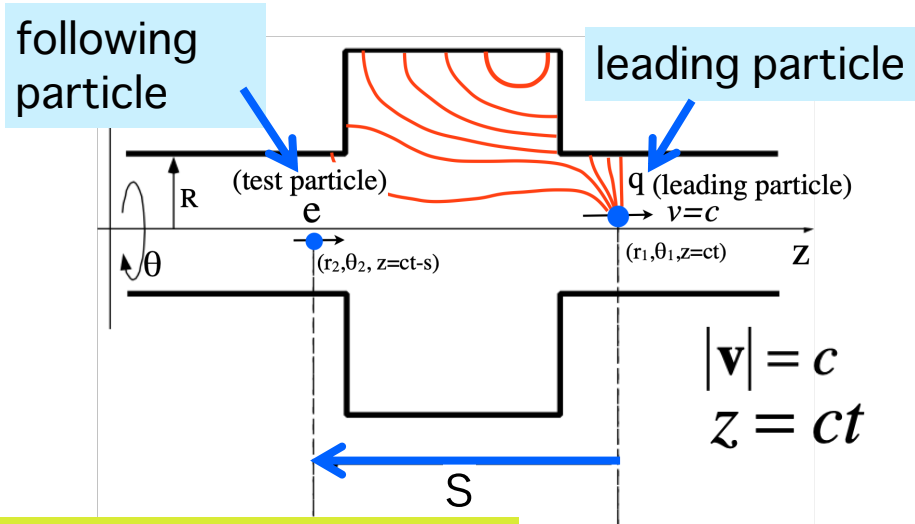
radiation damping

$$\propto e^{-\alpha_{rd}t}$$

stable condition

$$\alpha_{\mu} < \alpha_{rd}$$

Wake function and coupling impedance



Force felt by following particle

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \begin{aligned} F_{\parallel}(r_2, \theta_2, z, t) &= eE_z \\ F_{\perp}(r_2, \theta_2, z, t) &= e(\mathbf{E} + c\mathbf{e}_z \times \mathbf{B}) \end{aligned}$$

Wake Function
$$\mathbf{W}(s) = \frac{1}{eq} \int_{-\infty}^{\infty} \mathbf{F} \left(t = \frac{s+z}{c} \right) dz$$

$$\mathbf{W}(s < 0) = 0$$

No field before leading particle passing

Longitudinal: $W_{\parallel}(s)$

Transvers: $\mathbf{W}_{\perp}(s)$

Coupling Impedance

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} W_{\parallel}(s) e^{j\omega \frac{s}{c}} \frac{ds}{c}$$

$$Z_{\parallel}(-\omega) = Z_{\parallel}^*(\omega)$$

Fourier transform

Wake function Considering only longitudinal mode...

$$W_{\parallel}(s) = \frac{1}{eq} \int_{-\infty}^{\infty} eE_z \left(t = \frac{s+z}{c} \right) dz$$

Coupling Impedance of accelerating cavity

$$Z_{\parallel}(\omega) = Z_a(\omega) = \frac{R_{sh}/2Q_0}{\frac{1}{Q_L} + j \left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)}$$

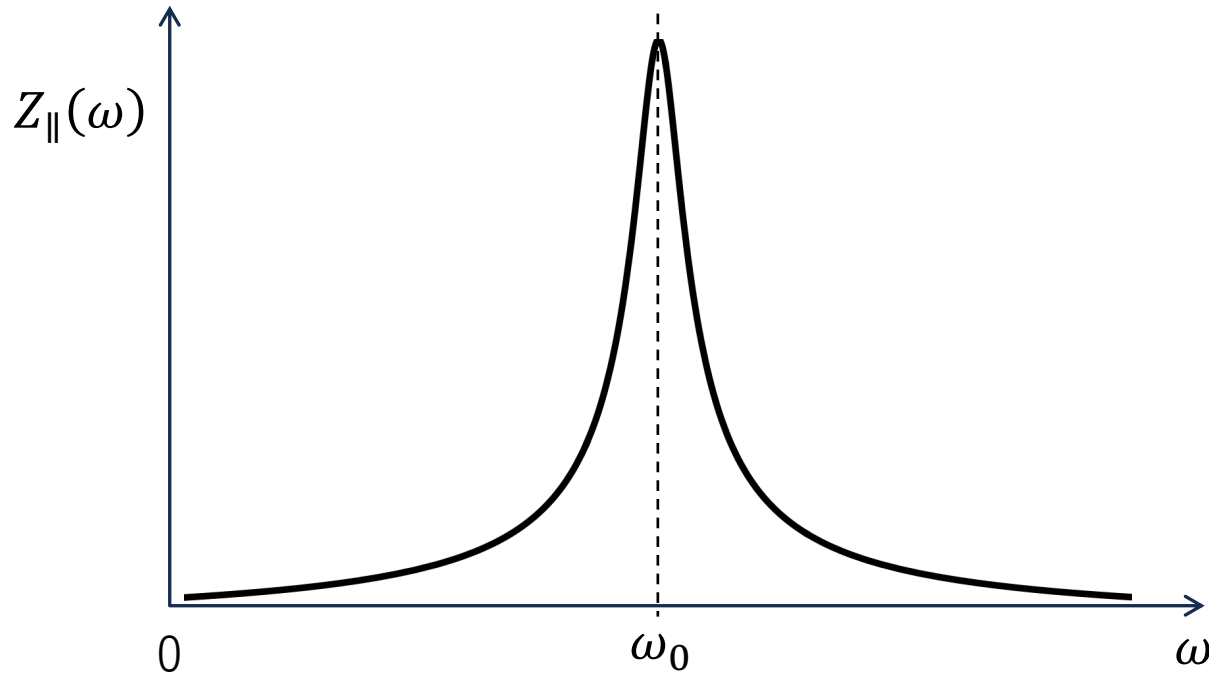
Wake function of accelerating cavity

$$W_{\parallel}(s) = \frac{\omega_0 R_{sh}}{2 Q_0} \cos \left(\omega'_0 \frac{s}{c} + \delta \right) e^{-\frac{\omega_0}{2Q_L} \frac{s}{c}}$$

Wake function and coupling impedance

Coupling Impedance of accelerating cavity

$$Z_{\parallel}(\omega) = Z_a(\omega) = \frac{R_{sh}/2Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)}$$

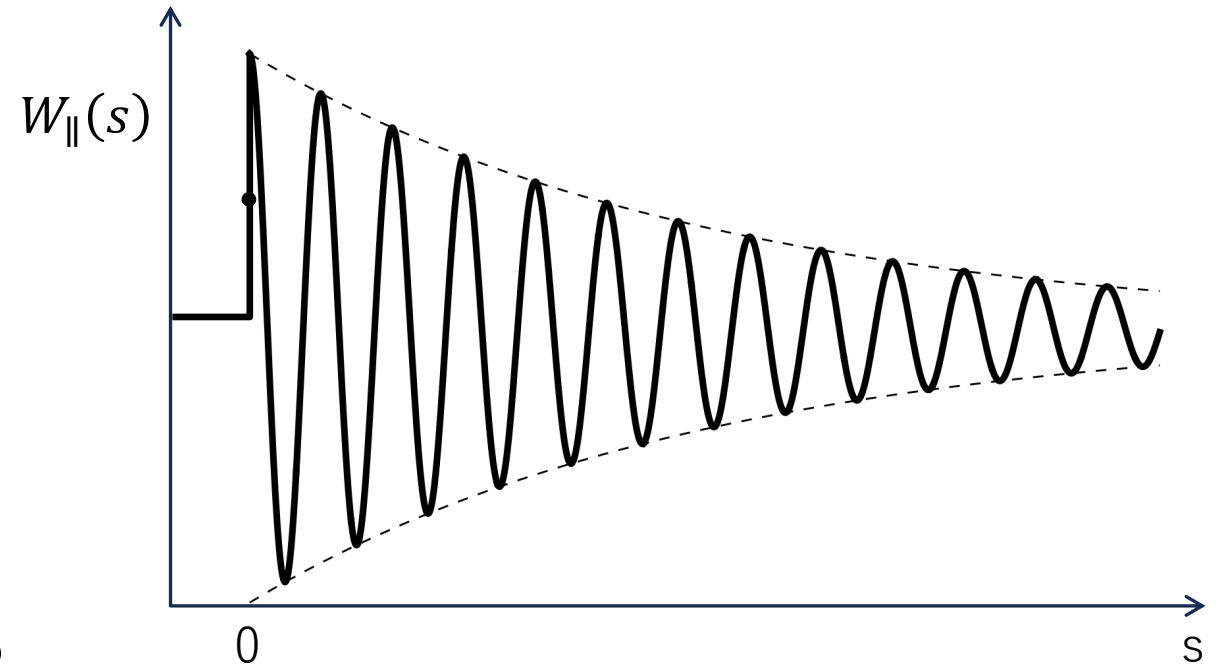


Fourier transform



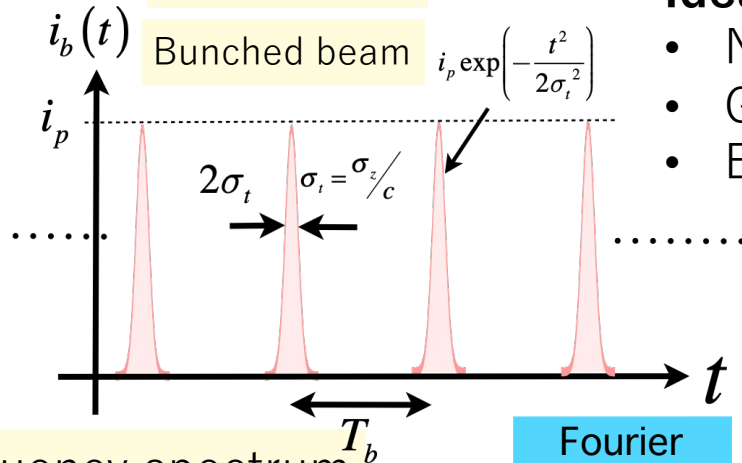
Wake function of accelerating cavity

$$W_{\parallel}(s) = \frac{\omega_0 R_{sh}}{2 Q_0} \cos\left(\omega'_0 \frac{s}{c} + \delta\right) e^{-\frac{\omega_0 s}{2Q_L c}}$$



Frequency spectrum of Beam

Bunch train



Ideal bunch train

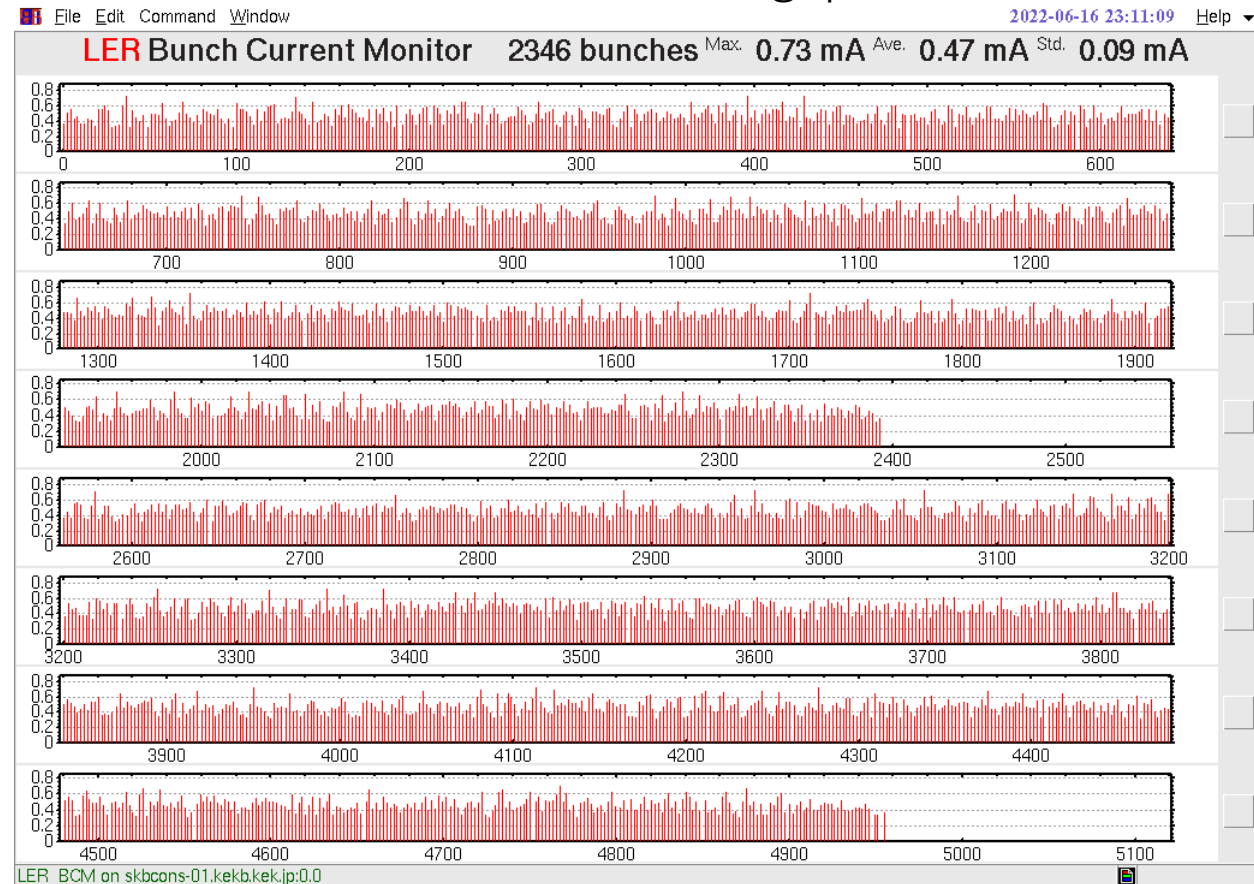
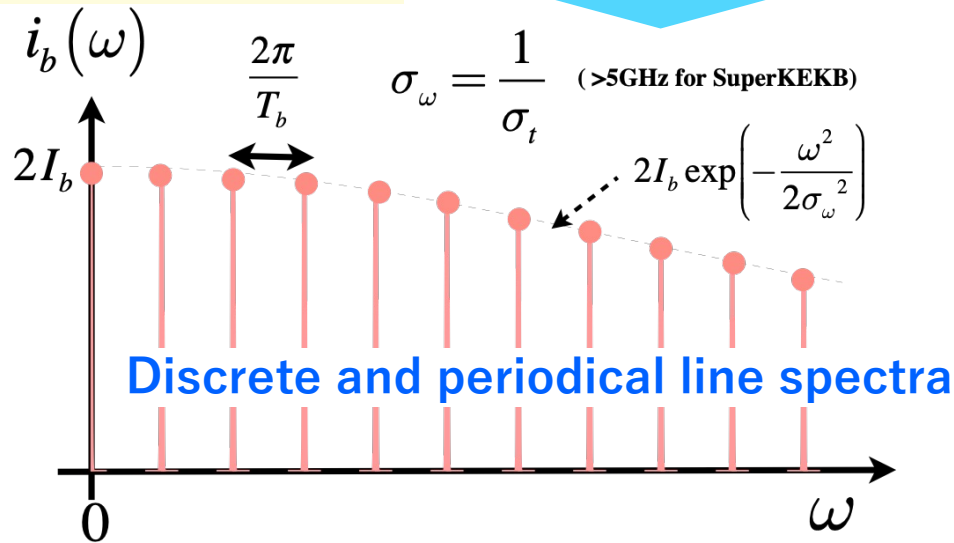
- No difference between all bunches
- Gaussian shape
- Equal spaced bunches

Real bunch train

- Not uniform bunch charge
- Not Equal spaced
- Abort gaps

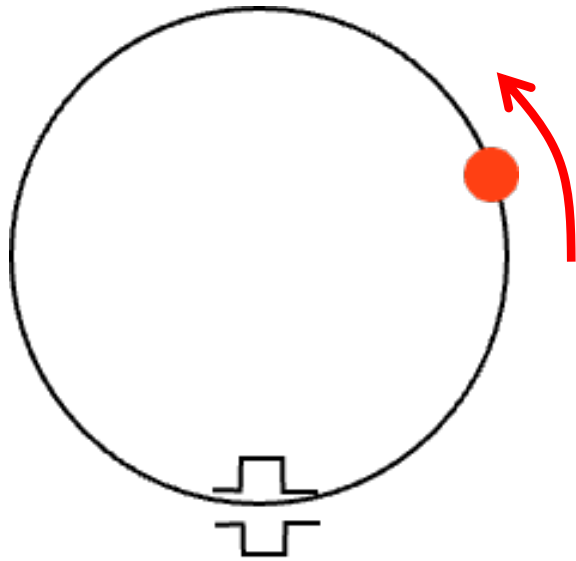
Frequency spectrum

Fourier transform



Frequency spectrum of Beam

One bunch (point charge) in a ring



$$f_{rev} = f_{rf} / h \sim 100 \text{ kHz}$$

RF frequency : $f_{rf} \sim 509 \text{ MHz}$

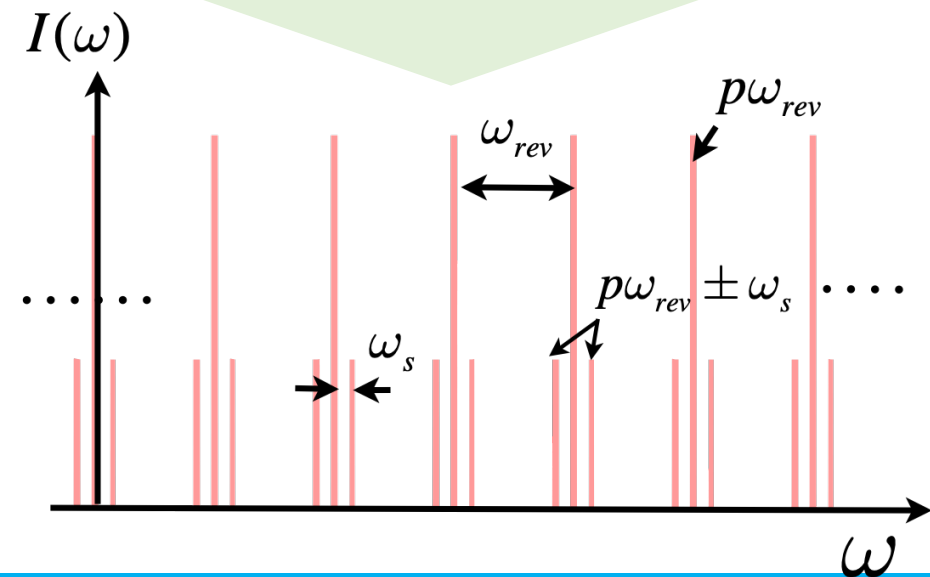
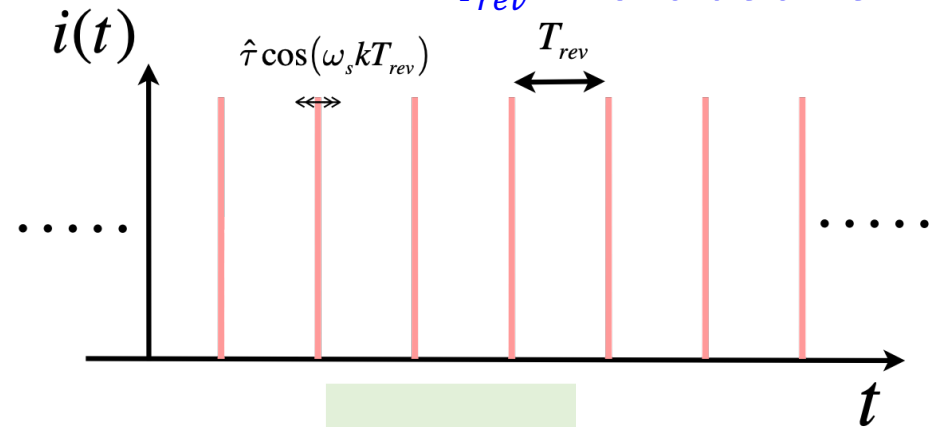
Revolution frequency : $f_{rev} = f_{rf} / h \sim 100 \text{ kHz}$

$$h = 5120$$

$$T_{rev} \sim 10 \mu\text{s}$$

Synchrotron frequency : $f_s \sim 2 \text{ kHz}$

T_{rev} : revolution time ($10 \mu\text{s}$)



Multi bunches in a ring

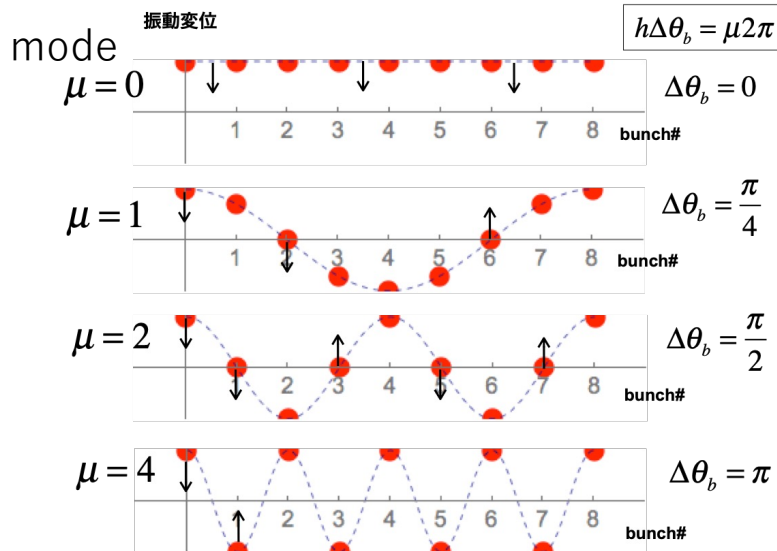
All RF buckets are filled by bunches (point charge, number of bunch = h , Synchrotron oscillation $\omega_s t$)

Phase difference of oscillation between neighbor bunches : $\Delta\theta_b$

Oscillation phase of n -th bunch : $\omega_s t + n\Delta\theta_b$

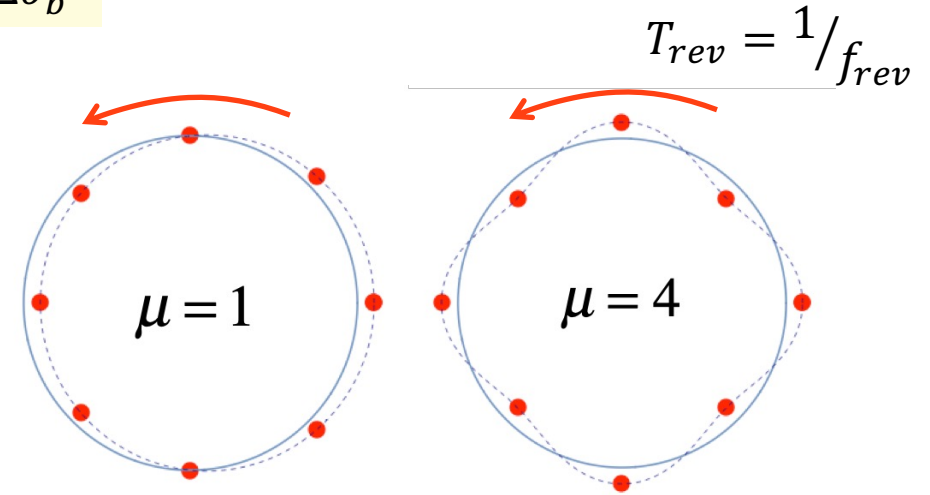
Returns to its own bunch after one turn : $n = h, h\Delta\theta_b = \mu 2\pi$

h=8



μ : mode number
 $0 \leq \mu \leq h - 1$

Bunch#0 and #8 mean the same bunch.



There are μ oscillation modes in one turn.

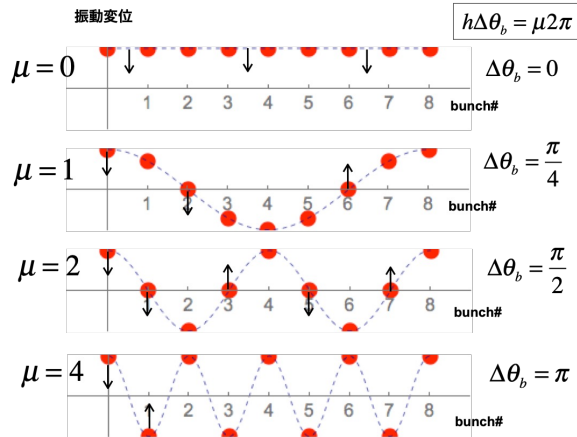
In the actual bunch, many particles are oscillating, resulting in a superposition of modes.

Since bunches are circulating a ring with revolution time of $T_{rev} = 1/f_{rev}$,
 beam spectrum has frequency of $\mu f_{rev} \pm f_s$

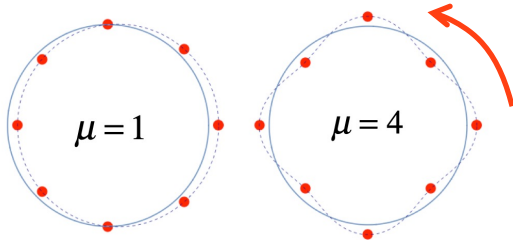
Frequency spectrum of Beam

All RF buckets are filled by bunches (number of bunches = h , oscillation mode μ)

h=8

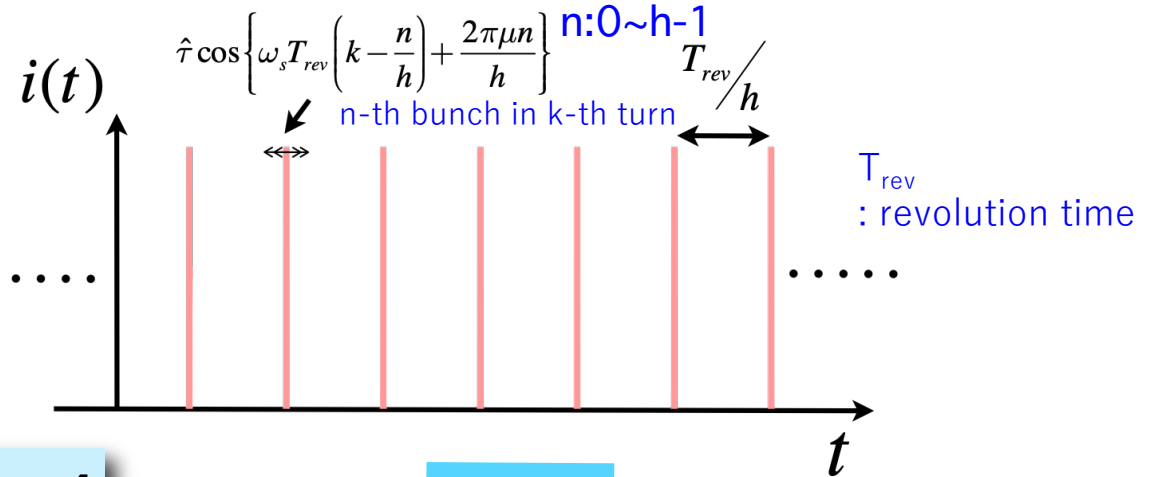


μ modes in one turn

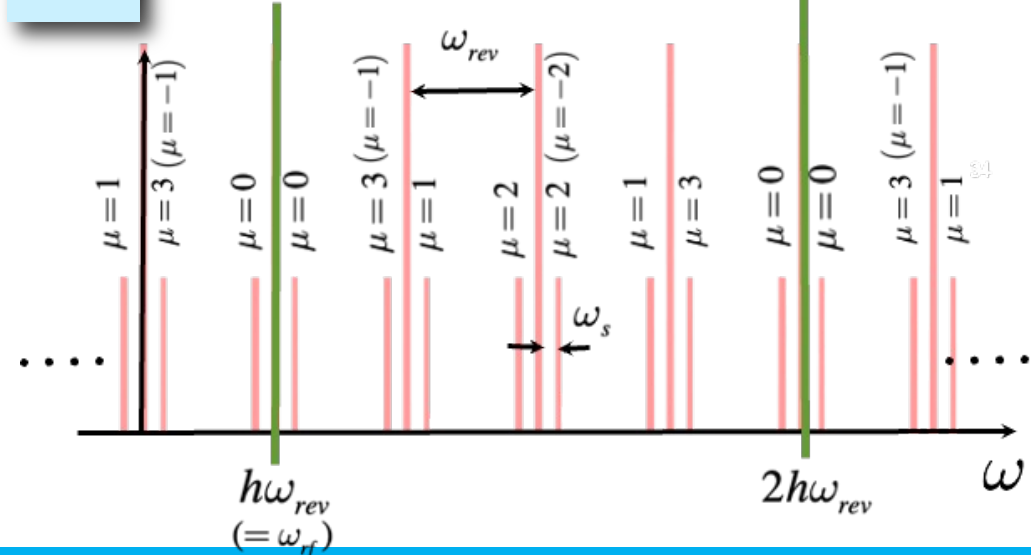


circulating with revolution time of T_{rev}

$$\mu f_{rev} \pm f_s$$



h=4



Evaluation of growth rate from impedance

$$\zeta(t) \propto e^{\alpha_\mu t} = e^{t/\tau_\mu}$$

growth rate

$$\alpha_\mu = \frac{1}{\tau_\mu} = AI_b \sum_{p=0}^{\infty} \left\{ \overset{\text{Excitation}}{f_p^{(\mu+)}} \operatorname{Re} Z\left(f_p^{(\mu+)}\right) - \overset{\text{Damping}}{f_p^{(\mu-)}} \operatorname{Re} Z\left(f_p^{(\mu-)}\right) \right\}$$

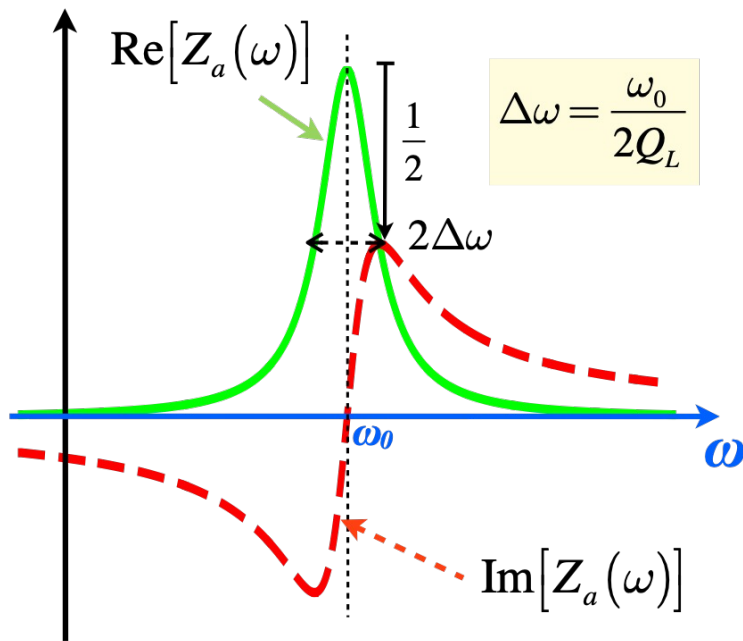
$$f_p^{(\mu+)} = phf_{rev} + \mu f_{rev} + f_s = pf_{rf} + \mu f_{rev} + f_s$$

$$f_p^{(\mu-)} = (p+1)hf_{rev} - \mu f_{rev} - f_s = (p+1)f_{rf} - \mu f_{rev} - f_s$$

$$hf_{rev} = f_{rf}$$

Coupling impedance of cavity

$$Z_{\parallel}(\omega) = Z_a(\omega) = \frac{R_{sh}/2Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)}$$



Growth rate of CBI due to accelerating mode

$$A(t) \propto e^{\alpha_{\mu} t} = e^{t/\tau_{\mu}}$$

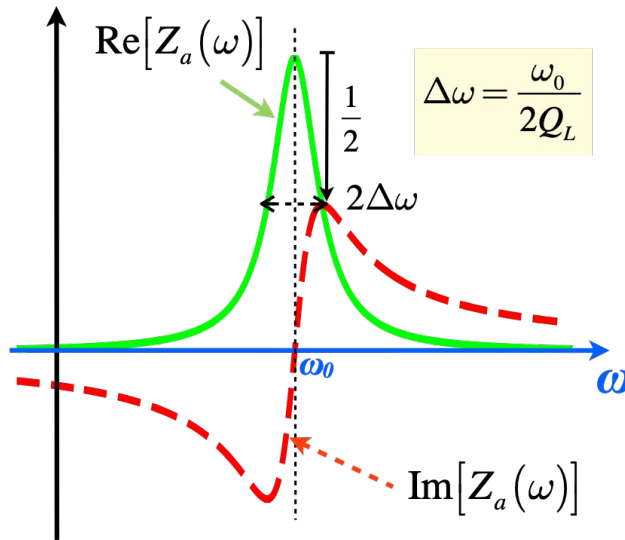
$$\frac{1}{\tau_{\mu}} = A I_b \sum_{p=0}^{\infty} \left\{ \overset{\text{excitation}}{f_p^{(\mu+)}} \text{Re} Z(f_p^{(\mu+)}) - \overset{\text{damping}}{f_p^{(\mu-)}} \text{Re} Z(f_p^{(\mu-)}) \right\}$$

$$f_p^{(\mu+)} = p h f_{rev} + \mu f_{rev} + f_s = p f_{rf} + \mu f_{rev} + f_s$$

$$f_p^{(\mu-)} = (p+1) h f_{rev} - \mu f_{rev} - f_s = (p+1) f_{rf} - \mu f_{rev} - f_s$$

Impedance of cavity

$$Z_{\parallel}(\omega) = Z_a(\omega) = \frac{R_{sh}/2Q_0}{\frac{1}{Q_L} + j\left(\frac{\omega_0 - \omega}{\omega} - \frac{\omega - \omega_0}{\omega}\right)}$$



Considering only around f_{rf} : $p=0, 1$

$$0 \leq \mu < h - 1$$

$$\mu = h - m \rightarrow \mu = -m$$

$$\frac{1}{\tau_{\mu}} = A I_b \left\{ \overset{p=1}{f_1^{(\mu+)}} \text{Re} Z(f_{rf} + f_s) - \overset{p=0}{f_0^{(\mu-)}} \text{Re} Z(f_{rf} - f_s) \right\} \mu = 0$$

$$\left\{ f_1^{(1+)} \text{Re} Z(f_{rf} + f_{rev} + f_s) - f_0^{(1-)} \text{Re} Z(f_{rf} - f_{rev} - f_s) \right\} \mu = +1$$

$$\left\{ f_1^{(2+)} \text{Re} Z(f_{rf} + 2f_{rev} + f_s) - f_0^{(2-)} \text{Re} Z(f_{rf} - 2f_{rev} - f_s) \right\} \mu = +2$$

$$\left\{ f_1^{(-1+)} \text{Re} Z(f_{rf} - f_{rev} + f_s) - f_0^{(-1-)} \text{Re} Z(f_{rf} + f_{rev} - f_s) \right\} \mu = -1 \quad (\mu = h - 1)$$

$$\left\{ f_1^{(-2+)} \text{Re} Z(f_{rf} - 2f_{rev} + f_s) - f_0^{(-2-)} \text{Re} Z(f_{rf} + 2f_{rev} - f_s) \right\} \mu = -2 \quad (\mu = h - 2)$$

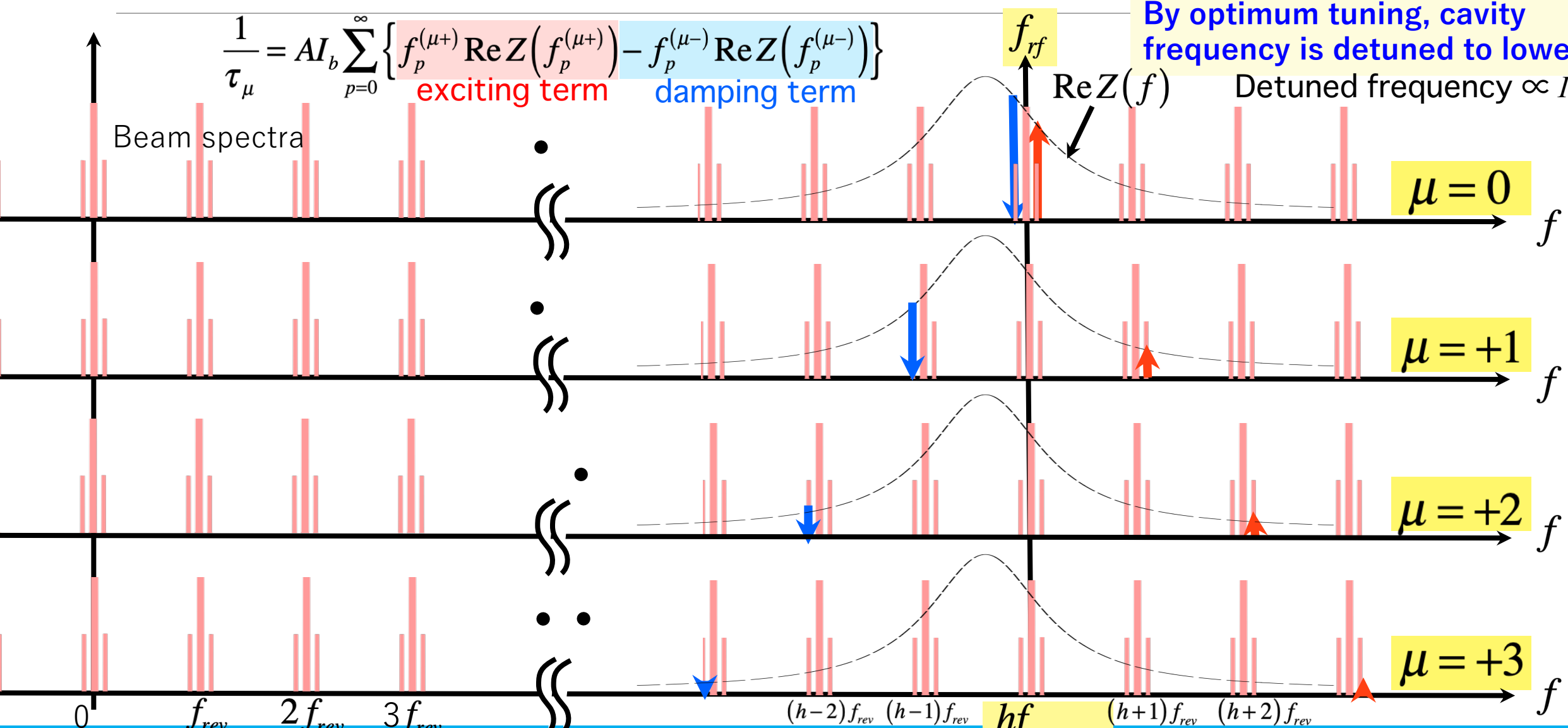
$N f_{rev} + f_s$: excitation effect

$N f_{rev} - f_s$: damping effect

Effect of CBI due to accelerating mode $\mu \geq 0$

$$\frac{1}{\tau_\mu} = AI_b \sum_{p=0}^{\infty} \left\{ \underbrace{f_p^{(\mu+)}}_{\text{exciting term}} \text{Re}Z(f_p^{(\mu+)}) - \underbrace{f_p^{(\mu-)}}_{\text{damping term}} \text{Re}Z(f_p^{(\mu-)}) \right\}$$

By optimum tuning, cavity frequency is detuned to lower.
 Detuned frequency $\propto I_b$

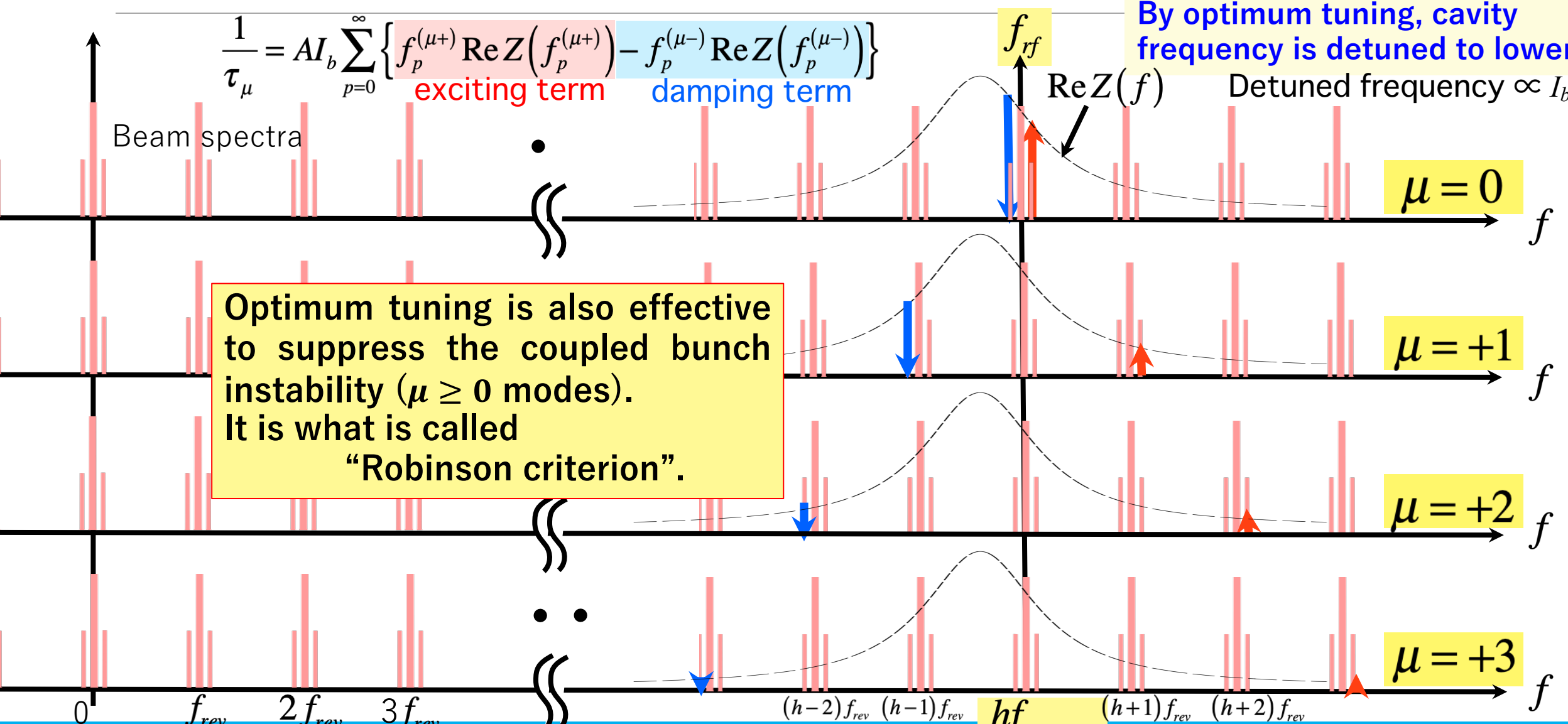


Effect of CBI due to accelerating mode $\mu \geq 0$

$$\frac{1}{\tau_\mu} = AI_b \sum_{p=0}^{\infty} \left\{ \underbrace{f_p^{(\mu+)}}_{\text{exciting term}} \text{Re}Z(f_p^{(\mu+)}) - \underbrace{f_p^{(\mu-)}}_{\text{damping term}} \text{Re}Z(f_p^{(\mu-)}) \right\}$$

By optimum tuning, cavity frequency is detuned to lower.

Detuned frequency $\propto I_b$



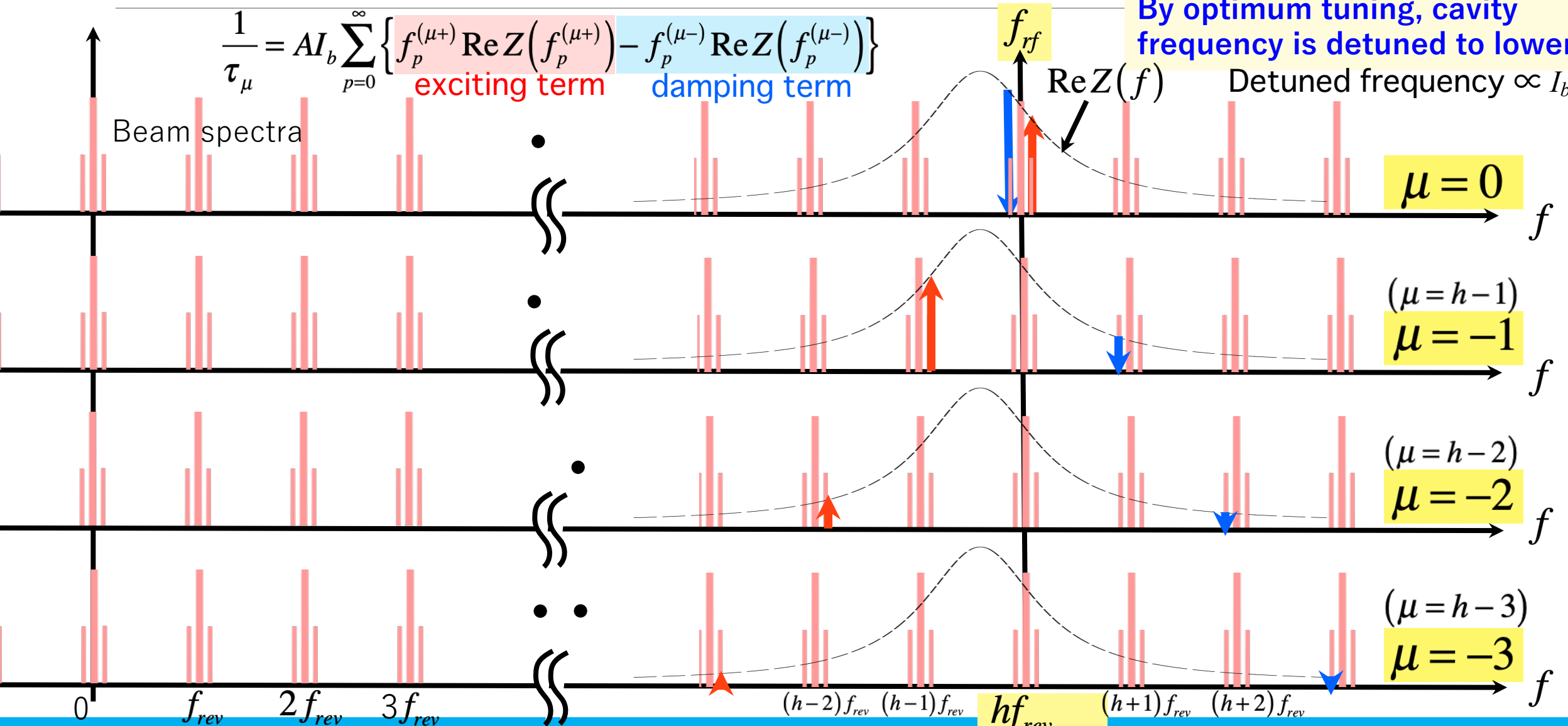
Optimum tuning is also effective to suppress the coupled bunch instability ($\mu \geq 0$ modes). It is what is called "Robinson criterion".

Effect of CBI due to accelerating mode $\mu \leq 0$

$$\frac{1}{\tau_\mu} = A I_b \sum_{p=0}^{\infty} \left\{ \underbrace{f_p^{(\mu+)}}_{\text{exciting term}} \text{Re}Z(f_p^{(\mu+)}) - \underbrace{f_p^{(\mu-)}}_{\text{damping term}} \text{Re}Z(f_p^{(\mu-)}) \right\}$$

By optimum tuning, cavity frequency is detuned to lower.

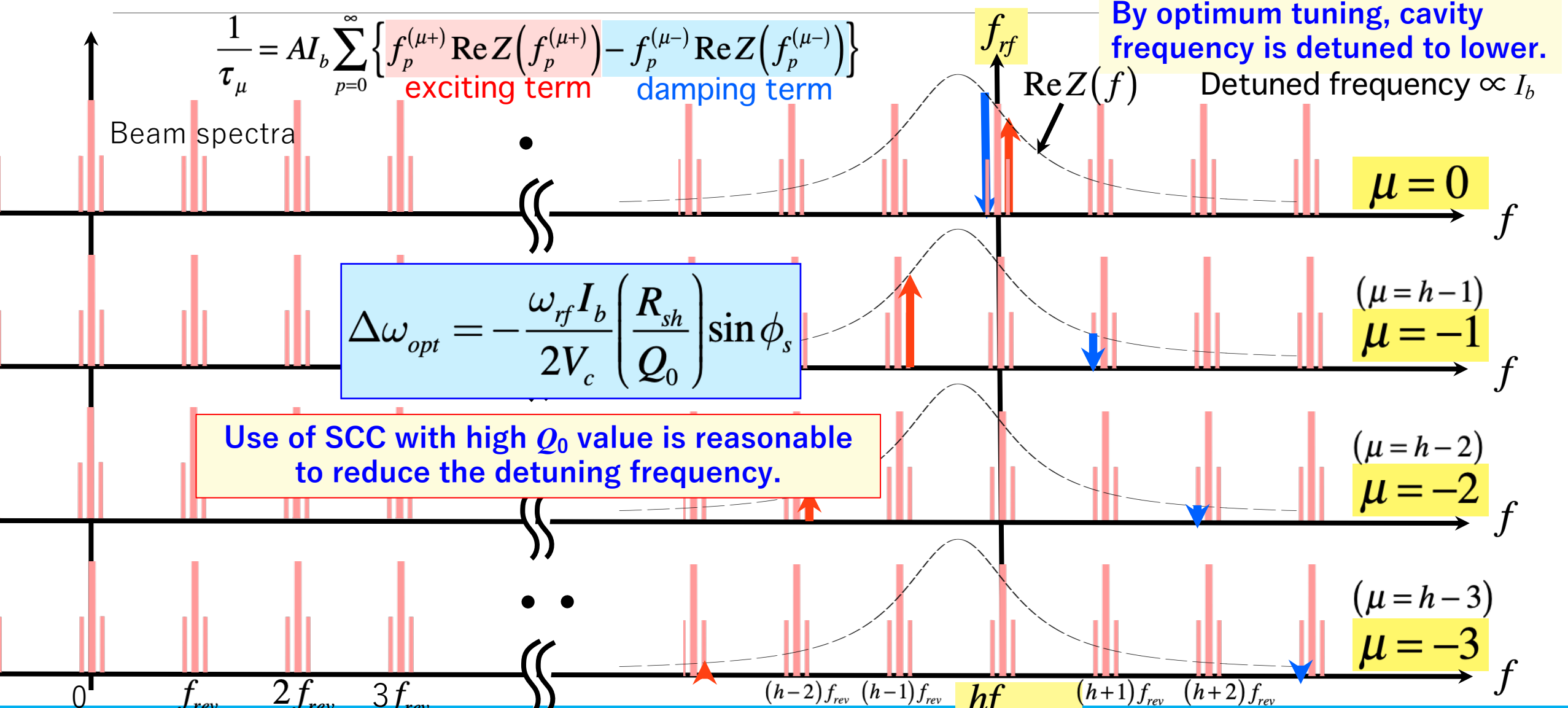
Detuned frequency $\propto I_b$



Effect of CBI due to accelerating mode $\mu \leq 0$

$$\frac{1}{\tau_\mu} = AI_b \sum_{p=0}^{\infty} \left\{ \underbrace{f_p^{(\mu+)}}_{\text{exciting term}} \text{Re}Z(f_p^{(\mu+)}) - \underbrace{f_p^{(\mu-)}}_{\text{damping term}} \text{Re}Z(f_p^{(\mu-)}) \right\}$$

By optimum tuning, cavity frequency is detuned to lower.
 Detuned frequency $\propto I_b$



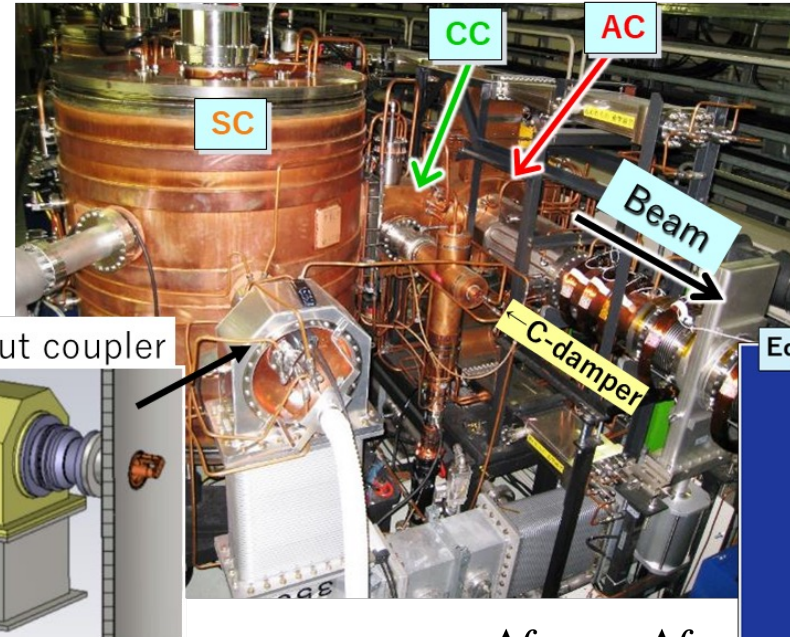
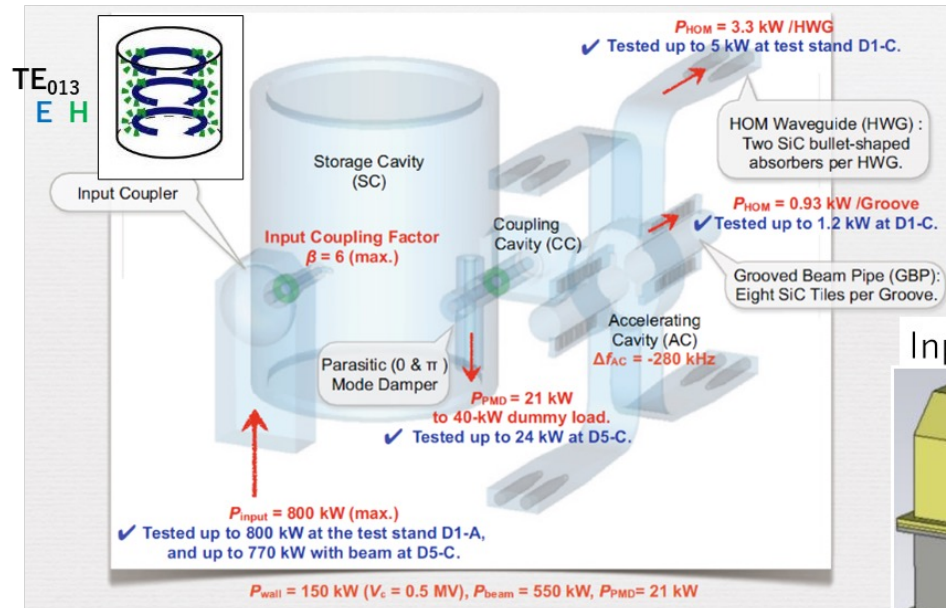
Use of SCC with high Q_0 value is reasonable to reduce the detuning frequency.

$$\Delta\omega_{opt} = -\frac{\omega_{rf} I_b}{2V_c} \left(\frac{R_{sh}}{Q_0} \right) \sin\phi_s$$

Reduction of detuning frequency

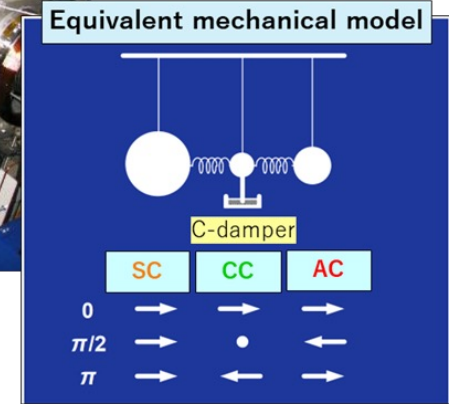
ARES : **A**ccelerator **R**esonantly coupled to **E**nergy **S**torage
 Unique cavity specialized for KEKB

T. Abe



Parameters	
Freq.	509 MHz
R_{sh}/Q_0	15 Ω
Q_0	$\sim 1.1 \times 10^5$
V_c (spec.)	0.5MV/cav.
P_{wall}	150kW

(60kW in AC, 90kW in SC)



- Three-cavity system is stabilized with $\pi/2$ mode operation
 - SC has large stored energy : $U_{sc}/U_{ac} = 9$
 - Optimum detuning of $f_{\pi/2}$ is reduced as $\Delta f_{\pi/2} = \Delta f_{ac}/(1 + U_{sc}/U_{ac})$
 - CBIs driven by the accelerating mode is suppressed.

$$\Delta f_{\pi/2} \sim \frac{\Delta f_a}{1 + \frac{U_s}{U_a}} \sim \frac{\Delta f_a}{10}$$

$$f_{rev} = f_{rf}/h \sim 100\text{kHz}$$

$$\Delta f_a \sim 280\text{kHz} \sim 3 \times f_{rev}$$



$$\Delta f_{\pi/2} \sim 28\text{kHz}$$

at design current

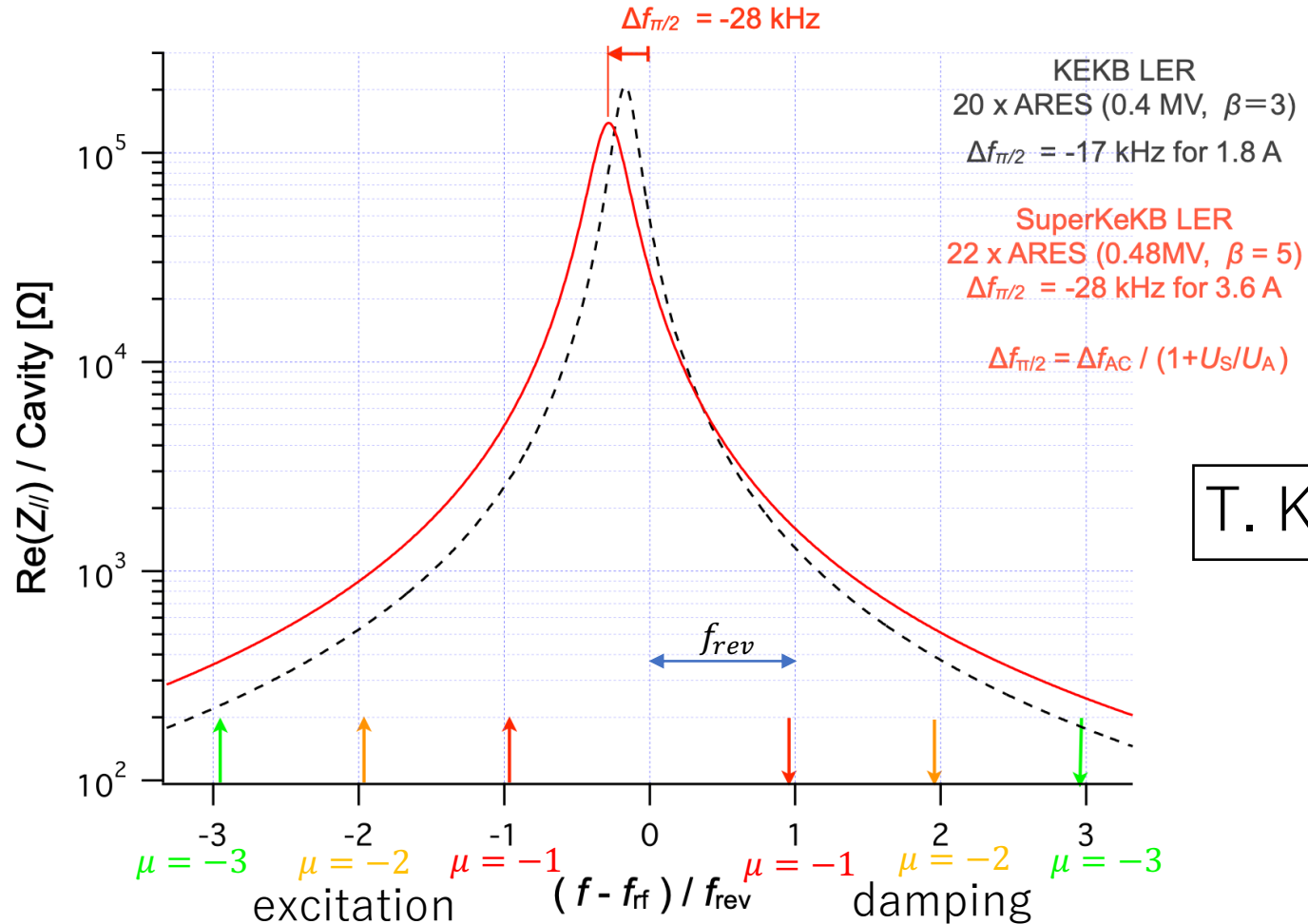
Impedance of ARES

$$\Delta f_a \sim 280\text{kHz}$$



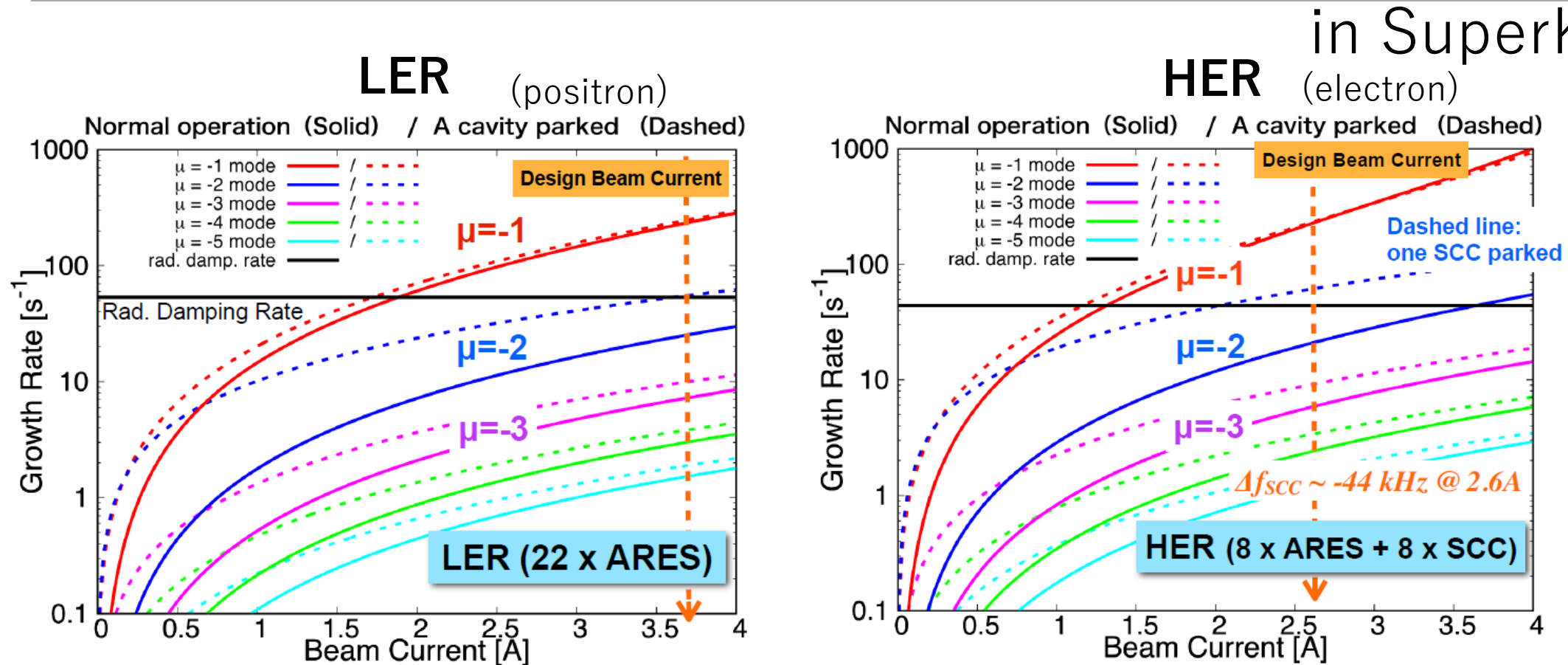
$$\Delta f_{\pi/2} \sim 28\text{kHz}$$

$$f_{rev} = f_{rf}/h \sim 100\text{kHz}$$



T. Kageyama

Estimation of the growth rates of CBI

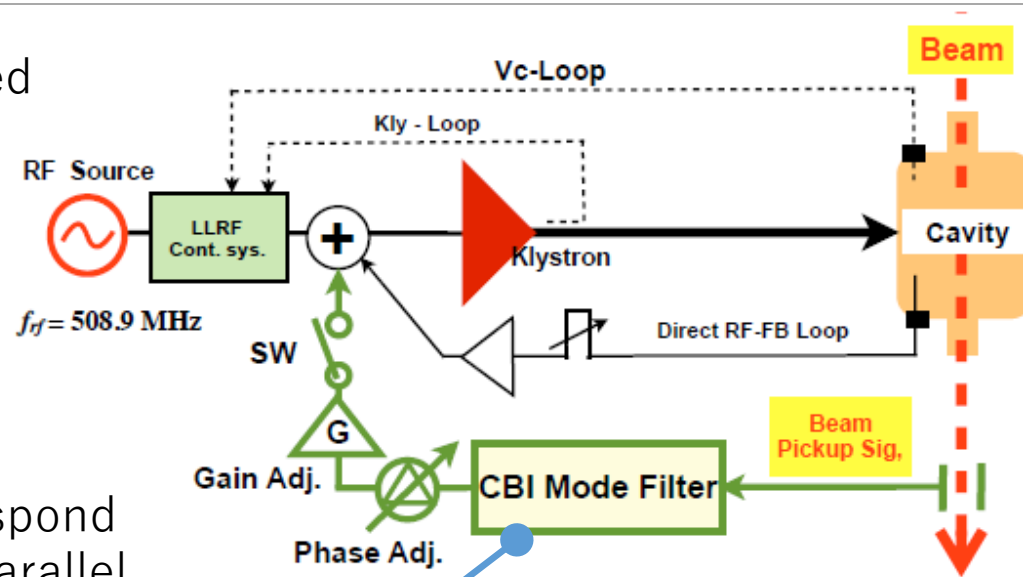
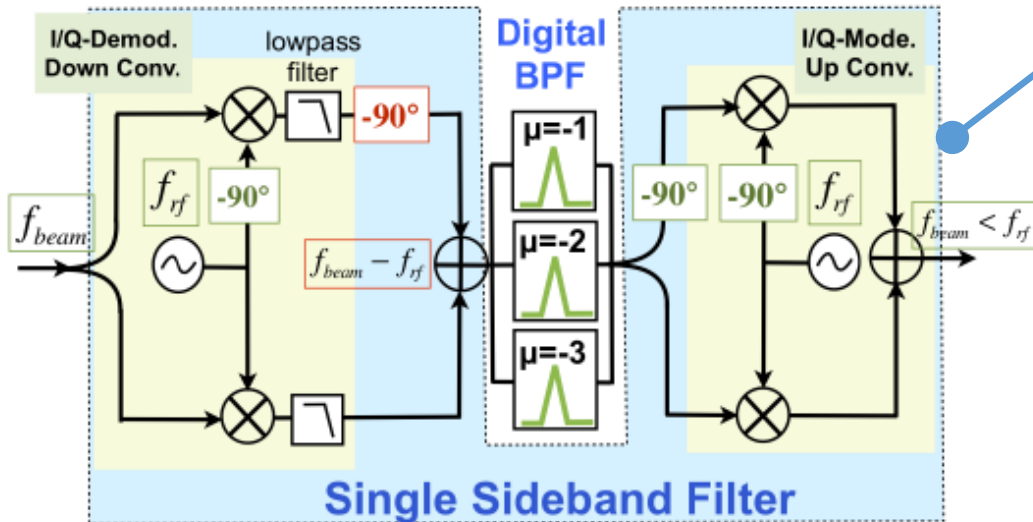


Threshold currents for $\mu=-1$ mode are quite below the design current.
 When there are parked cavities, $\mu=-2$ mode also has no margin.
 New CBI damper system has been developed and installed.

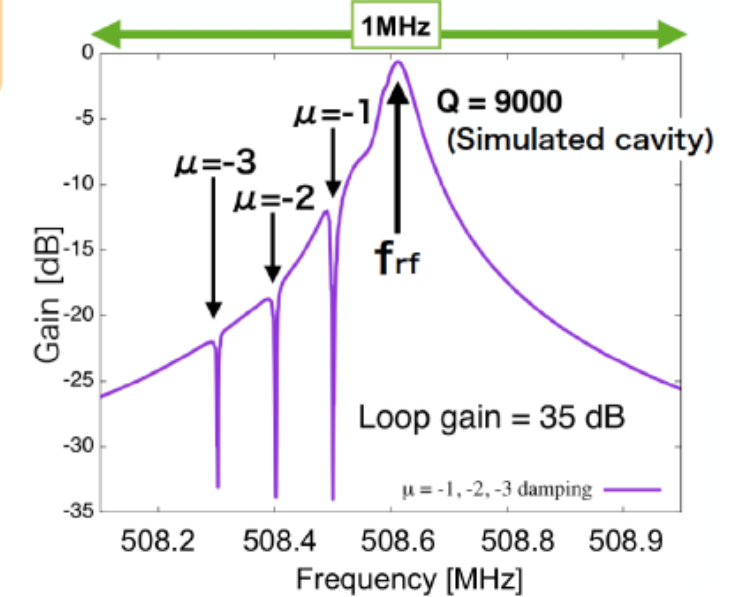
CBI damper system in SuperKEKB

The damper system is installed to ARES station with digital LLRF system.

The damper system can correspond to $\mu = -1, -2$ and -3 modes in parallel.



FB loop test result of the CBI mode filter for a simulant cavity

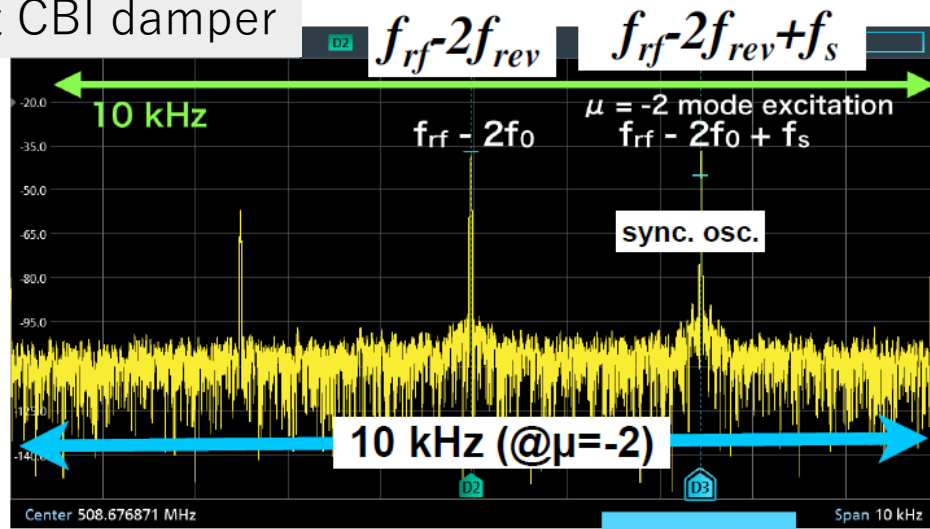


Impedance correspond $\mu = -1, -2$ and -3 modes are suppressed successfully.

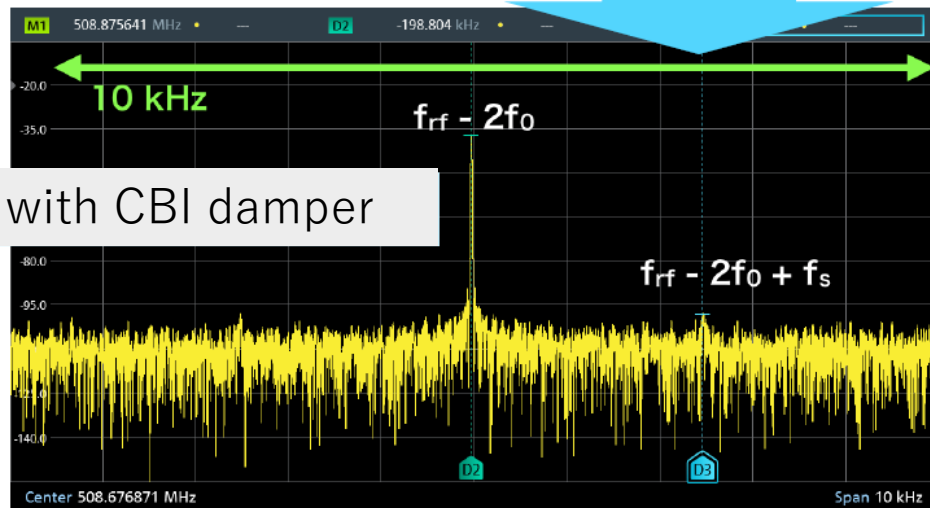
The new CBI damper system is working in SuperKEKB LER and HER.

Example of CBI damper operation

without CBI damper



with CBI damper

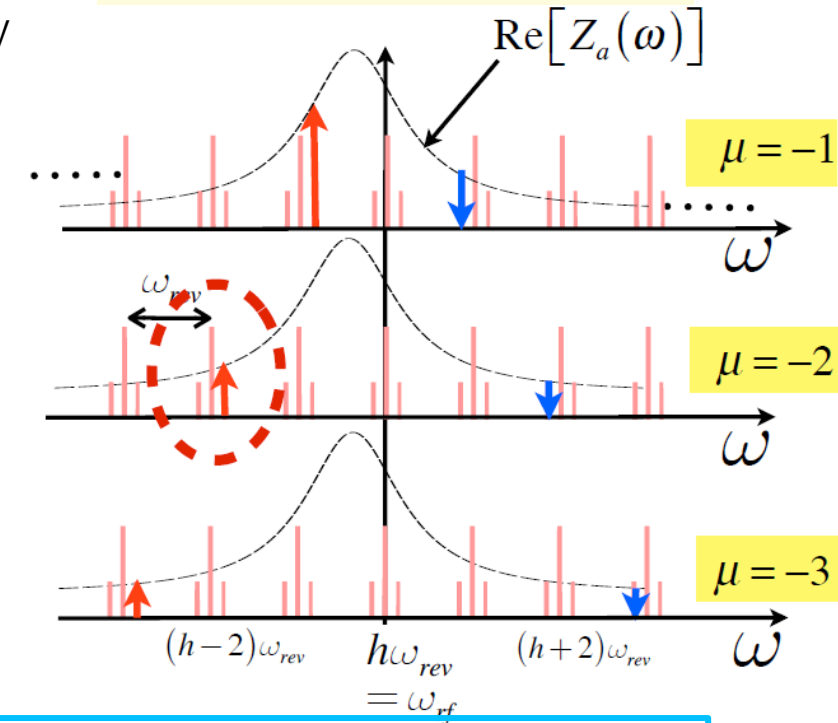


$\mu = -2$ mode was excited purposely by large detuning SCC.

Peak disappeared by CBI damper.

in SuperKEKB

By optimum tuning, cavity frequency is detuned to lower.



CBI is not a problem with this damper systems in beam operation so far.
 $(I_{b,max} : 1.46 \text{ A for LER and } 1.14 \text{ A for HER})$

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Static Robinson Instability

K.Akai, PRAB 25, 102002 (2022).

T.Yamaguchi et al., PRAB 26, 044401 (2023).

This is another type of longitudinal instability related to the accelerating mode.

This instability arises from the **coherent synchrotron oscillation** where all bunches oscillate in the **same phase (zero-mode)**.

This instability limits the **maximum beam current** stored in high-current ring accelerators.

Coherent oscillation (zero-mode)

V_b shifts with the bunch phase
No contribution for restoring force

Only V_k contributes to the restoring force.
 θ_k is considered as the synchronous phase.

Restoring force decreases and
synchrotron frequency decreases.

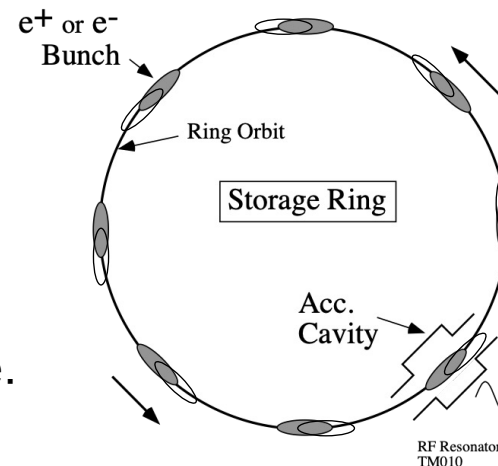
$$\begin{aligned} \text{When } \omega_s \rightarrow \omega'_s, \\ \left(\frac{\omega'_s}{\omega_s}\right)^2 &= \frac{V_k \sin \theta_k}{V_c \sin \phi_s} = \frac{V_{kr} \cos \psi_{opt} \sin(\phi_s + \psi_{opt})}{V_c \sin \phi_s} \\ &= \frac{1 - \{(V_{br}/V_c) \cos \phi_s\}^2}{1 + \{(V_{br}/V_c) \sin \phi_s\}^2} \end{aligned}$$

$\omega'_s > 0$

Stable condition

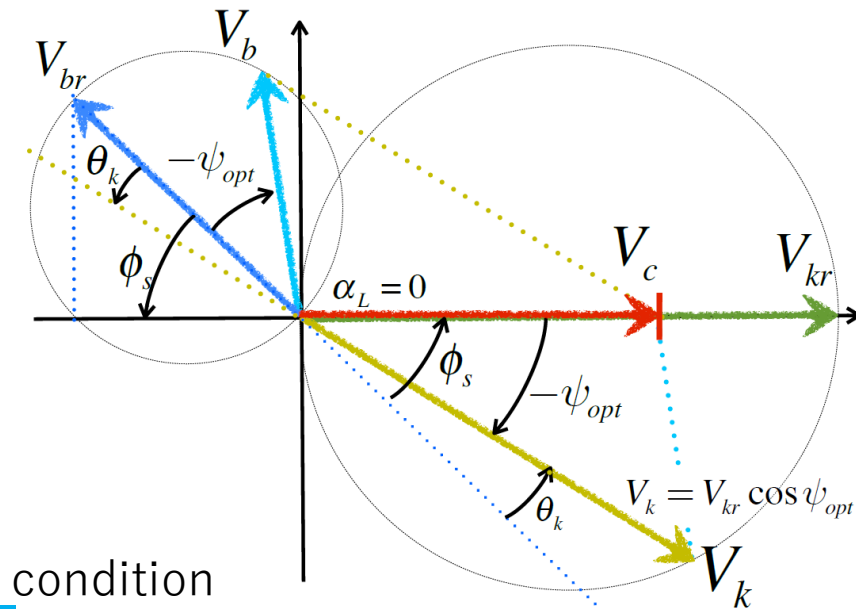
$$\begin{aligned} V_{br} \cos \phi_s &< V_c \\ \frac{P_b}{P_c} &< \beta + 1 \end{aligned}$$

satisfied in the optimum coupling condition



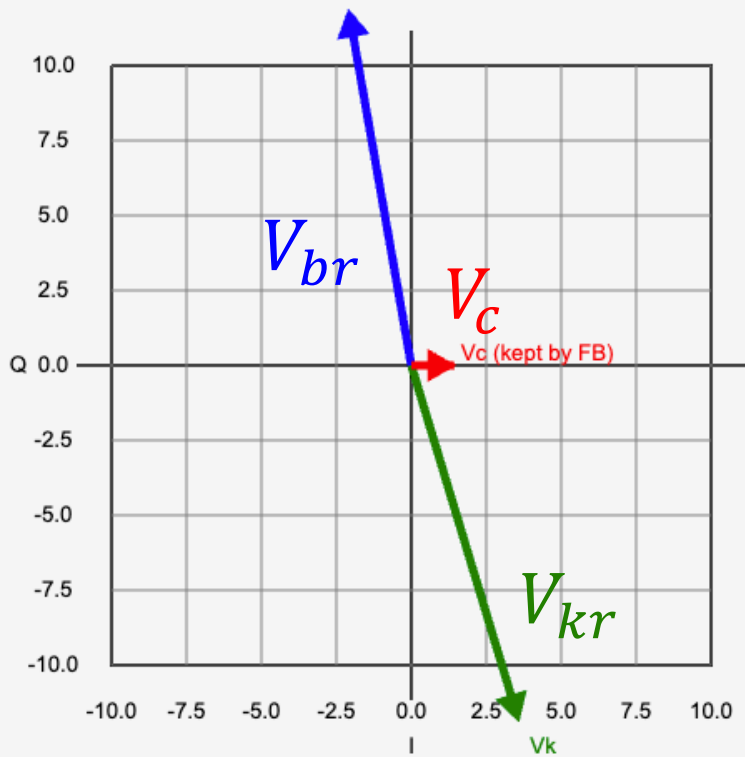
Coherent synchrotron oscillation
(all bunches oscillate in the same phase)

Principle of phase stability dose not work.



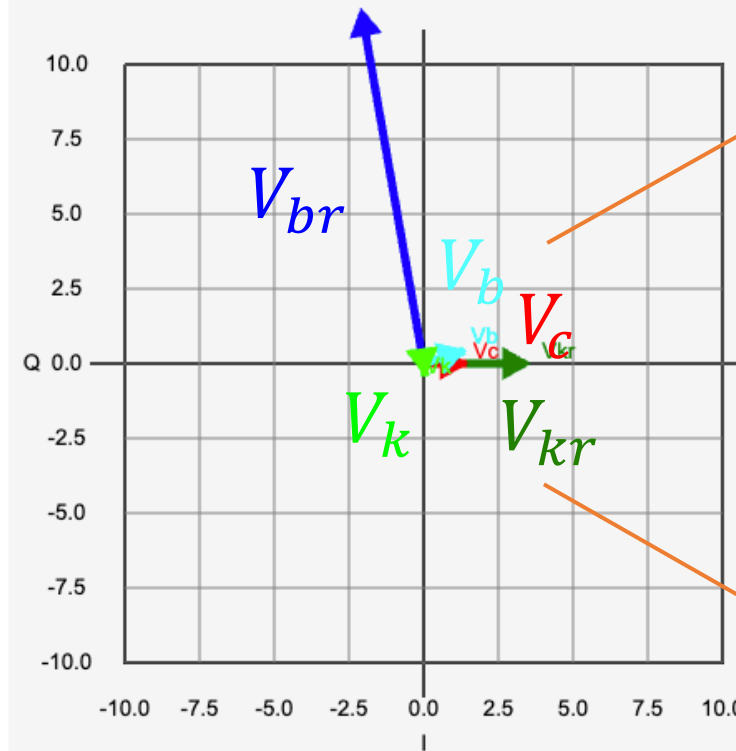
Efficiency by optimum tuning

Example of SuperKEKB, SCC operating parameters in $I_b=2.6$ A



Beam Current: 2600 mA

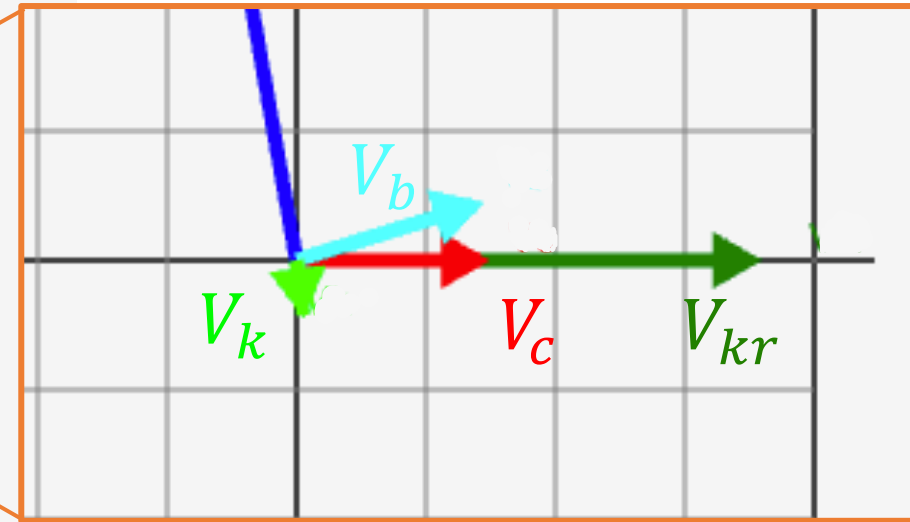
without Optimum tuning



Beam Current: 2600 mA

with Optimum tuning

zoomed up



V_c is generated by V_b !!

If V_b cannot contribute to restoring force of the synchrotron oscillation, V_k alone cannot maintain stable oscillation.

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Necessity of HOM damping

The beam also excites the RF field of higher-order modes (HOMs) in the cavity.

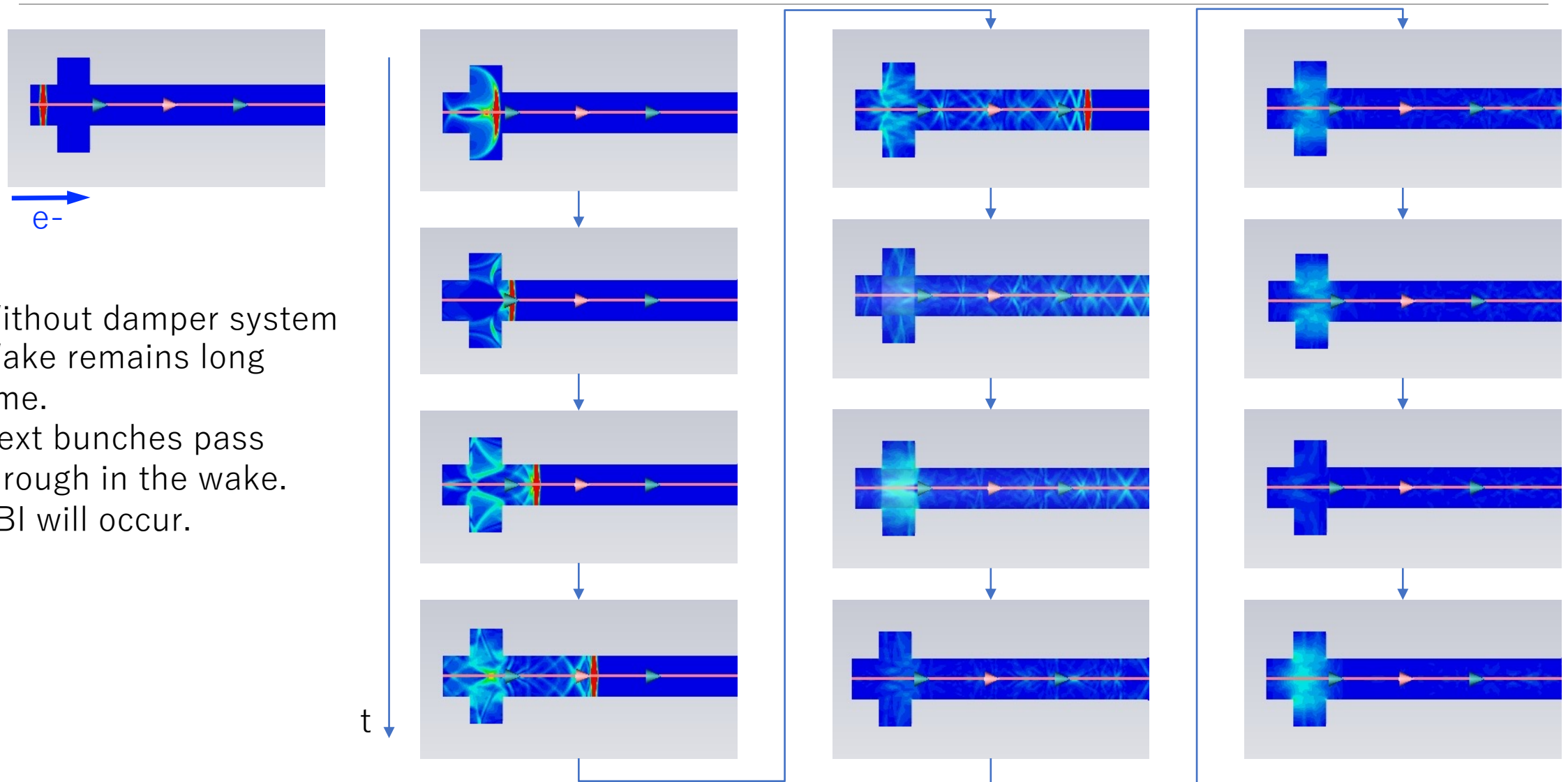
HOM fields accumulate in multi passes of bunches and finally causes instability and loss of the beam.

Since the beam-induced field is proportional to the beam intensity, the problem due to HOMs becomes more serious in high-intensity accelerators.

In addition, the beam in high-current storage rings is distributed in a large number of bunches.

HOMs have to be damped sufficiently to avoid multi-bunch instabilities.

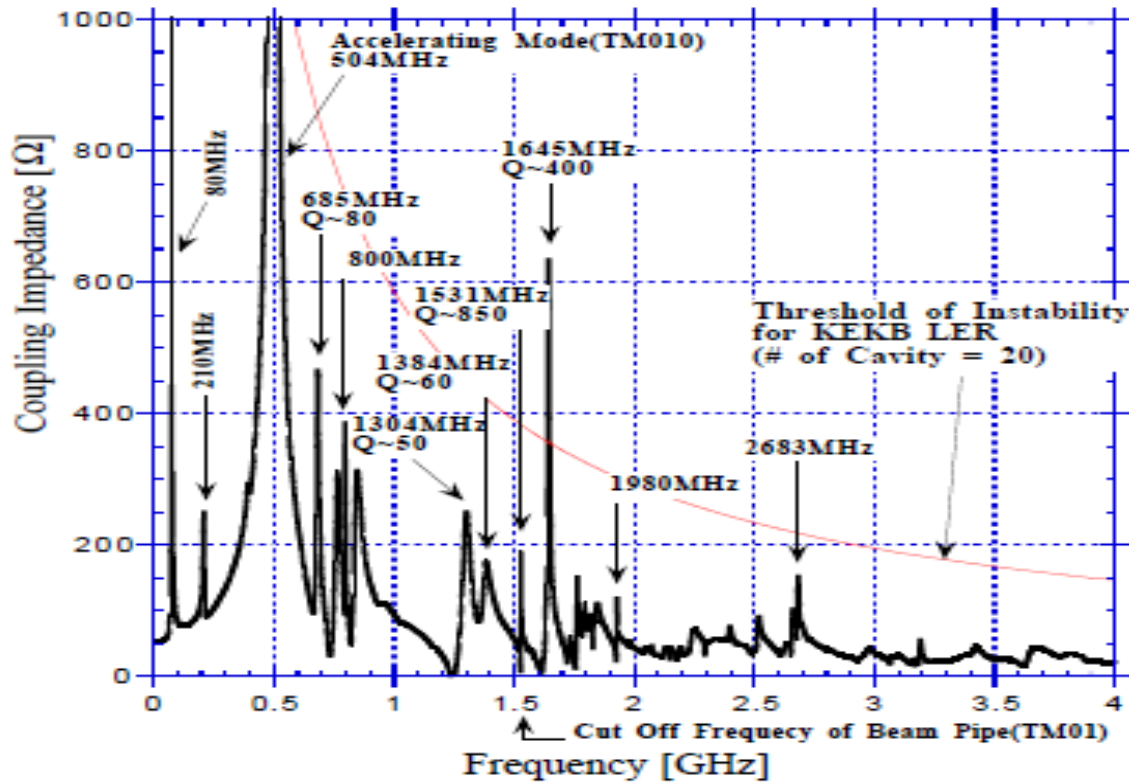
Wakefield in pillbox



- Without damper system
- Wake remains long time.
- Next bunches pass through in the wake.
- CBI will occur.

Reduction of coupling impedance of cavity

Example of Longitudinal Coupling Impedance
(KEKB-LER, ARES, T. Kobayashi et al, 1997)



(ARES, HOM-damped cavity with SiC damper)

In principle, the beam instability could be avoided by just tuning a few dangerous modes at safe frequencies (somewhere between the driving frequencies of the instabilities).

But, in a large ring with high current beam, it is unrealistic to tune all dangerous modes at the same time.

In SuperKEKB,

- $f_{rev} \sim 100$ kHz (beam spectra are everywhere)
- Various bunch filling pattern (beam spectrum is not fixed)

The impedance of longitudinal and transverse HOMs must be sufficiently reduced to satisfy $\tau_g^{-1} < \tau_{rd}^{-1}$.

τ_{rd}^{-1} : Radiation damping rate

τ_g^{-1} : Growth rate of instability
due to each HOM mode

The evaluation of the growth rate is the same as the CBI.

How to Reduce Impedance of Cavity

The **reduction of R/Q and Q_L of dangerous HOM modes** in cavity is important.

How to reduce R/Q and Q_L of HOMs in SC cavities,

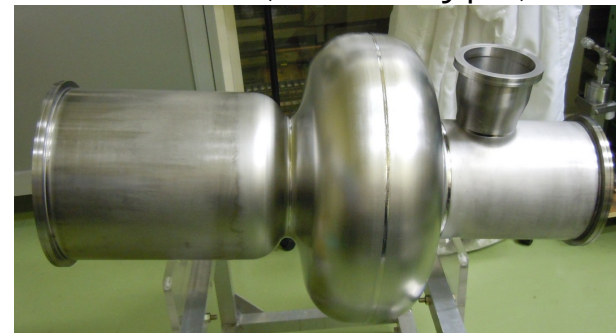
1. Use of couplers (antennas, loops, waveguides) to extract HOMs
2. Design of HOM damped structure

- single-cell
- large-diameter beam pipes with absorbers

To damp lower frequency dipole modes,

- further enlarge beam pipe (KEKB)
- fluted beam pipe (CESR-B)

TPS (KEKB-type)

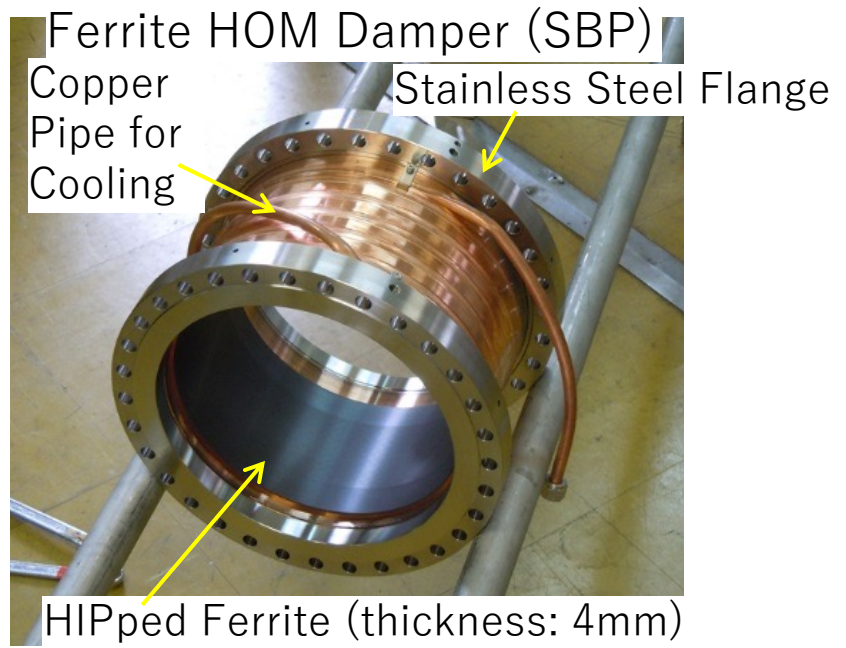
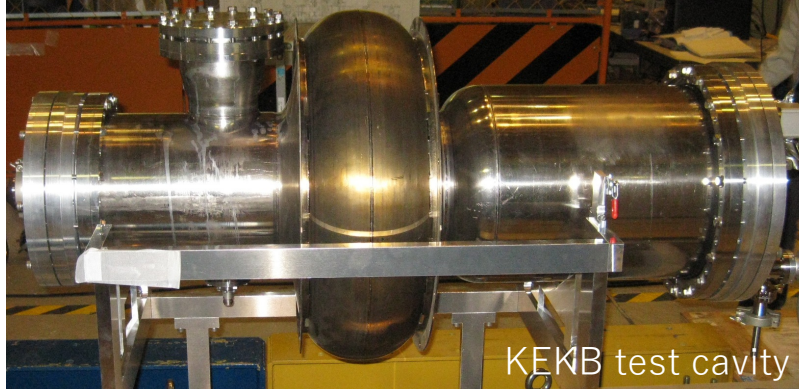
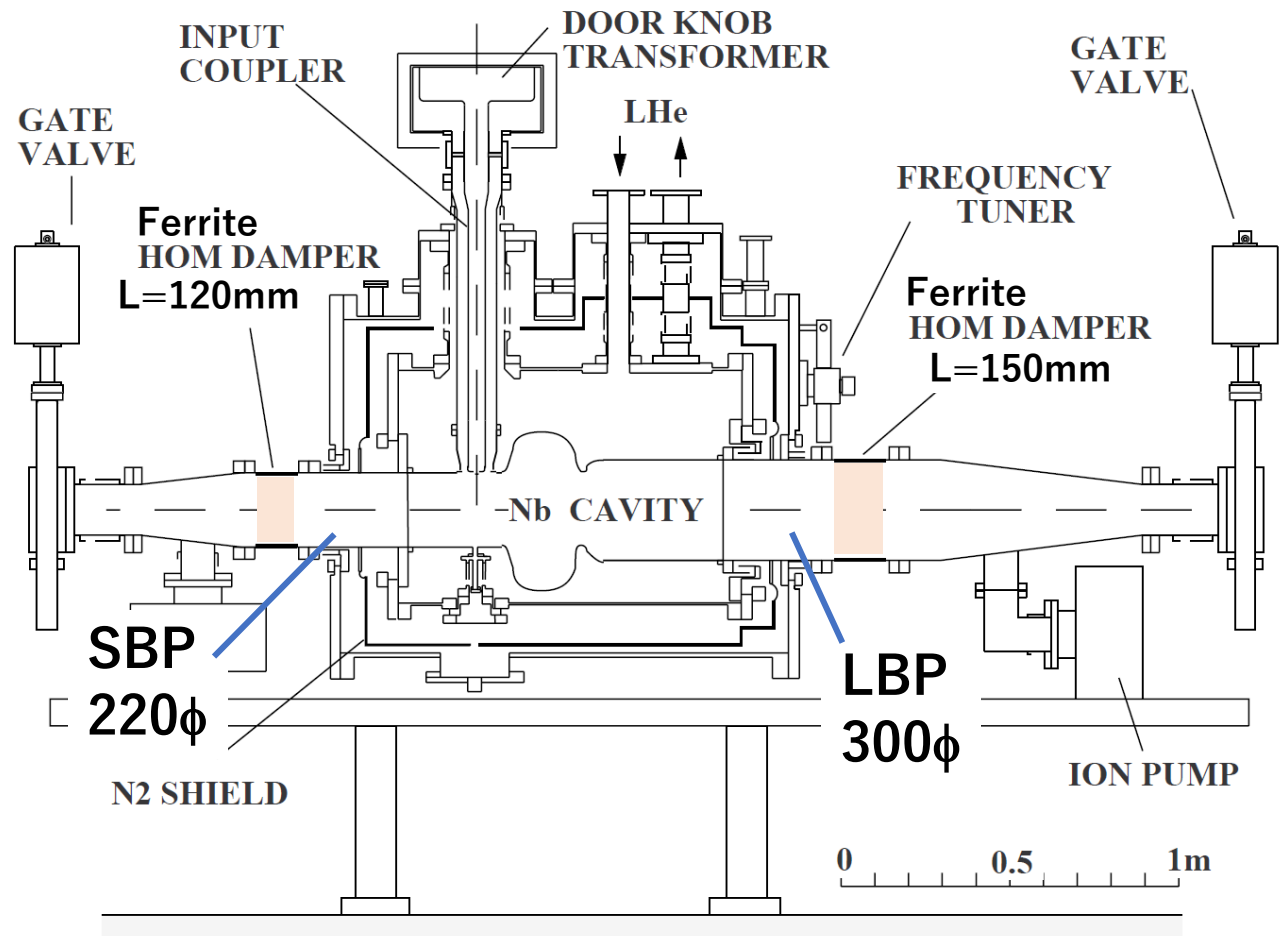


CESR



SCC is designed for KEKB and still operating in SuperKEKB.

Optimized cell shape and ferrite absorbers



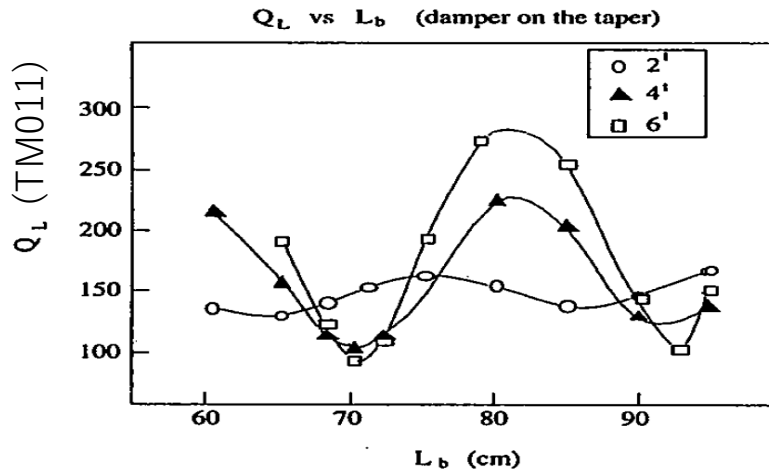
The HOM damping characteristics of absorbers depend on the geometrical parameters.

Optimization of ferrite dampers

Optimized parameters

- Distance from cavity
- Length and thickness of ferrite
- Taper shape between dampers and beam duct
- etc...

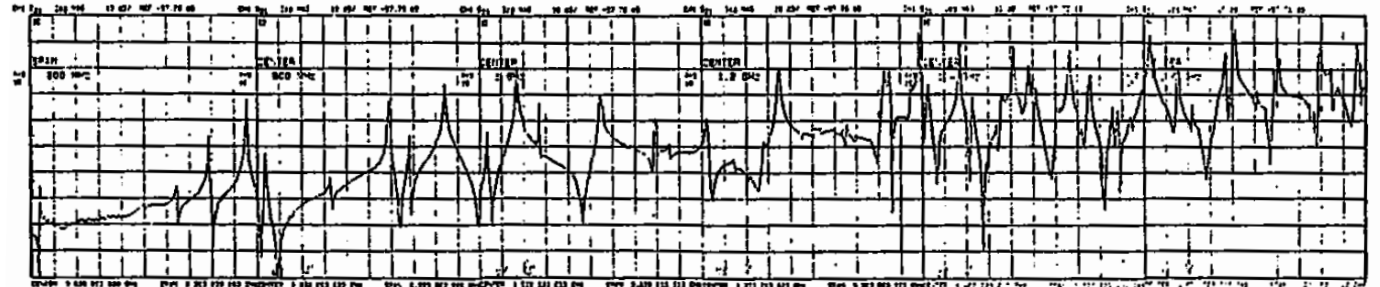
Example of Q_L calculation by SEAFISH



Distance between cavity cell and ferrite

HOM spectra obtained by network analyzer

a) Al model cavity **without Ferrite**

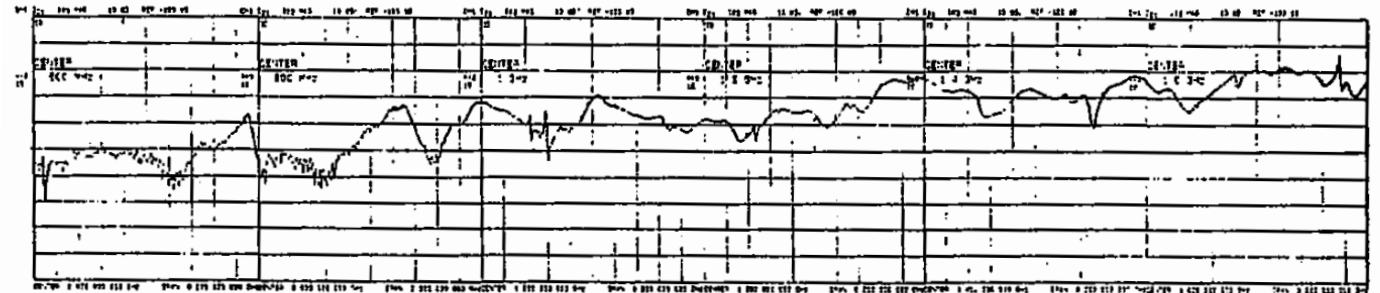


0.5 GHz

3 GHz

optimization

b) Nb cavity **with Ferrite**



0.5 GHz

3 GHz

Many modes are damped by optimization.

Optimization of ferrite dampers

Measured frequencies and Q_L values of HOM modes with optimized ferrite dampers

(a) Monopole

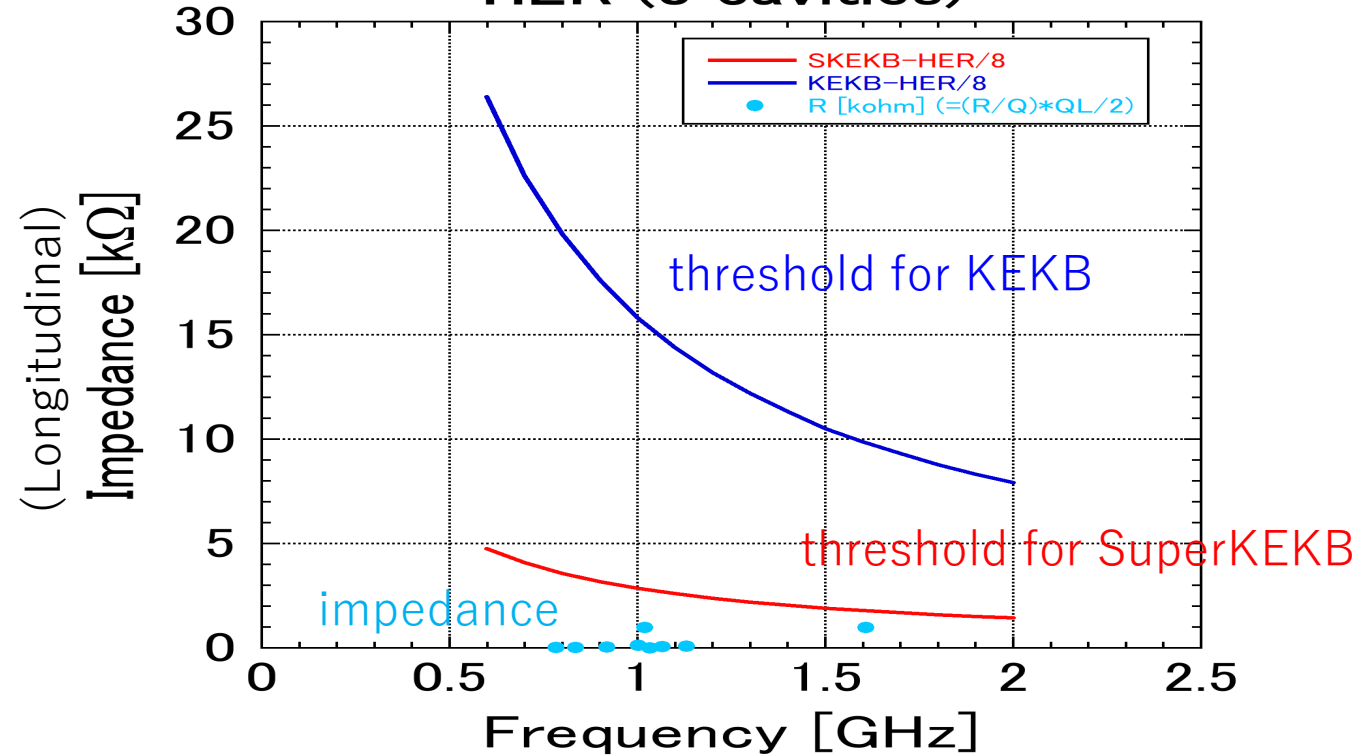
mode	Frequency	R/Q	Q
LBP/TM ₀₁	782.7460	0.29000	196
LBP/TM ₀₁	834.3830	0.32200	100
LBP/TM ₀₁	918.9360	1.20460	60
LBP/TM ₀₁	1002.5400	5.79200	42
TM ₀₁₁	1018.3800	11.58400	170
TM ₀₂₀	1032.7500	1.48260	15
SBP/TM ₀₁	1065.8199	1.23480	106
SBP/TM ₀₁	1130.9900	2.20000	74
TM ₀₃₀	1607.5200	6.48800	300

(b) Dipole

Mode	MHz	$(R/Q)'$	Q
LBP-TE ₁₁	606.2090	1.84	97
LBP-TE ₁₁	628.6900	33.78	90
LBP-TE ₁₁	654.0472	39.59	129
LBP-TE ₁₁	684.9373	152.47	86
TM ₁₁₀	701.3156	245.03	150
SBP-TE ₁₁	813.2880	6.34	74
TE ₁₁	1023.6659	2.96	50

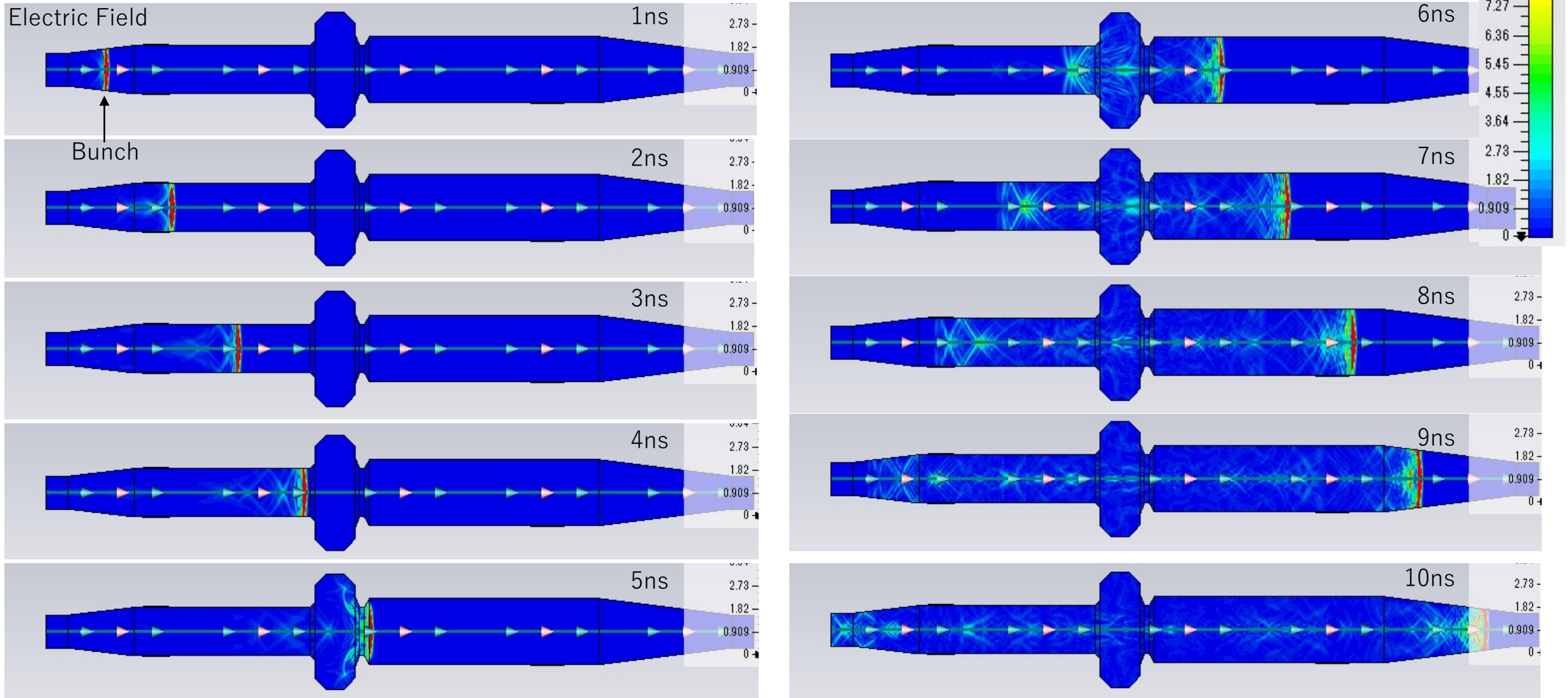
R/Q and $(R/Q)'$ are calculated values.

Impedance Threshold for KEKB and SuperKEKB HER (8 cavities)



The impedance of HOM modes were successfully reduced by the HOM damped structure and optimization of the ferrite dampers.

Wake field in KEKB SC cavity



HOM load estimation

- HOM power damped by ferrite damper changes to heat load.
 - To estimate the heat load of ferrite damper, broad-band HOMs have to be also considered.
 - The **shorter-length bunch** has higher frequency modes.
 - Higher frequency HOM modes can be excited.
- ➔ Total heat load becomes larger.

Loss Factor k

$$k = \sum_n k_n = \sum_n \frac{\omega_n}{4} \left(\frac{R}{Q} \right)_n$$

(A sum of individual loss factors of HOM modes)

Total power loss of beam bunches

$$P_{total} = k \cdot q \cdot I_b$$

(Bunch charge q
Average beam current I_b)

example
100mA, 1V/pC, 1nC
=> 100W

$$P_{total}[\text{kW}] = k[\text{V/pC}] \frac{(I_b[\text{mA}])^2}{N_b \cdot f_{rev} [\text{kHz}]}$$

(Number of bunches N_b
Revolution frequency f_{rev})

HOM load can be estimated by Loss Factor of the cavity system.

We can get the loss factor including dampers by using simulation codes such as CST particle studio.

Contents

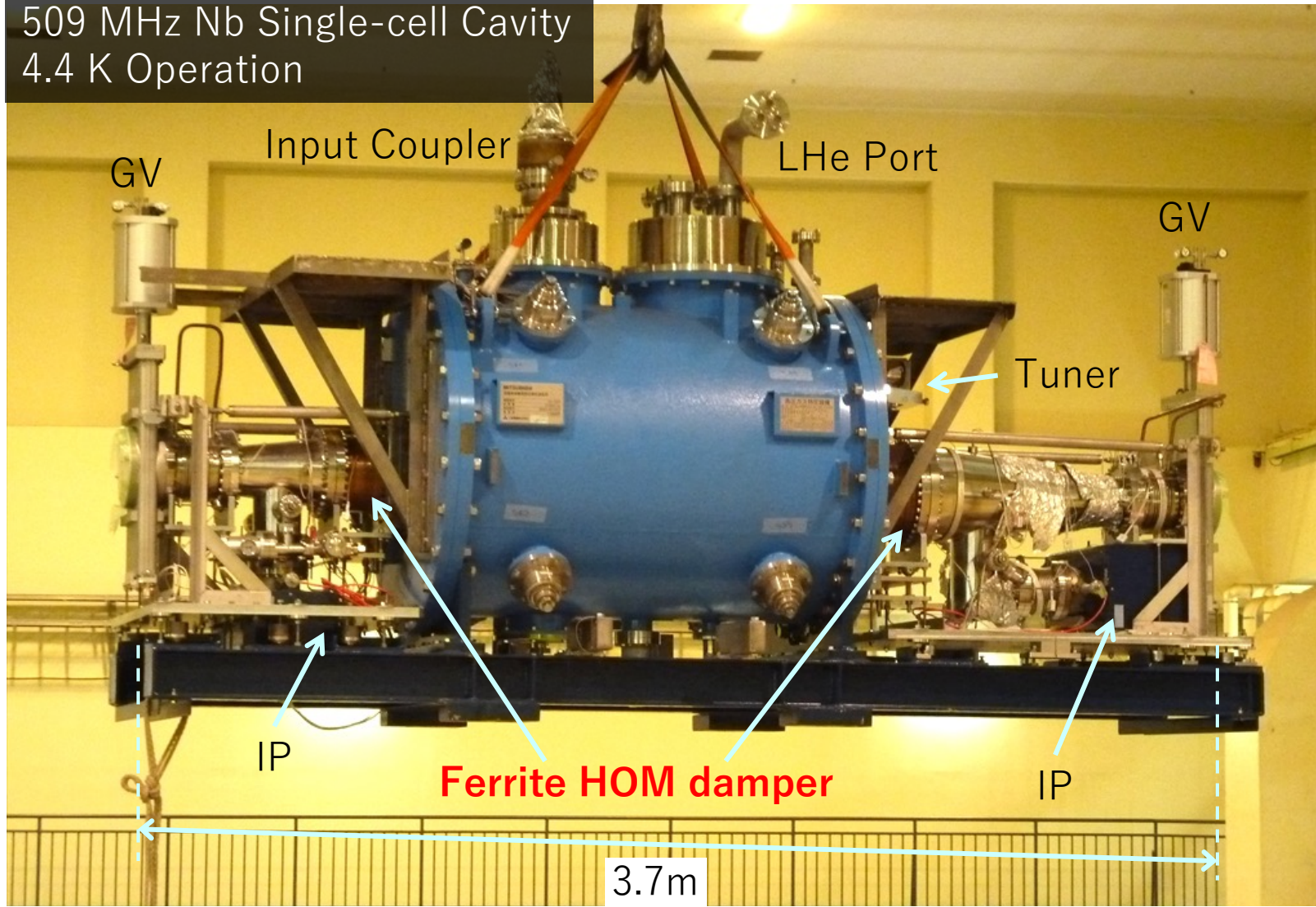
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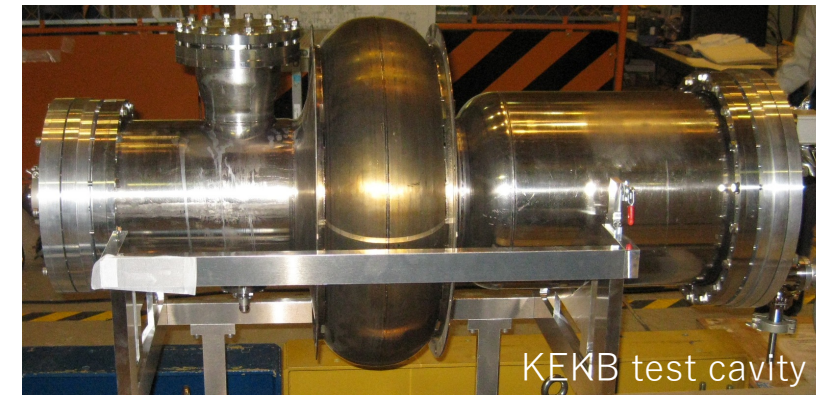
- ◆ HOM
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SCC Module of SuperKEKB

509 MHz Nb Single-cell Cavity
4.4 K Operation

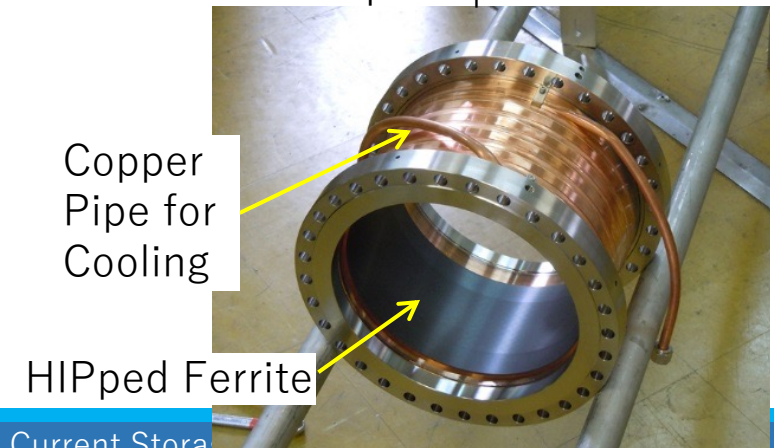


8 cavities in the electron ring



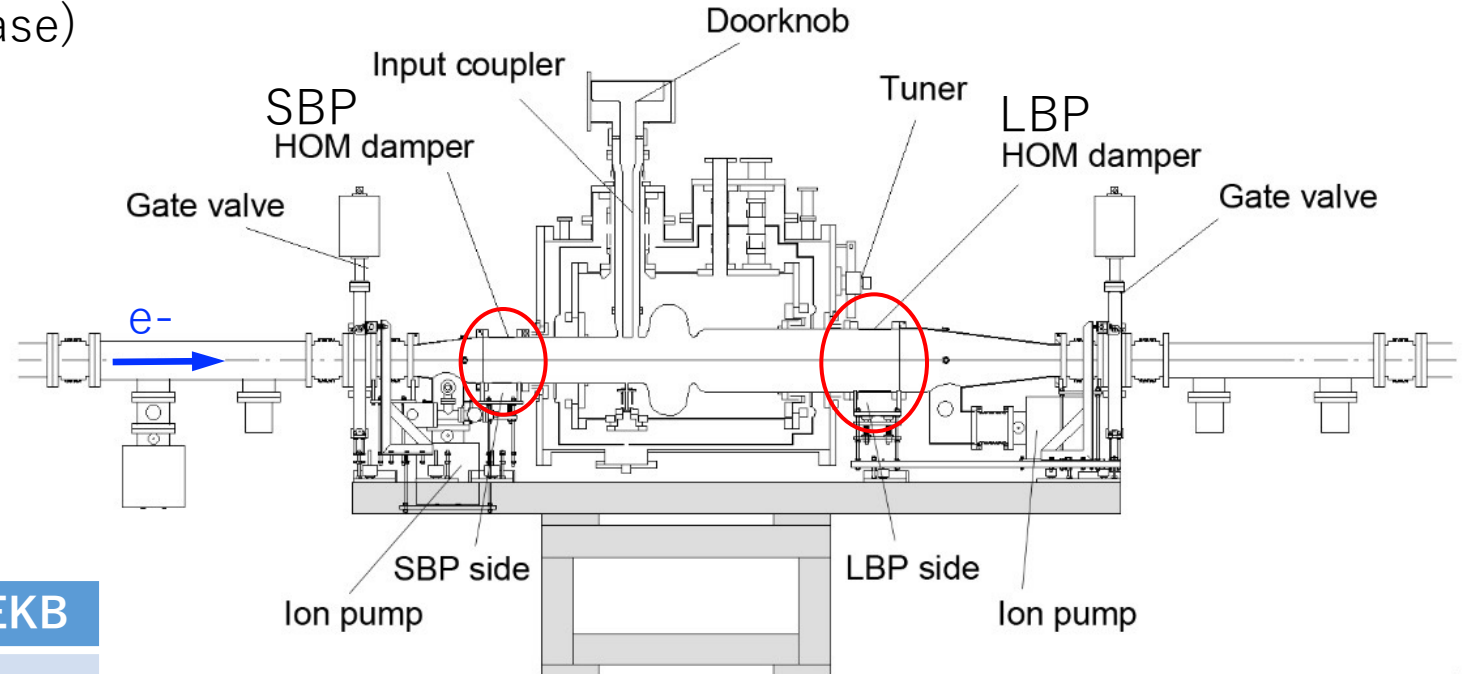
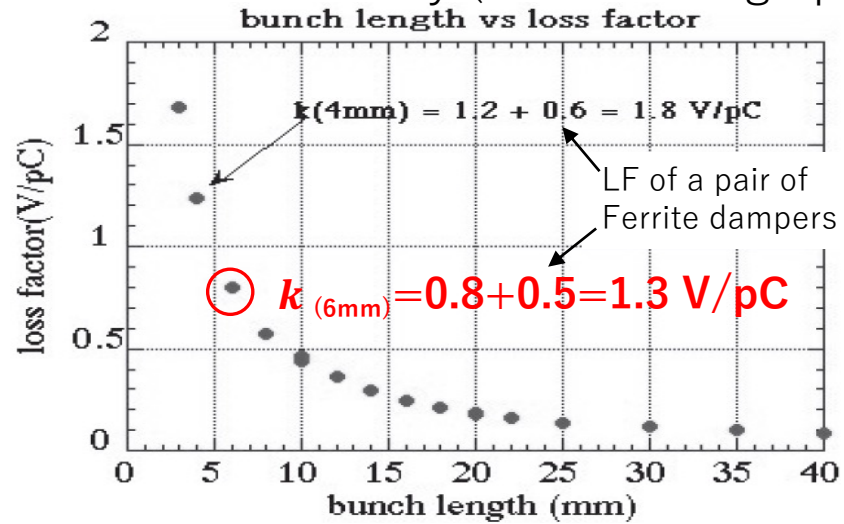
A Pair of Ferrite HOM dampers

- SBP damper : $\phi 220 \times t4 \times L120$
- LBP damper : $\phi 300 \times t4 \times L150$



HOM power in KEKB operation

Loss Factor of Cavity (calc. in design phase)

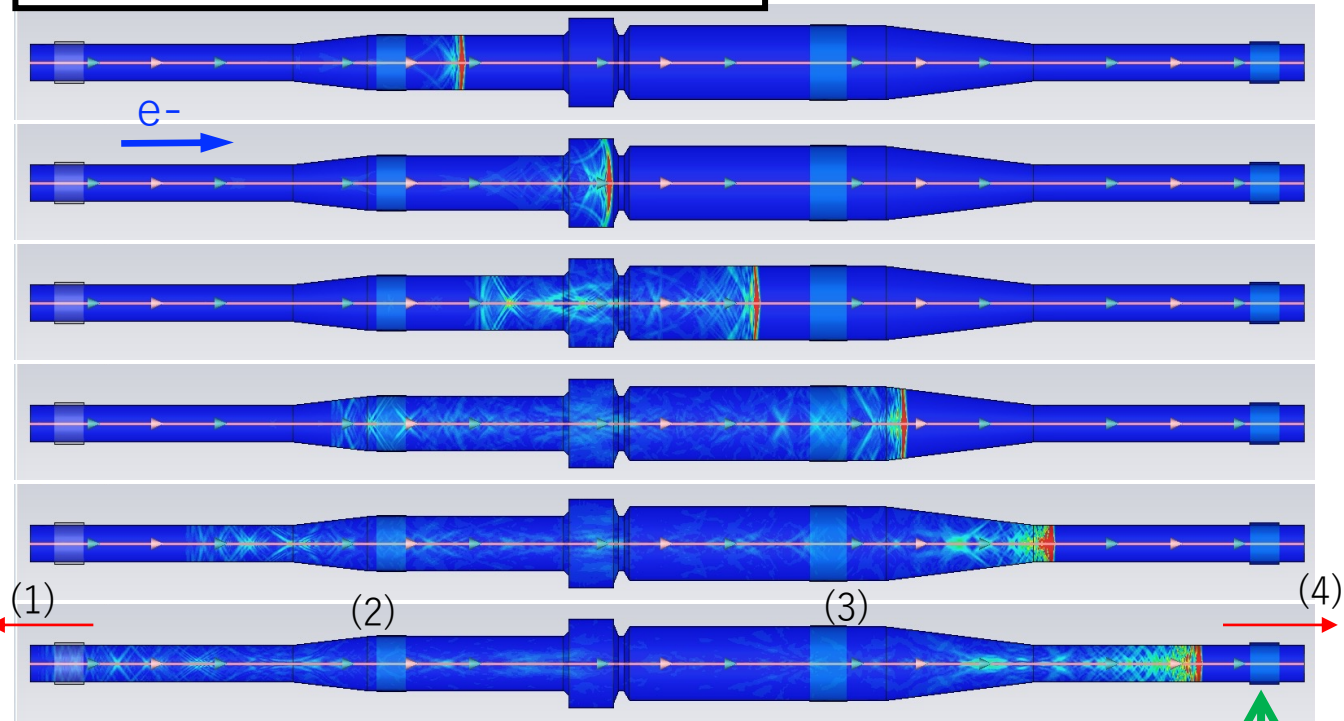


KEKB Operation Parameters		SuperKEKB
Beam current	1.4 A	2.6 A
Bunch charge	~10 nC	10.4 nC
Bunch length	6 mm	5 mm
Loss factor	1.3 V/pC	1.38 V/pC
Power loss of beam in a cavity module	18 kW <small>calculated</small>	37 kW
Absorbed power by a pair of ferrite dampers	16 kW <small>measured</small>	(design) LF : calc. by CST

- The **calculated power loss** of beam from the loss factor and the **measured absorbed power** by the ferrite dampers **were roughly matched**. The loss factors were almost consistent.
- In SuperKEKB, enormous large HOM power is expected due to twice high beam current and shorter bunch length. Further measures are required.

Study of HOM power for SuperKEKB

Wake field simulation using CST Particle Studio



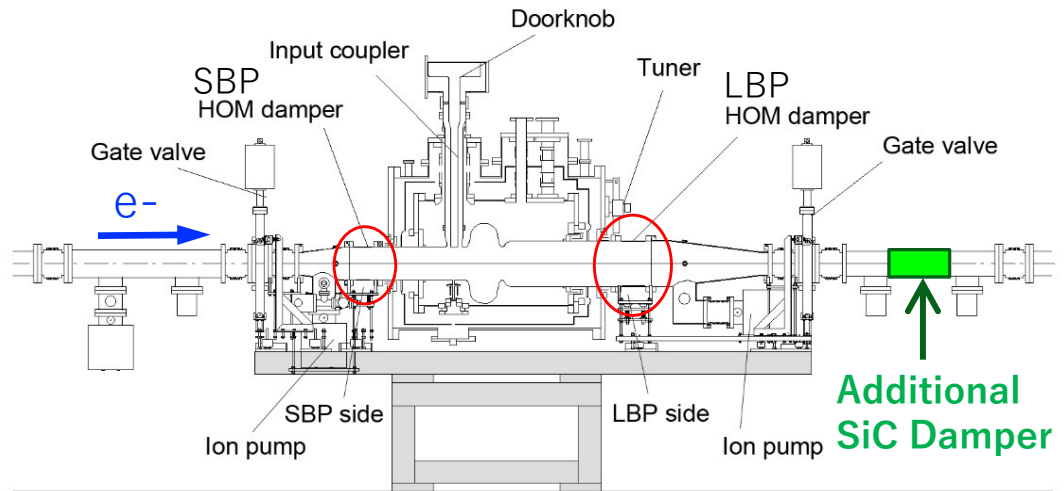
Power ...	ratio
(1) emit through upstream pipe	0.03
(2) absorbed by SBP ferrite damper	0.23
(3) absorbed by LBP ferrite damper	0.32
(4) emit trough downstream pipe	0.41

Additional SiC Damper

- Much HOM power emit through the downstream beam pipe.
- The emitted power becomes the additional load of the next cavity's ferrite dampers.

➡ Additional SiC Damper

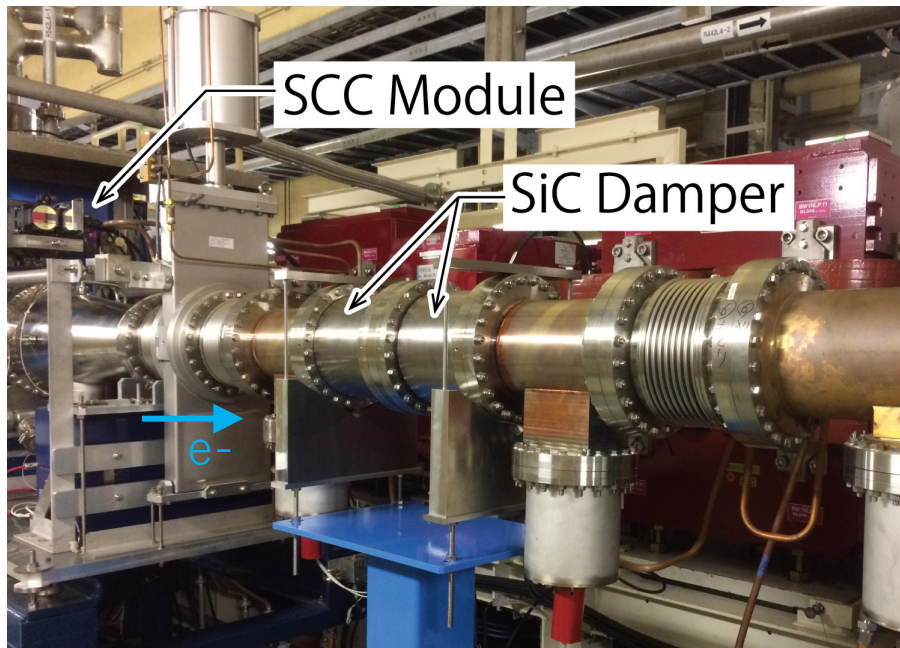
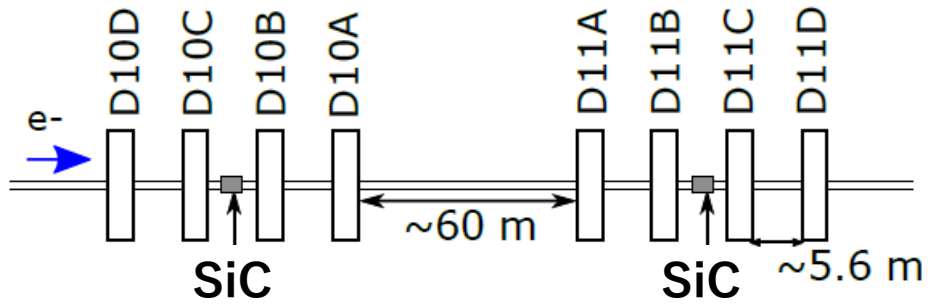
- The emission power is reduced to one-third in the simulation study.



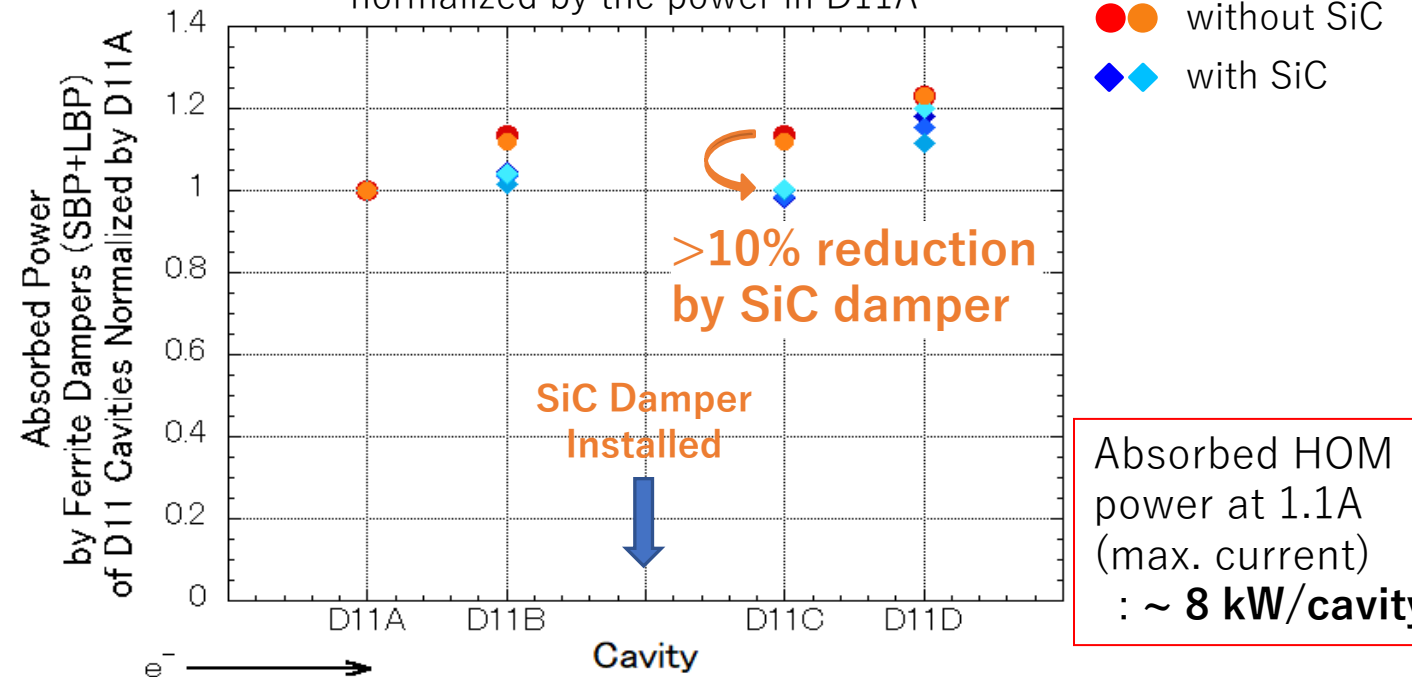
Additional SiC Damper

Results of Beam Test with SiC Damper

Layout of 8 SCCs and SiC dampers

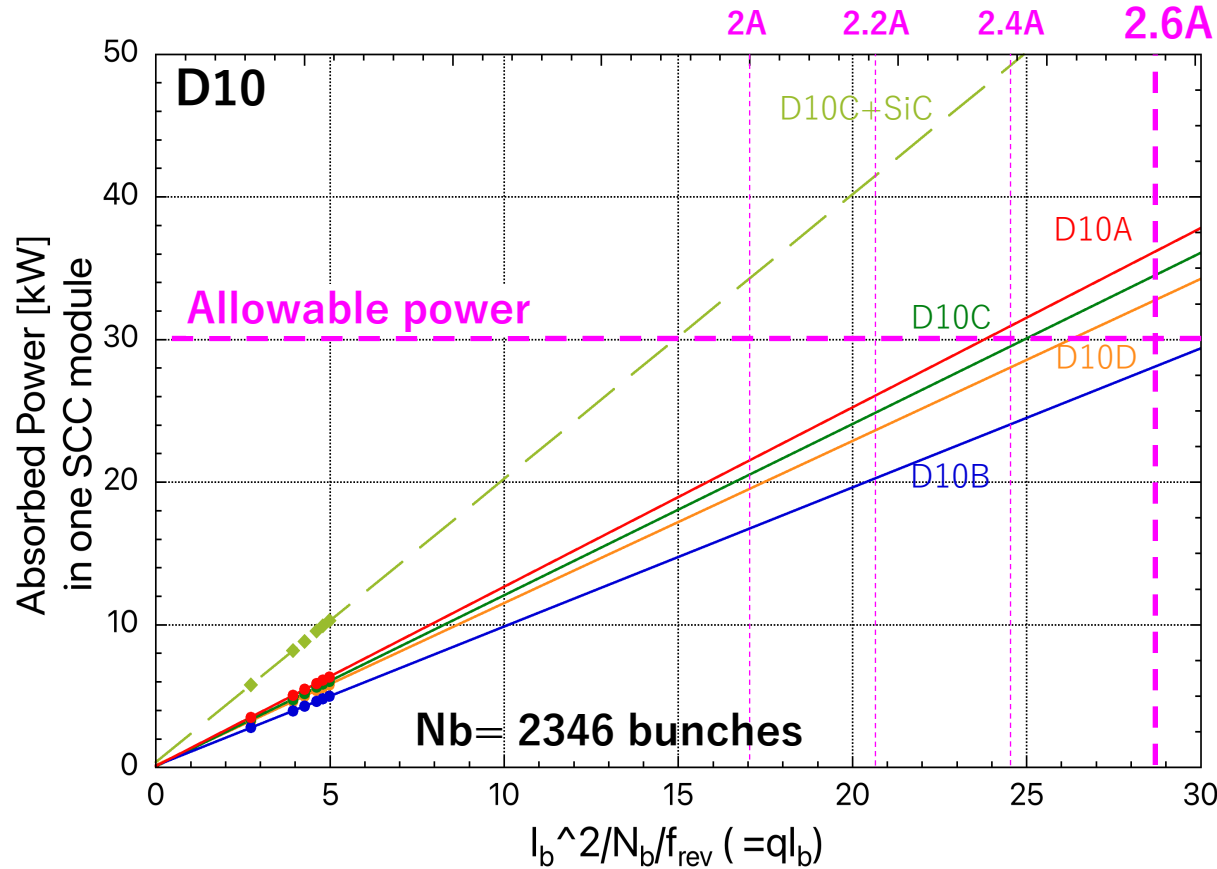


Absorbed power by a pair of Ferrite dampers in D11 cavities normalized by the power in D11A



Two set of SiC dampers have been installed to SCC section for beam test. **The HOM power absorbed by the ferrite dampers of downstream cavities (D11C in plot) were reduced >10% after SiC damper installation. It was confirmed that the additional SiC damper is effective to reduce the load of downstream cavities.** For the future high current operation, SiC dampers will be installed to all SCC modules.

Evaluation of HOM power in higher beam current

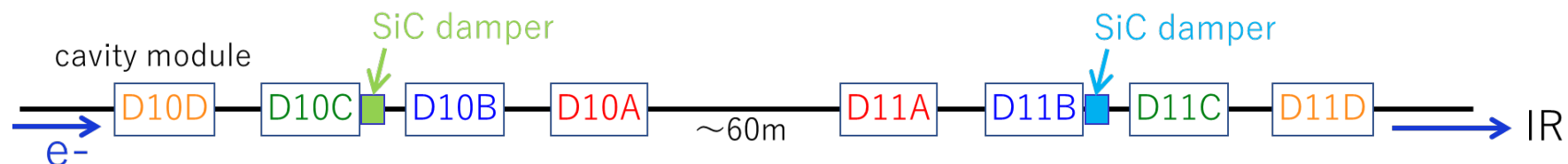


The slope of linear fit is the apparent loss factor.
Equivalent Loss Factor (Eq.LF) k_{eq} [V/pC]

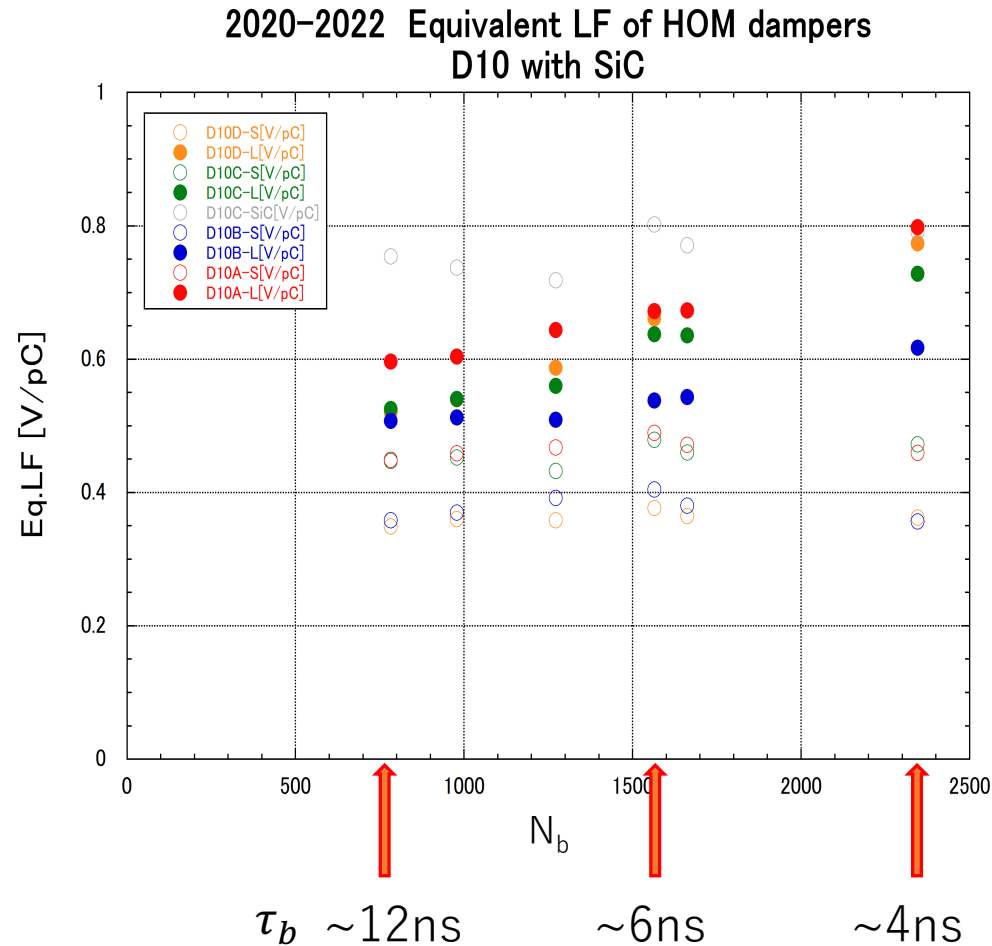
$$k_{eq} \text{ [V/pC]} = \frac{P_{abs} \text{ [kW]} \cdot f_{rev} \text{ [kHz]}}{(I_b \text{ [mA]})^2 / N_b}$$

The absorption power is expected to be higher than the allowable level for the ferrite dampers in the design current.

More SiC dampers or some other measures are necessary.



Consideration of Build-up field



- In recent beam operation, bunch interval dependence was observed in the absorbed power.

Build-Up of HOM fields

Decay time of HOM fields ($\tau_{d,n}$)

> Bunch interval (τ_b)

- In addition to evaluating the loss factor, the effects of the build-up of HOM fields must be considered.

example of KEKB SCC:

$$f_{HOM} = 1\text{GHz}, Q_{L,1\text{GHz}} = 100 \quad \tau_{d,1\text{GHz}} \sim 30\text{ns}$$

see Okada-san's poster in SRF2023
6/26 (Mon.) MOPMB068

Summary

- This lecture explained the optimum tuning, optimum coupling, CBI and HOM damping, which are more important for accelerators with the high current beams, by showing the actual examples of KEKB and SuperKEKB operation.
- In designing your SRF system, the beam-cavity interaction should be well understood, and the LLRF control system and the high-power supply system should also be well considered.
- The parameters of SRF system should have sufficient margins. Your system may be used by the next generation of researchers. The margins will help them.
- This lecture dealt mostly with qualitative matters. The underlying equations and other aspects are explained in detail in many textbooks. Please refer them.

Thank you for your attention.

-
- ◆ T. Kobayashi, “RF system (2)” in OHO’19 (2019), in Japanese.
 - ◆ K. Akai, “RF system” in OHO’94 (1994), in Japanese.
 - ◆ S.Kurokawa et.al., Proc. of the Asian accelerator school, Huairou and Beijing, China, 1999, “Physics and Engineering of High-Performance Electron Storage Rings and Application of Superconducting Technology” WORLD SCIENTIFIC, Feb. 2002.
 - K. Akai, “RF System for Electron Storage Rings”
 - T. Furuya, “Superconducting Cavity”
 - ◆ K. Akai, “RF Issues for High Intensity Factories”, in Proc. of EPAC’96, Sitges, Spain, Jun. 1996, TUX03A.
 - ◆ T.Tajima, “Development of Higher-Order-Mode (HOM) Absorbers for KEKB Superconducting Cavities”, KEK Report 2000-10(2000).

 - ◆ Wilson, Slater, Pedersen, Chao, etc