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The Frequency Shift and Q of Disordered Superconducting RF Cavities

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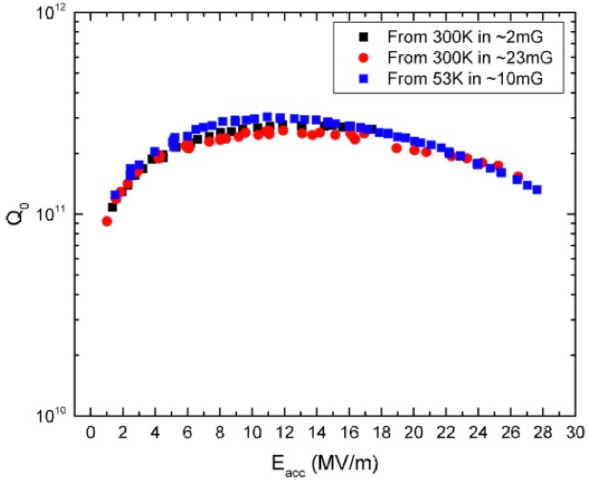
Introduction

- Niobium superconducting radio-frequency (SRF) cavities are high quality electromagnetic (EM) resonators.
- Nb SRF cavities with unprecendented quality factors, $Q{\sim}10^{11}$, have been achieved by infusing Nitrogen into the Nb surface [1-3].

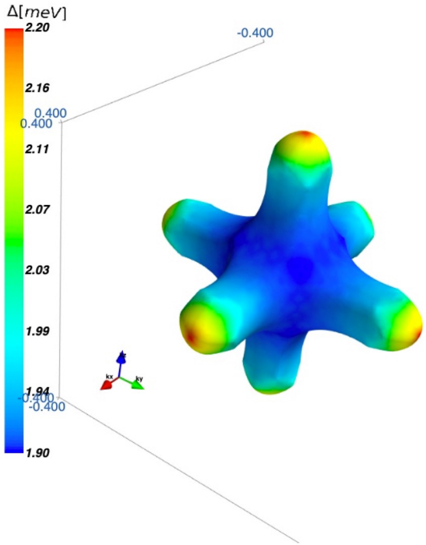


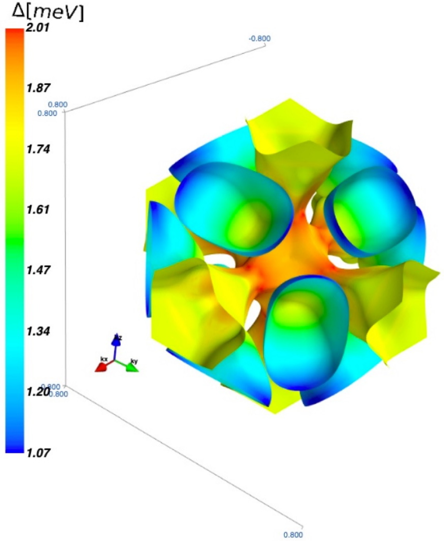
Mechanisms leading current limits in Q are not fully understood

[1] A. Grassellino et al., Supercond. Sci. Technol. **26**, 102001 (2013). [2] A. Romanenko et al., Appl. Phys. Lett. **105**, 234103 (2014). [3] A. Romanenko et al., Phys. Rev. Applied **13**, 034032 (2020).

Introduction

- T_c equation: $$\ln\frac{T_{c0}}{T_c}=\mathcal{A}\sum_{n=0}^{\infty}\left(\frac{1}{n+\frac{1}{2}}-\frac{1}{n+\frac{1}{2}+\frac{1}{2}\frac{1/\tau}{2\pi T_c}}\right),\ \mathcal{A}=\lim_{T\to T_c}\frac{\langle|\Delta(\mathbf{p})|^2\rangle-|\langle\Delta(\mathbf{p})\rangle|^2}{\langle|\Delta(\mathbf{p})|^2\rangle}.$$
- Transition temperature for pure Nb: $T_{c_0} = 9.33$ K.
- Gap anisotropy factor: $A = 0.037$.





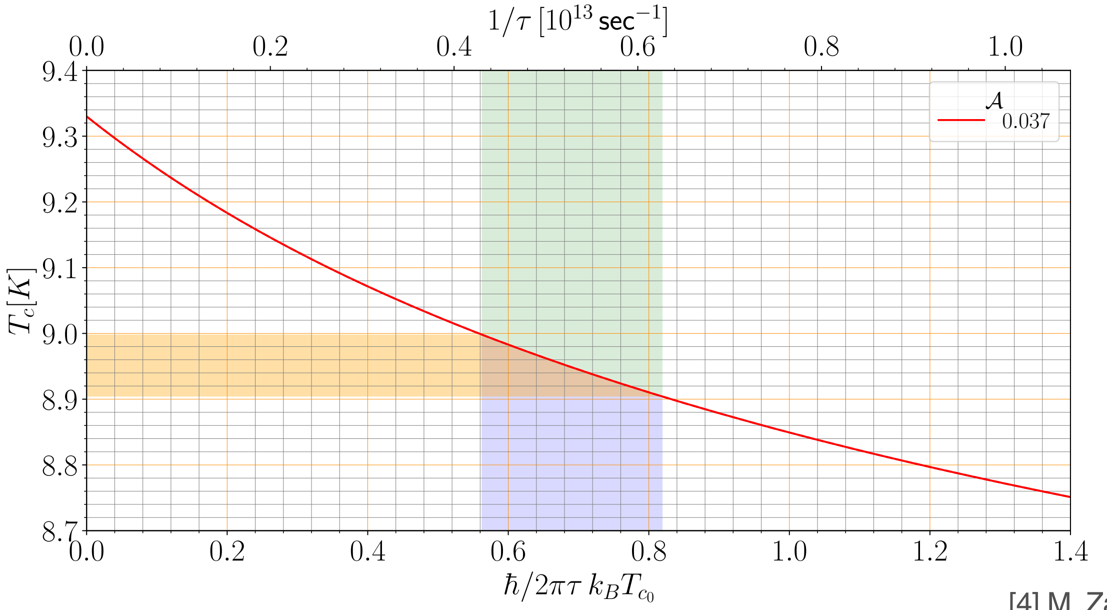
T_c is suppressed due to the gap anisotropy and impurities.

These parameters are obtained from the LDA calculation [4].

[4] M. Zarea, HU, and J. A. Sauls, arXiv:2201.07403.

Introduction

- T_c equation: $$\ln \frac{T_{c_0}}{T_c} = \mathcal{A} \sum_{n=0}^{\infty} \left(\frac{1}{n+\frac{1}{2}} - \frac{1}{n+\frac{1}{2}+\frac{1}{2}\frac{1/\tau}{2\pi T_c}} \right), \quad \mathcal{A} = \lim_{T \to T_c} \frac{\langle |\Delta(\mathbf{p})|^2 \rangle - |\langle \Delta(\mathbf{p}) \rangle|^2}{\langle |\Delta(\mathbf{p})|^2 \rangle}.$$



T_c can be suppressed from $T_{c_0} = 9.33$ K to $T_c \simeq 8.9 - 9.0$ K, which corresponds to T_c of N-doped Nb SRF cavities [4].

[4] M. Zarea, HU, and J. A. Sauls, arXiv:2201.07403.

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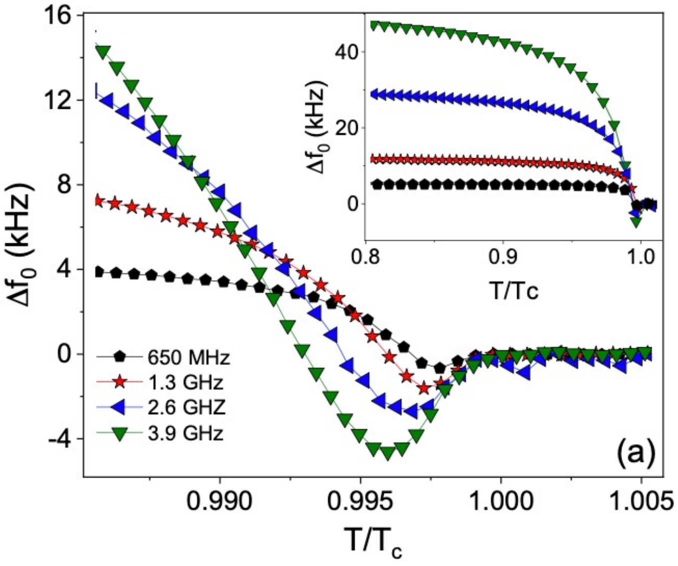
Introduction

- Bafia et al. measured the frequency shift of N-doped Nb SRF cavities near T_c in detail [5].
- The high-Q SRF cavities have "dip" feature in the frequency shift.



Important to study the N impurity effects on the cavity wall

Essential for improvements in performance of quantum sensors and processors

[5] D. Bafia et al., arXiv:2103.10601.

Introduction

- We have developed numerical methods to calculate Q and frequency shift based on nonequilibrium theory of superconductivity, including the role of impurity disorder [6], combined with Slater's method for solving Maxwell's equations for the EM fields confined in a cavity [7].

- We present theoretical results for the EM response of N-doped Nb SRF cavities as a function of disorder, temperature, and mode frequency.

- Our theoretical results are in good agreement with experimental results on both the T_c and frequency shift reported in Ref. [5].

[5] D. Bafia et al., arXiv:2103.10601. [6] D. Rainer and J. A. Sauls, "Superconductivity: From Basic Physics to New Developments", ch. 2, pp. 45–78, World Scientific, Singapore (1994). [7] J. C. Slater, Rev. Mod. Phys. **18**, 441 (1946).

Formalism

- Conductivity $\sigma = \sigma_1 + i\sigma_2$ calculated based on nonequilibrium theory of superconductivity: [6]

$$\sigma = \frac{\sigma_D}{i\omega\tau}\int_{-\infty}^{\infty}\frac{d\varepsilon}{4\pi i}\left\{\tanh\left(\frac{\varepsilon-\omega/2}{2T}\right)\right.$$
$$\times\frac{-2\pi}{D^{\mathrm{R}}(\varepsilon+\omega/2)+D^{\mathrm{R}}(\varepsilon-\omega/2)+1/\tau}\left[\frac{\varepsilon^2-\omega^2/4+\Delta^2}{D^{\mathrm{R}}(\varepsilon+\omega/2)D^{\mathrm{R}}(\varepsilon-\omega/2)}+1\right]$$
$$+\left[\tanh\left(\frac{\varepsilon+\omega/2}{2T}\right)-\tanh\left(\frac{\varepsilon-\omega/2}{2T}\right)\right]$$
$$\left.\times\frac{-\pi}{D^{\mathrm{R}}(\varepsilon+\omega/2)+D^{\mathrm{A}}(\varepsilon-\omega/2)+1/\tau}\left[\frac{\varepsilon^2-\omega^2/4+\Delta^2}{D^{\mathrm{R}}(\varepsilon+\omega/2)D^{\mathrm{A}}(\varepsilon-\omega/2)}+1\right]\right\},$$

$$D^{\mathrm{R,A}}(\varepsilon)\equiv\sqrt{\Delta^2-(\varepsilon\pm i0^+)^2},$$

σ_D: Drude conductivity.

[6] D. Rainer and J. A. Sauls, “Superconductivity: From Basic Physics to New Developments”, ch. 2, pp. 45–78, World Scientific, Singapore (1994).

Formalism

- Surface impedance $Z_s = R_S + iX_s$ obtained by solving Maxwell's equations on interface between vacuum and superconductor:

$$\frac{R_s}{R_n} = \frac{\sigma_{n1}^{1/2}}{(\sigma_1^2+\sigma_2^2)^{1/4}}\left[\cos\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right) - \sin\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right)\right],$$
$$\frac{X_s}{R_n} = \frac{\sigma_{n1}^{1/2}}{(\sigma_1^2+\sigma_2^2)^{1/4}}\left[\cos\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right) + \sin\left(\frac{1}{2}\arctan\frac{\sigma_2}{\sigma_1}\right)\right].$$

Normal-state resistance R_n, reactance X_n, and conductivity σ_{n1}:

$$R_n = X_n = \frac{4\pi}{\omega_p c}\sqrt{\frac{\pi f}{\tau}}, \quad \sigma_{n1} = \frac{\sigma_D}{1+(\omega\tau)^2},$$ ω_p: plasma frequency.

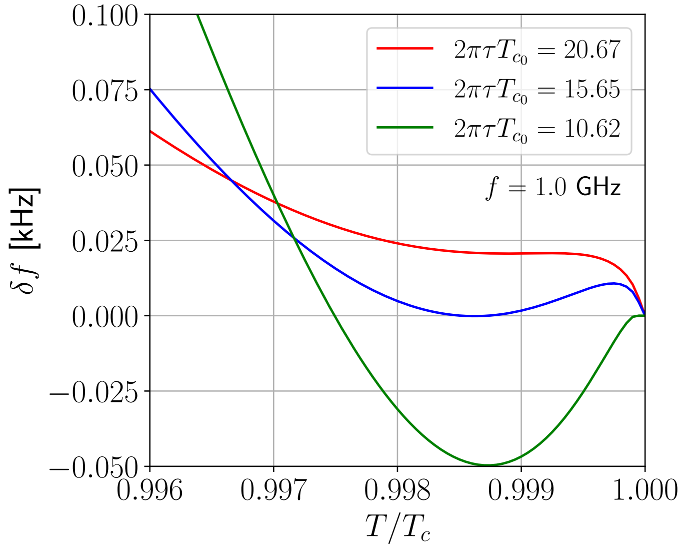
- Quality factor Q and frequency shift δf of the SRF cavities calculated from Maxwell's equations in a hollow cavity based on Slater's method: [7]

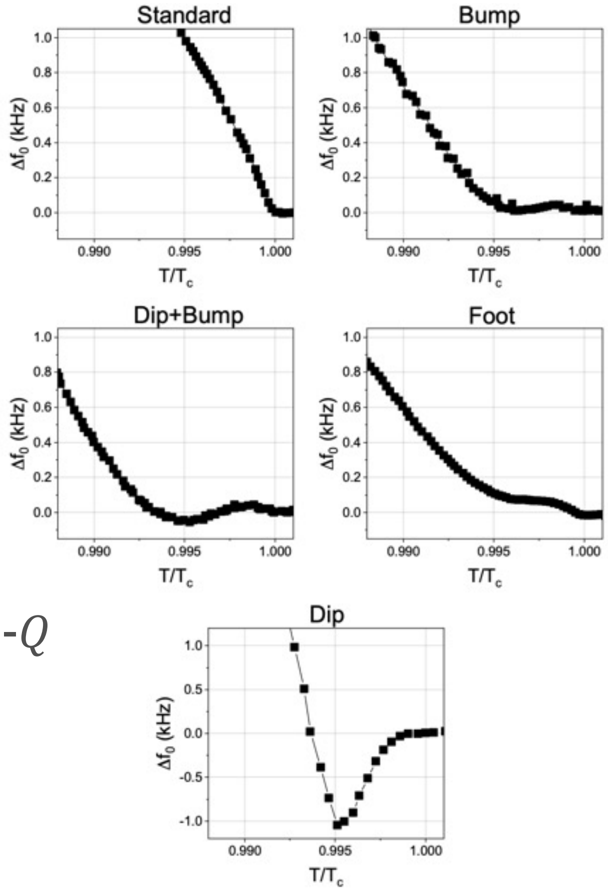
$$Q = \frac{G}{R_s}, \quad \delta f = \frac{f}{2G}(X_n - X_s), \quad G = \frac{8\pi^2 f}{c^2}\int_V \mathbf{H}^2 dv \Big/ \int_S \mathbf{H}^2 da.$$

[7] J. C. Slater, Rev. Mod. Phys. **18**, 441 (1946).

Theoretical Calculation of Frequency Shift

[5] D. Bafia et al., arXiv:2103.10601.





- The green line has the “dip” feature as seen in the high-Q cavities.
- The red and blue lines are the same behavior as the experimental data called the “foot” and “bump” feature, respectively [5].

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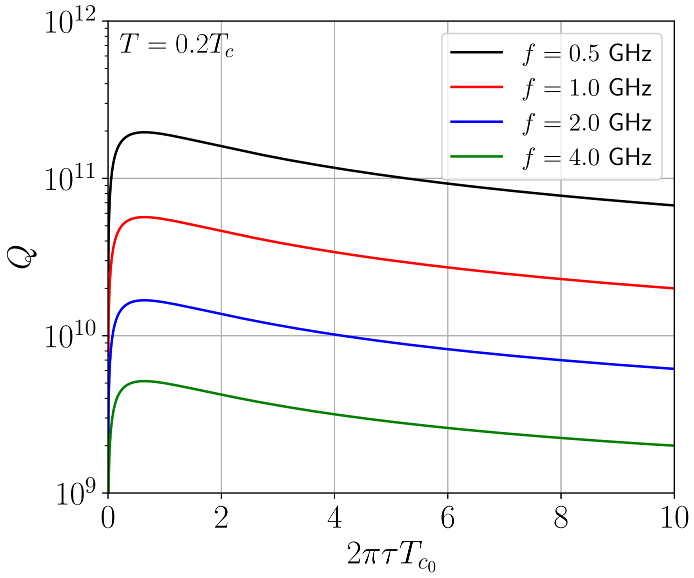
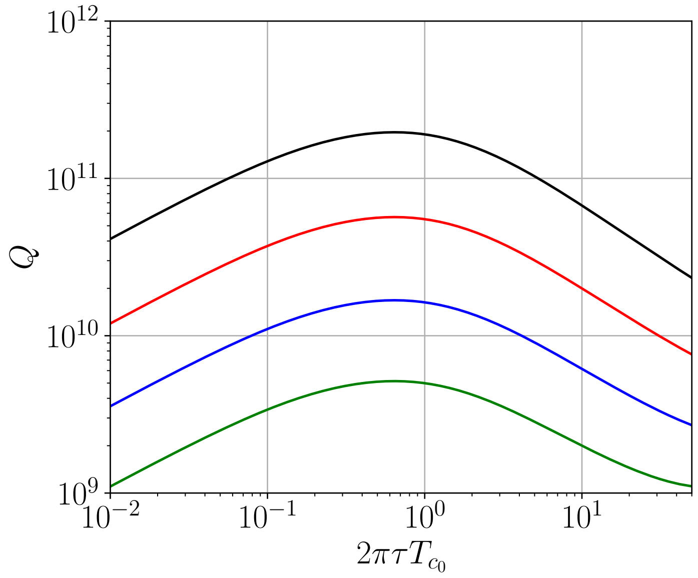
Theoretical Calculation of Quality Factor





- The quality factor in cavities with intermediate disorder is the largest.
- It becomes rather small in the too dirty cavities due to the pair breaking.

Comparison & Analysis of N-doped Nb SRF Cavities

- Bafia et al. measured the frequency shift of Nb SRF cavities near T_c in detail [5].

- Calculation for homogeneous SRF cavities does not fit well with the peak position and dip width of the frequency shift in the experimental data.

- Bafia et al. found that T_c varies depending on where it is measured. The SRF cavities are inhomogeneous for T_c and τ.

- We consider this T_c spread in our calculations.

[5] D. Bafia et al., arXiv:2103.10601.

Superconducting Gap with Spread in T_c

- Gap energy with spread in T_c: [8]

$$\Delta(T) = \Delta_0 \sqrt{\int_{-\infty}^{\infty} dT_c\, \rho(T_c) \tilde{\Delta}^2(T, T_c)}, \quad \Delta_0 = \pi e^{-\gamma} T_c^{\mathrm{ave}},$$

$$\tilde{\Delta}(T, T_c) = \tanh\left(\frac{\pi T_c}{\Delta_0}\sqrt{\frac{8}{7\zeta(3)}\frac{T_c - T}{T}}\right)\Theta(T_c - T).$$

- Gaussian distribution of T_c: [8]

$$\rho(T_c) = \frac{1}{\sqrt{2\pi} T_c^{\mathrm{SD}}} \exp\left[-\frac{1}{2}\left(\frac{T_c - T_c^{\mathrm{ave}}}{T_c^{\mathrm{SD}}}\right)^2\right].$$

Average of T_c: $T_c^{\mathrm{ave}} = (T_c^{\mathrm{max}} - T_c^{\mathrm{min}})/2.$

Standard deviation of T_c: $T_c^{\mathrm{SD}} = (T_c^{\mathrm{ave}} - T_c^{\mathrm{min}})/3.$

T_c^{max} and T_c^{min} are fitting parameters

[8] HU, M. Zarea, and J. A. Sauls, arXiv:2207.14236.

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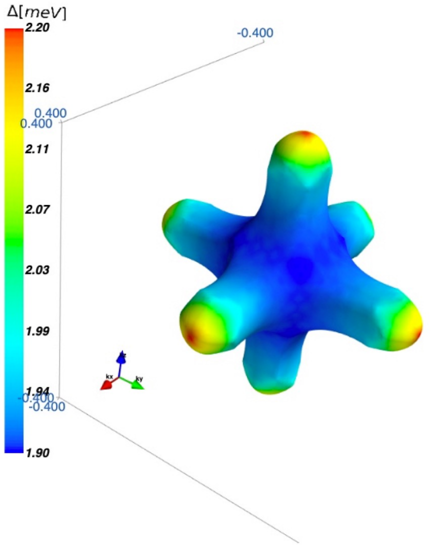
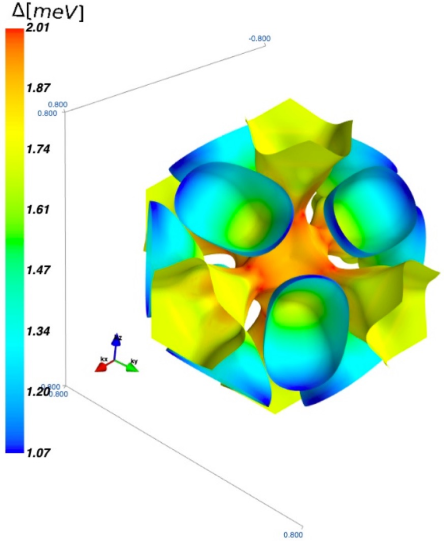




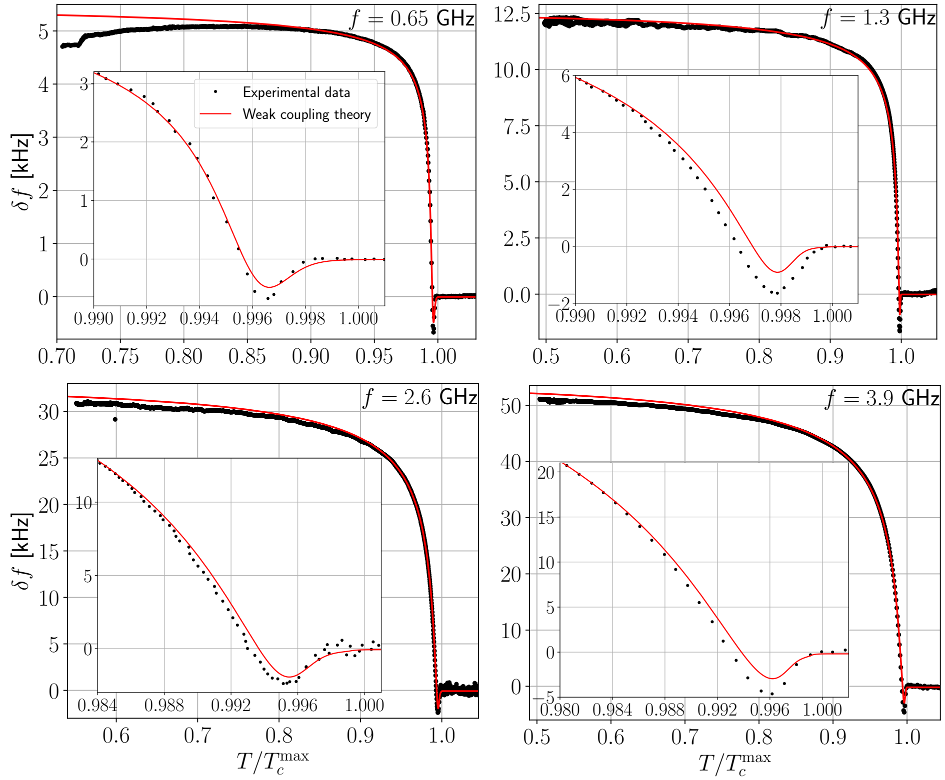
These parameters are obtained from the LDA calculation [4].

This T_c spread comes from inhomogeneity of impurities.

[4] M. Zarea, HU, and J. A. Sauls, arXiv:2201.07403.

Comparison & Analysis of N-doped Nb SRF Cavities

- Frequency shift of the N-doped Nb SRF cavities with the different frequency: [5]



Our theoretical lines are fitted well with the experimental data!

Changes in the resonant frequency of order $10\ \mathrm{Hz}$ for GHz SRF cavities over temperature ranges of $0.001 T_c$

[5] D. Bafia et al., arXiv:2103.10601.

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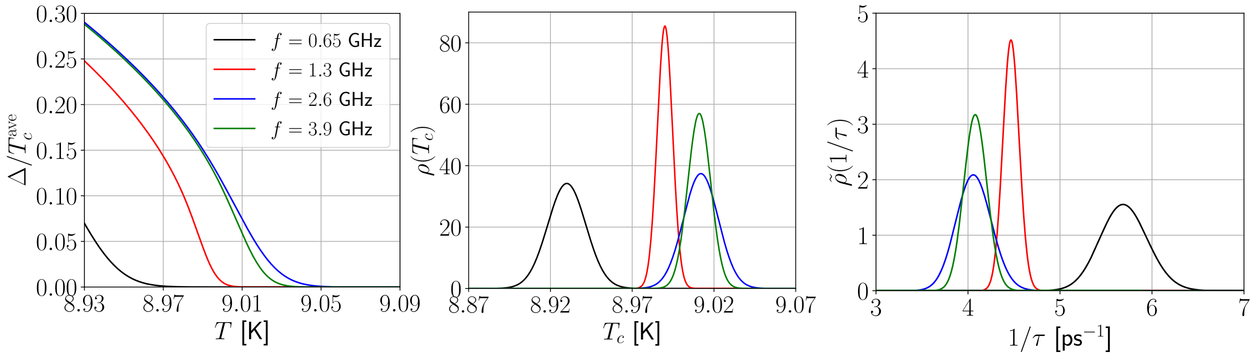
Comparison & Analysis of N-doped Nb SRF Cavities

- Gap energy near T_c and Gaussian distributions of T_c and $1/\tau$:



- Used $T_c^{\rm max}$ and $T_c^{\rm min}$, and calculated R_n, and the corresponding experimental data: [5]

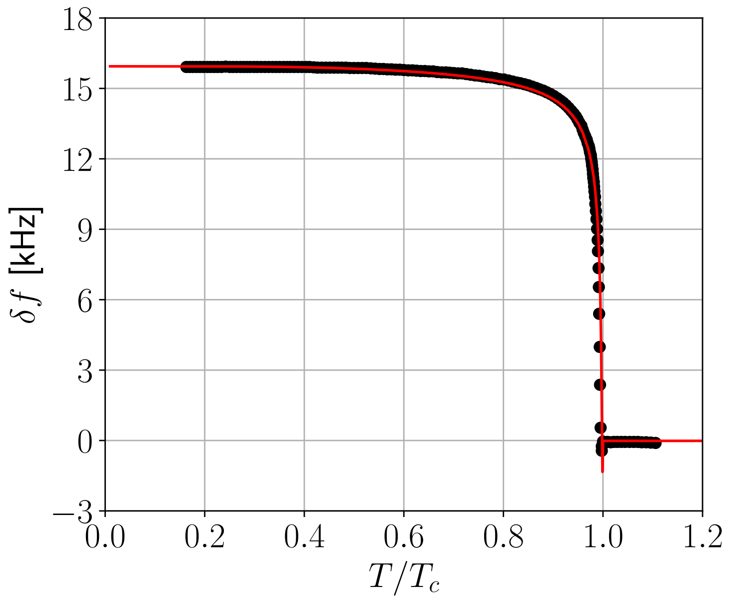
	Theory				Experiment			
f [GHz]	0.65	1.3	2.6	3.9	0.65	1.3	2.6	3.9
$T_c^{\rm max}$ [K]	8.965	9.004	9.044	9.032	9.005	8.907	9.081	9.165
$T_c^{\rm min}$ [K]	8.895	8.976	8.980	8.990	8.975	8.87	9.041	9.15
R_n [mΩ]	4.471	5.601	7.554	9.272	4.364	5.425	6.95	8.93

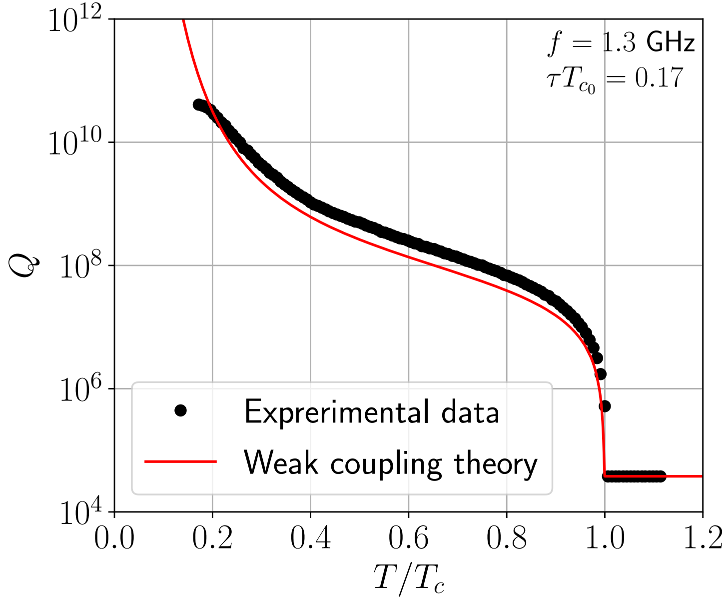
[5] D. Bafia et al., arXiv:2103.10601.

Our theory is in good agreement with the experimental data!

Comparison & Analysis of N-doped Nb SRF Cavities

- Quality factor and frequency shift of the Nb SRF cavity: [5]





The calculation of Q is not perfect but in reasonable agreement with the experimental data.

[5] D. Bafia et al., arXiv:2103.10601.

Frequency Shift Anomaly Near T_c

To understand the dip in the frequency shift, we express the conductivity as [8]

$$\sigma_1 = \sigma_{1n} + \delta\sigma_1, \quad \sigma_2 = \sigma_{2n} + \delta\sigma_2.$$

$\delta\sigma_1$ and $\delta\sigma_2$ are the small deviations from σ_{1n} and σ_{2n}, respectively.

Assuming $\sigma_{1n} \gg \sigma_{2n}$, we obtain the frequency shift near T_c as

$$\delta f = \frac{f R_n}{4G\sigma_{1n}}(\delta\sigma_1 - \delta\sigma_2).$$

Since $\delta\sigma_2$ in dirty superconductors is larger than $\delta\sigma_1$, the frequency shift becomes negative near T_c.

[8] HU, M. Zarea, and J. A. Sauls, arXiv:2207.14236.

Summary

- We showed that the quality factor has a peak of upper convexity as a function of the quasiparticle-impurity scattering rate, with the largest Q in cavities with intermediate disorder.
- We presented theoretical results for the effects of inhomogeneous disorder on the transition temperature and frequency shift of SRF cavities and our calculations are in good agreement with the experimental results [8].
- Since the sensitivity to the axion and EH signal depends on not only Q but also the frequency shift, it is important to study the frequency shift in detail.

We thank D. Bafia, A. Grassellino, A. Romanenko, and J. Zasadzinski for many discussions on their experimental work on SRF cavities.

[8] D. Bafia et al., arXiv:2103.10601.

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