Superconducting Non-Elliptical Cavities (TEM Cavity Designs)

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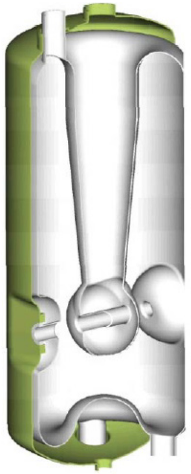
Outline

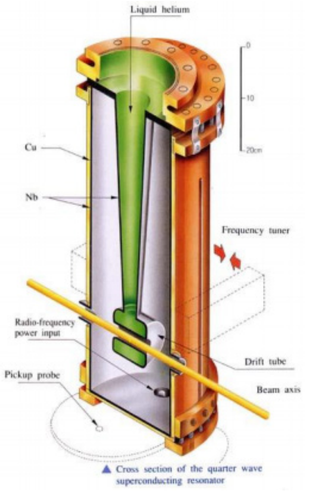
- Why Non-Elliptical Cavities?
- Electromagnetic modes
- Non-Elliptical accelerating cavities
  - Basic principles
  - Types of cavities
  - Current accelerator applications
- Non-Elliptical deflecting and crabbing cavities
  - Basic principles
  - Types of cavities
  - Current collider applications
- Design issues for non-elliptical cavities

World of Superconducting Non–Elliptical Cavities

RF Cavities of interesting shapes for particle acceleration

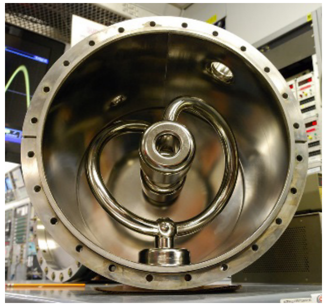
Quarter Wave Cavities





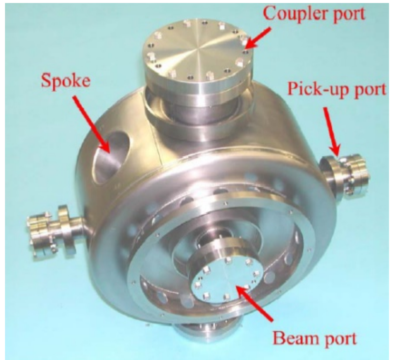
Half Wave Cavities

Split Ring Resonator

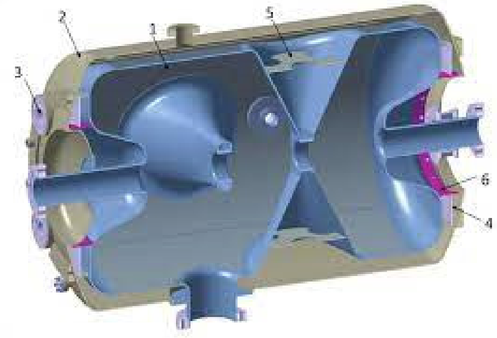


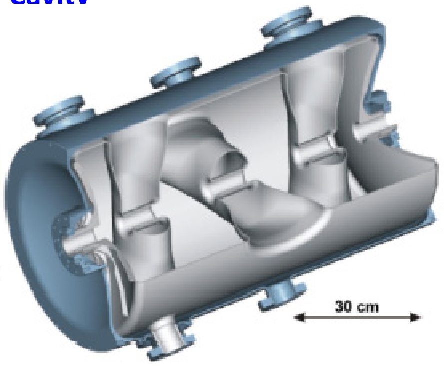
Superconducting RFQ Cavity

Single Spoke Cavities

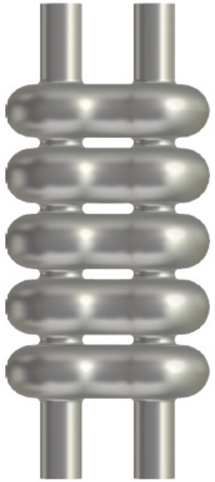


Multi Spoke Cavity





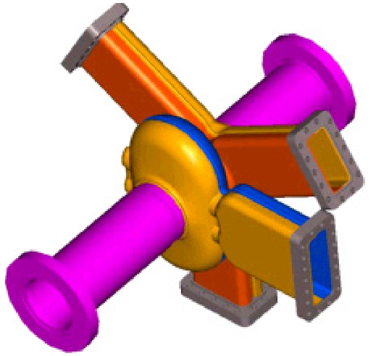
Twin Axis Cavity

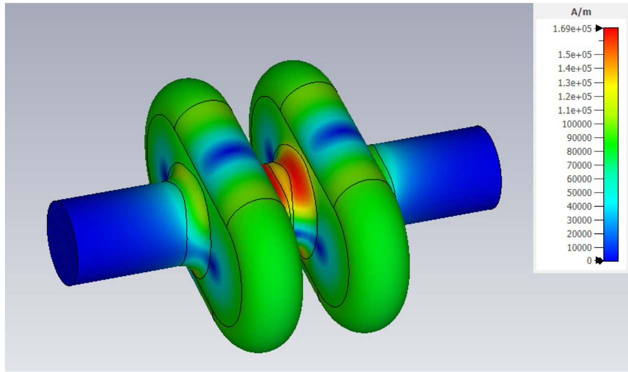


World of Superconducting Non–Elliptical Cavities

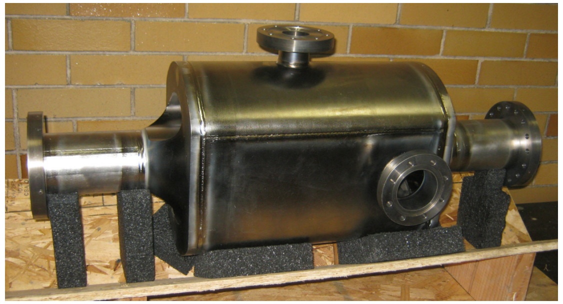
RF Cavities of interesting shapes for deflecting and crabbing applications

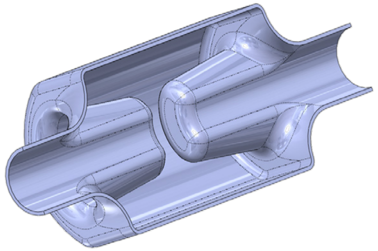
Squashed Elliptical Cavities

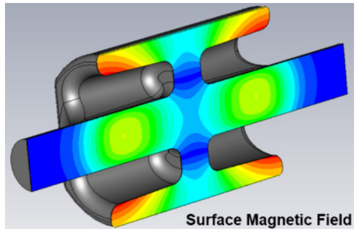




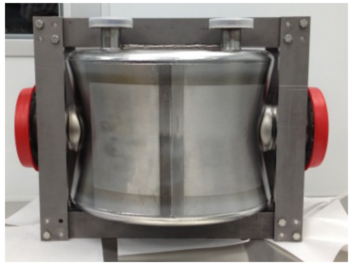
4-Rod Cavity

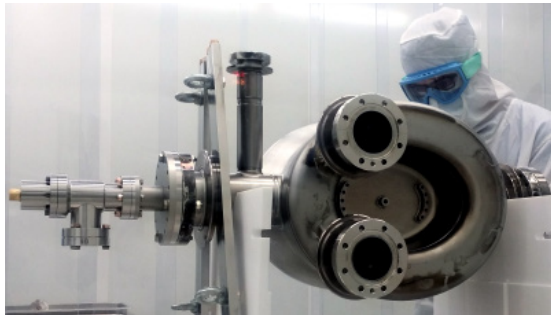






Double Quarter Wave Cavity





RF-Dipole Cavities

Electron vs Proton/Heavy Ion Acceleration

- Electrons and protons are distinctly different due to mass of the particles
  - Electrons → 0.511 MeV/c^2
    - 550 keV – γ = 2.08, β = 0.88
    - 550 MeV – γ = 1077, β = 1
  - Protons → 938 MeV/c^2
    - 550 keV – γ = 1.0005, β = 0.032
    - 550 MeV – γ = 1.59, β = 0.78
    - 550 GeV – γ = 587.4, β = 1

$$E_T = \gamma m_0 c^2 = \text{KE} + E_0$$

$$\text{KE} = (\gamma - 1) m_0 c^2$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

- For heavy ions: Mass number (A) >1

$$E_T = \text{KE} + AE_0$$

$$\text{KE} = (\gamma - 1) A m_0 c^2$$

$$p = \gamma\beta A m_0 c = \frac{\gamma\beta A E_0}{c}$$

- Protons → $\boldsymbol{A}$ = 1 and q = 1

$$\text{KE} = qV_{eff}\cos\phi \qquad B\rho = mv = \frac{p}{q} = \frac{\gamma\beta E_0}{c}$$

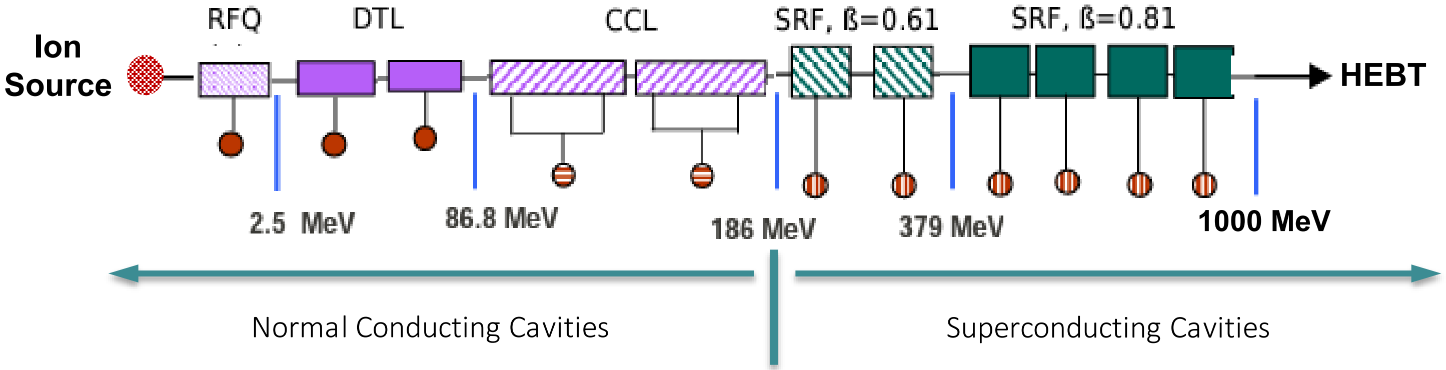
- Heavy ions → $\boldsymbol{A}$ >1 and q

$$\text{KE} = \frac{q}{A}V_{eff}\cos\phi \qquad B\rho = p = \frac{A}{q}\frac{\gamma\beta E_0}{c}$$

Why Non-Elliptical Cavities?

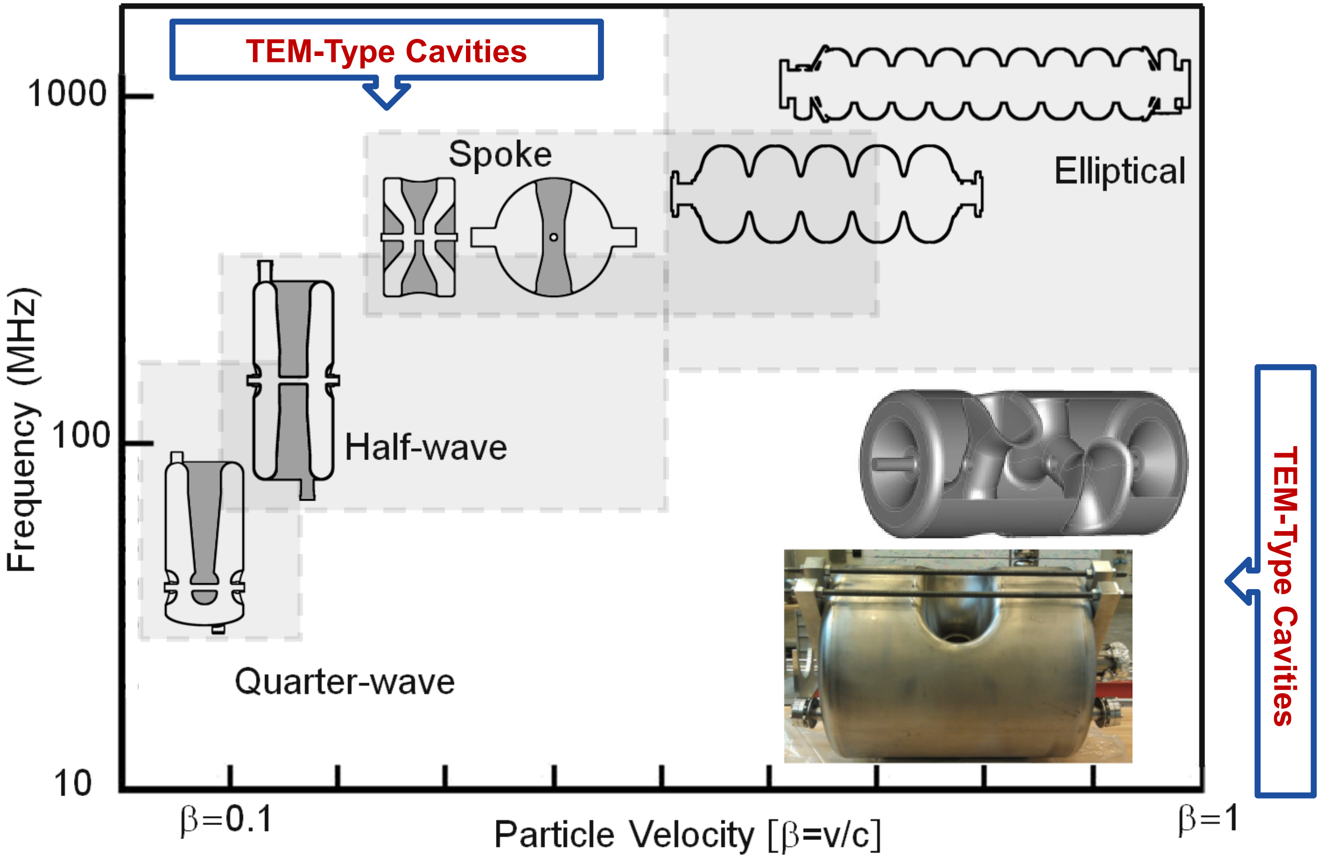
1. To accelerate Protons/Heavy Ions → Need $\beta<1$ (β=v/c<1) accelerating cavities

- In accelerating electrons, all cavities are designed at $\beta=1$
- In Hadron acceleration
  - Velocity of the particle beam increases with energy
  - Depends on the synchronization of the particle bunches and rf voltage in the cavity
  - Require various types of cavities each optimized to accelerate different velocity ranges
- Elliptical cavities has intrinsic problem as β goes down
  - Due to mechanical problems, multipacting, low RF efficiency
- Solution: Use of TEM-type cavities



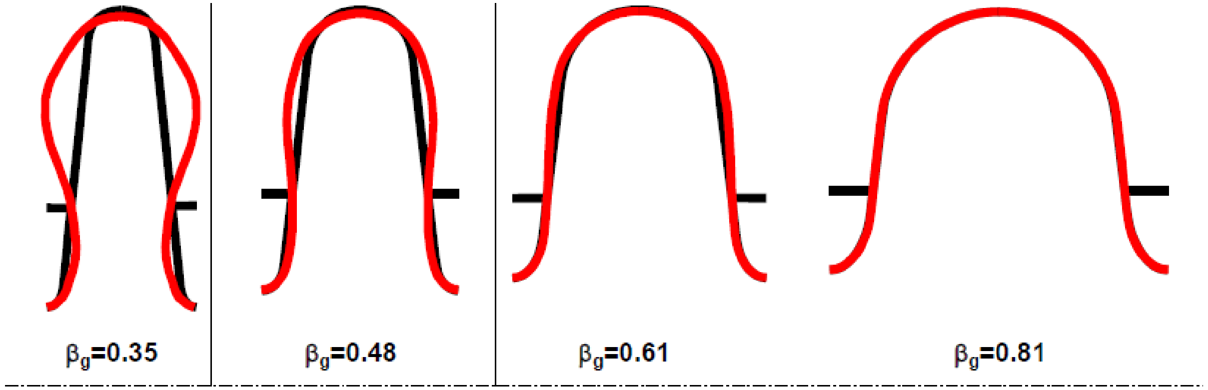
Proton/Heavy Ion Acceleration

- Protons/Heavy Ions require acceleration by many cavities to reach velocities approaching speed of light
- Cavities are designed for specific regions of velocity



Limitation on β=1 Elliptical Cavities

- Elliptical cavities have been designed for β>0.5 for cw applications and β>0.6 for pulsed high energy acceleration
- At very low β elliptical cavities start to look like bellows
  - In π mode cell-to-cell distance $\sim \beta\lambda/2$ and cavity diameter is $\sim \lambda$
  - Ratio of cavity length/diameter $\sim \beta/2$



- Low rf efficiency
- Poor mechanical stability
- Possibility of strong multipacting

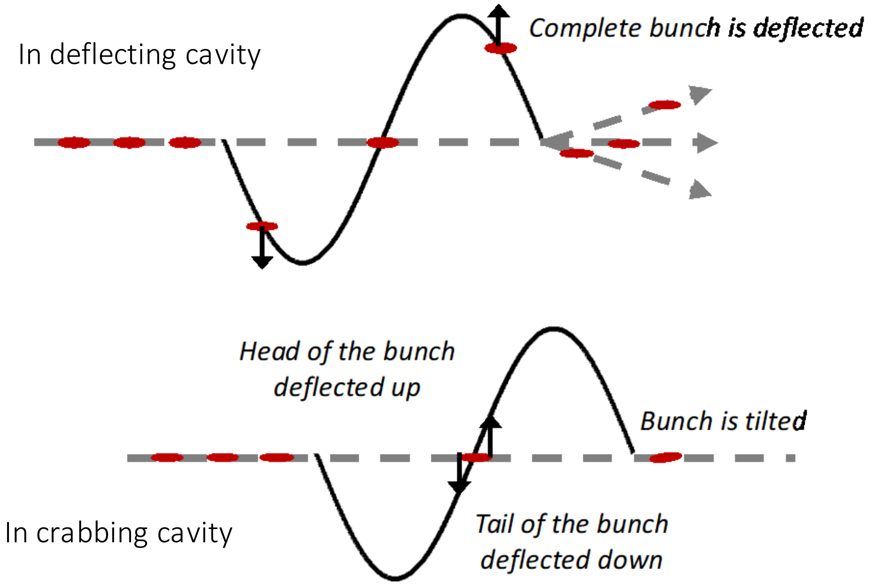
- Will work in CW applications
- Pessimistic in pulsed applications

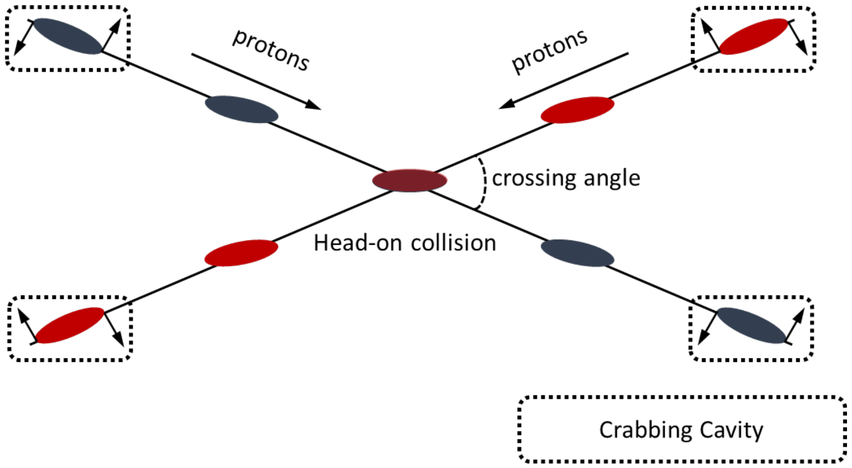
Suitable for all CW and pulsed applications

Why Non-Elliptical Cavities?

2. To deflect or crab a beam

- Requires to provide a transverse kick to the beam
- Standard elliptical cavities operating in TM_{010} mode only produce a longitudinal gradient
- Deflecting/crabbing cavities operate mostly at β=1
- Solution: Use of TE_{11}-like mode, TM_{11}-type mode, or TEM-type mode cavities





Applications of Non-Elliptical Cavities

Application	Maximum β	Beam	Maximum Current	Operation
Linacs for nuclear physics research	~ 0.2 (0.5)	Light & Heavy Ions	~ 1 μA	cw
Drivers for radioactive ion beam (RIB) facilities and accelerator driven systems (ADS)	~ 0.3 – 0.9	Light & Heavy Ions	~ 0.1 – 30 mA	cw
Linacs for radioisotope production	~0.3	p, d	~ 1 – 10 mA	cw
Neutron spallation sources	~1	p	~ 10 – 100 mA	pulsed
Accelerators for material irradiation	~0.3	d	~ 100 mA	cw
Compact high β linacs (proposed)	1	e	~ 1 mA	cw
Deflecting and crabbing applications	1	e, p	~ 1 A	cw

Applications of Non-Elliptical Cavities

- Superconducting technology allows cw and high duty cycle operation
  - Also allows increase bore (transverse acceptance) as highest shunt impedance is not essential and TEM cavities allow lower frequencies with the associated larger longitudinal acceptance

- Drivers – RIB production (ISOL, fragmentation) (ions), ADS (transmutation, energy) (p, H-, d), Spallation neutron sources (p, H-)
  - Longer machines typically, large velocity swing, several cavity regimes
  - Treat as almost fixed gradient machines
  - Beam loss (halo) an issue, careful beam dynamics required, favor symmetric rather than asymmetric cavities
  - Beam loading is typically an important consideration

- Post-accelerators (Radioactive ion beam and nuclear physics) (ions)
  - Shorter machines typically, broad velocity acceptance
  - Utilize maximum cw gradient to improve performance and/or reduce cost
  - Short independently phased cavities give flexibility to beam delivery
  - Beam loading typically not an issue

Advantages of Non-Elliptical Cavities

- Traditionally low β superconducting resonators were quarter wave (or split rings) used as post-accelerators for heavy ion tandems serving the nuclear physics community (ATLAS, INFN-LNL)

- Increased interest in Radioactive Ion Beam (RIBs) has created a renaissance in low and medium β superconducting cavity development in the last 20 years for both post-accelerators and drivers – ISAC-II, NSCL-ReA, FRIB

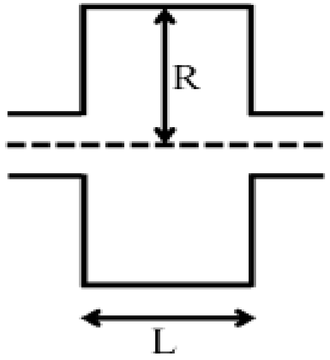
- High duty cycle driver linacs are now being built with superconducting sections beginning at lower β values (SPIRAL-II, C-ADS, IFMIF, ESS)
  - Rise in performance of spoke cavities and half-wave resonators (HWR)
  - Shapes are being optimized for performance with more emphasis on forming
  - Clean room assembly, high pressure water rising and separated vacuum cryostats are now standard

- Spoke resonators are now being investigated at velocities at or near β=1 for compact machines

- Deflecting/crabbing cavities have seen a rise in interest due to high performance, compact designs for applications such as crabbing cavities for LHC high luminosity upgrade

Electromagnetic Fields

- Resonance cavity mode types: TM type, TE type, TEM type
- For a cylindrical geometry (Simplest form of a resonant cavity)



$$\vec{E}(x,y,z,t)=\vec{E}(x,y)e^{j(kz-\omega t)}$$

$$\vec{H}(x,y,z,t)=\vec{H}(x,y)e^{j(kz-\omega t)}$$

TM Modes

- Modes with longitudinal electric fields and no transverse magnetic fields

$$\text{TM Modes:}\begin{cases} E_z = E_0 \cos\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x_{mn}r}{R}\right)\cos(m\phi)\,, \\ E_r = -E_0\frac{p\pi R}{Lx_{mn}}\sin\left(\frac{p\pi z}{L}\right) J'_m\left(\frac{x_{mn}r}{R}\right)\cos(m\phi)\,, \\ E_\phi = E_0\frac{mp\pi R^2}{rLx_{mn}^2}\sin\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x_{mn}r}{R}\right)\sin(m\phi)\,, \\ H_z = 0\,, \\ H_r = jE_0\frac{m\omega R^2}{c\eta r x_{mn}^2}\cos\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x_{mn}r}{R}\right)\sin(m\phi)\,, \\ H_\phi = jE_0\frac{\omega R}{c\eta x_{mn}}\cos\left(\frac{p\pi z}{L}\right) J'_m\left(\frac{x_{mn}r}{R}\right)\cos(m\phi)\,, \end{cases}$$

$$\omega_{TM_{mnp}} = c\sqrt{\left(\frac{x_{mn}c}{R}\right)^2+\left(\frac{p\pi}{L}\right)^2}$$

x_{mn} is the n^th^ root of J_m

TE Modes

- Modes with longitudinal magnetic fields and no transvers electric fields

$$\text{TE Modes:}\begin{cases} H_z = H_0 \sin\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x'_{mn}r}{R}\right)\cos(m\phi)\,, \\ H_r = H_0\frac{p\pi R}{Lx'_{mn}}\cos\left(\frac{p\pi z}{L}\right) J'_m\left(\frac{x'_{mn}r}{R}\right)\cos(m\phi)\,, \\ H_\phi = -H_0\frac{mp\pi R^2}{rL(x'_{mn})^2}\cos\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x'_{mn}r}{R}\right)\sin(m\phi)\,, \\ E_z = 0\,, \\ E_r = jH_0\frac{m\eta\omega R^2}{cr(x'_{mn})^2}\sin\left(\frac{p\pi z}{L}\right) J_m\left(\frac{x'_{mn}r}{R}\right)\sin(m\phi)\,, \\ E_\phi = jH_0\frac{\eta\omega R}{cx'_{mn}}\sin\left(\frac{p\pi z}{L}\right) J'_m\left(\frac{x'_{mn}r}{R}\right)\cos(m\phi)\,, \end{cases}$$

$$\omega_{TE_{mnp}} = c\sqrt{\left(\frac{x'_{mn}c}{R}\right)^2+\left(\frac{p\pi}{L}\right)^2}\,.$$

x'_{mn} is the n^th^ root of J'_m

Modes in a Pill Box Cavity

SRF 2023 Tutorial –

SRF Fundamental/Nb Material, Akira Miyazak

- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Mode of interest for acceleration in elliptical cavities
  - Frequency depends only on radius, independent of length
- TM_{0np}
  - Monopole modes that can couple to the beam and exchange energy
- TM_{1np}
  - Dipole modes that can deflect the beam
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam
- TEM modes → For coaxial geometries
  - Transverse Electro Magnetic (TEM) mode

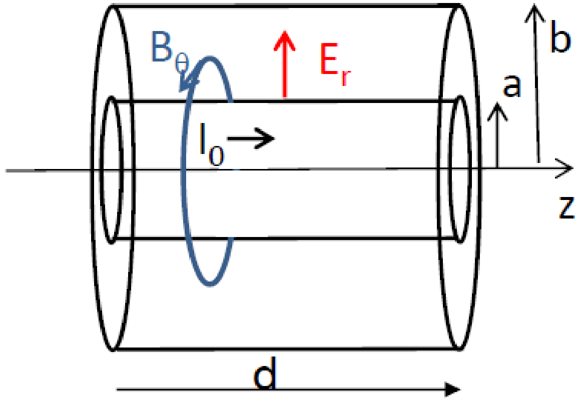
Coaxial Resonator (TEM Mode)

- Consider a coaxial geometry with grounded plates at the ends with inner radius a, outer radius b and length d
- A standing wave occurs with E_r vanishing at the end walls at $z=0$ and $z=d$
- Fields components

$$B_\theta = \frac{\mu_0 I_0}{\pi r}\cos\frac{p\pi z}{d}e^{j\omega t}$$

$$E_r = -2j\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{I_0}{2\pi r}\sin\frac{p\pi z}{d}e^{j\omega t}$$

$$\text{where } \omega = k_z c = \frac{p\pi c}{d},\quad p = 1,2,3,\ldots$$



- Peak voltage on the inner conductor is found by integrating the radial electric field between the grounded outer conductor and the inner conductor

$$\hat{V}(z) = \int_a^b E_r(z)dr$$

$$\hat{V}(z) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{I_0}{\pi}\ln\left(\frac{b}{a}\right)\sin\frac{p\pi z}{d}$$

Types of Superconducting Non-Elliptical Cavities

TM Type

- Accelerating cavities

Twin axis cavity
TM$_{110}$-like mode

- Deflecting and crabbing cavities

Squashed elliptical cavity
TM$_{110}$-like mode

TE Type

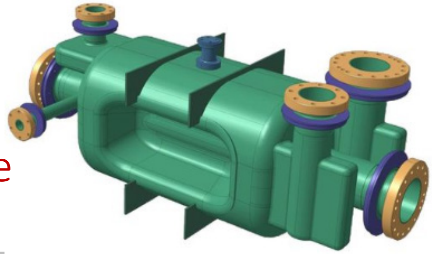
- Accelerating cavities

RF quadrupole cavity
TE$_{21}$-like mode

- Deflecting and crabbing cavities

Double quarter wave cavity
TE$_{11}$-like mode

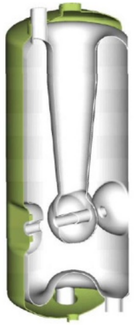
RF-dipole cavity
TE$_{11}$-like mode



TEM Type

- Accelerating cavities

Quarter Wave Cavity

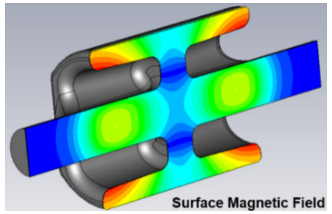


Half Wave Cavity

Spoke Cavity

- Deflecting and crabbing cavities

4-Rod Cavity



Designing Non-Elliptical Cavities

- No universal design for any of the cavity geometries
- Has many degrees of freedom with many parameters for optimization
- *May lead to complicated designs*

BUT IT IS MORE FUN !!

- Non-elliptical cavities are 3D geometries
- Requires 3D simulations to optimize cavity designs
- Available simulation packages
  - ✓ CST Studio, HFSS, ACE3P Code Suite, COMSOL

NON-ELLIPTICAL ACCELERATING CAVITIES

(1) BASIC PRINCIPLES

(2) TYPES OF NON-ELLIPTICAL ACCELERATING CAVITIES

  i. TEM-Type Cavities
  ii. TM-Type Cavities

(3) DESIGN CONSIDERATIONS

BASIC PRINCIPLES OF NON-ELLIPTICAL ACCELERATING CAVITIES

RF Acceleration

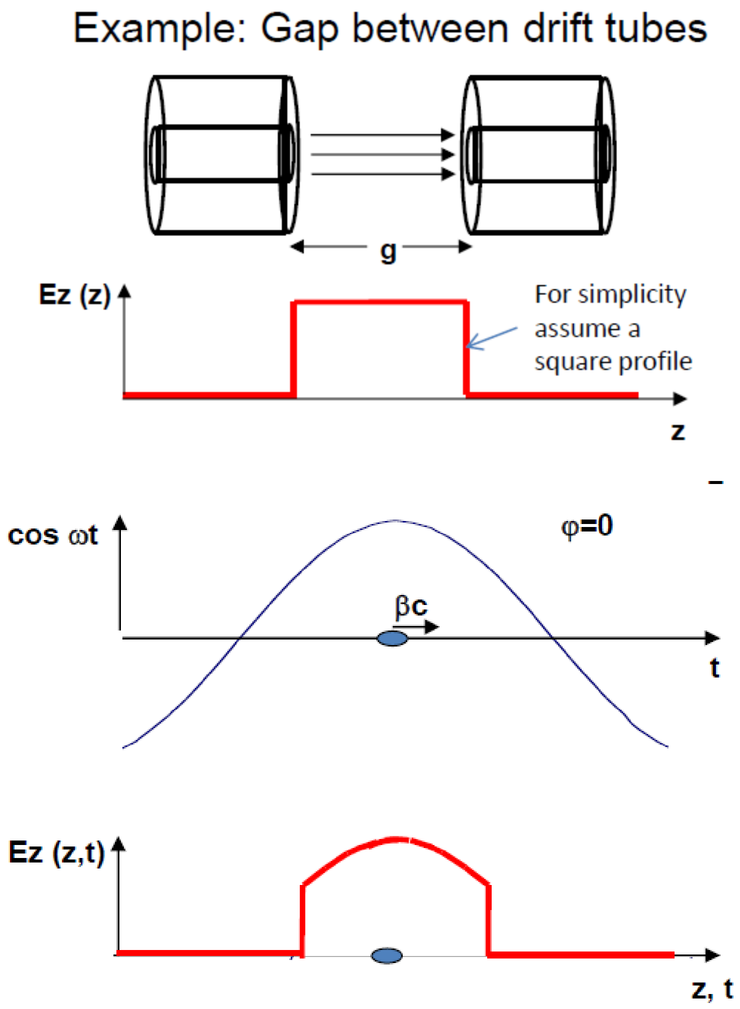
- A standing wave is established in the resonator with a time varying E_z field on axis
- For a particle travelling on axis with velocity βc and sees a field that is the product of spatial variation and time modulation

$$E_z(z,t) = E_z(\rho = 0, z)\cos(\omega t + \varphi)$$

  - φ is a constant (rf phase) that defines the time of arrival of the particle with respect to the rf time modulation
  - A phase corresponds to the maximum acceleration that can be given to the particle (on crest acceleration)

- One can calculate the accelerating voltage (V_{eff}) imparted to the particle by

$$V_{eff}(\varphi) = \int_{-\infty}^{\infty} E_z(\rho = 0, z)\cos(\omega t(z) + \varphi)dz$$



Accelerating Voltage and Gradient

$$V_{eff}(\varphi)=\int_{-\infty}^{\infty}E_z(\rho=0,z)\cos(\omega t(z)+\varphi)dz$$

- Time and position are linked through the velocity (assuming β doesn't change)

$$t=\frac{z}{v}=\frac{z}{\beta c},\qquad c=f\lambda \text{ and } \omega=2\pi f$$

$$V_{eff}(\varphi)=\int_{-\infty}^{\infty}E_z(\rho=0,z)\cos\left(\frac{2\pi z}{\beta\lambda}+\varphi\right)dz$$

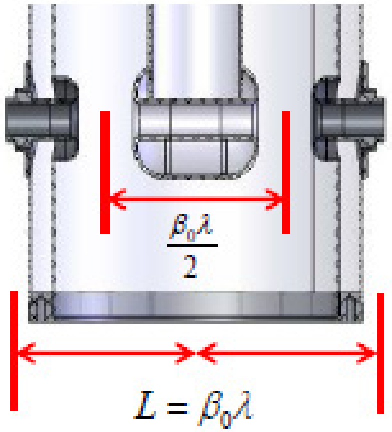
$$V_{eff}(\varphi)=\int_{-\infty}^{\infty}E_z(\rho=0,z)\left[\cos\left(\frac{2\pi z}{\beta\lambda}\right)\cos\varphi-\sin\left(\frac{2\pi z}{\beta\lambda}\right)\sin\varphi\right]dz$$

- Since E_z is typically an even function this simplifies to

$$V_{eff}(\varphi)=\int_{-\infty}^{\infty}E_z(\rho=0,z)\cos\left(\frac{2\pi z}{\beta\lambda}\right)dz\cos\varphi=V_c\cos\varphi$$

$$\text{where } V_c=\int_{-\infty}^{\infty}E_z(\rho=0,z)\cos\left(\frac{2\pi z}{\beta\lambda}\right)dz$$

is called the accelerating voltage (also V_c=V$_{acc}$=V$_{eff}$(0))



- Accelerating gradient (E_a) – Gives the effective voltage gain [MV/m]

$$E_a=\frac{V_{eff}}{L}\text{ where } L=n\frac{\beta_0\lambda}{2},\text{ n is the number of cells}$$

$$\text{where } V_{eff}=\int_{-\infty}^{\infty}E_z(z,t)dz \text{ with } \beta=\beta_0 \text{ and } \varphi=0$$

- Very important to specify length in defining E_a

Energy Gain, Transit Time Factor, Velocity Acceptance

- Energy gain: $$\Delta W = q \int_{-\infty}^{+\infty} E(z)\cos(\omega t + \phi)\, dz$$
- Velocity acceptance: $$\Theta = \frac{\int_{-\infty}^{\infty} E(z)\cos\left(\frac{\omega z}{\beta c}\right) dz}{\int_{-\infty}^{\infty} E(z)\cos\left(\frac{\omega z}{\beta_0 c}\right) dz}$$
- Velocity acceptance is the reduction of the energy gain associated with the time dependence of the field
- Transit time factor: Time variation of the field during particle transit through the gap

$$T(\beta) = \frac{\int_{-\infty}^{\infty} E(z)\cos\left(\frac{\omega z}{\beta_0 c}\right) dz}{\int_{-\infty}^{\infty} E(z) dz}$$

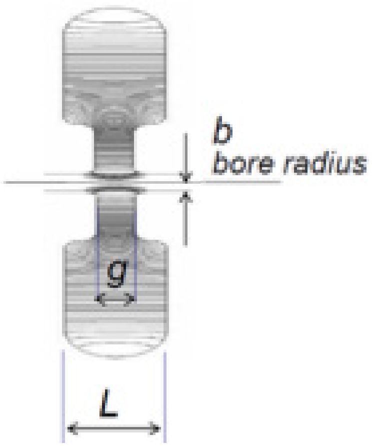
Achievable voltage at $\beta=\beta_0$

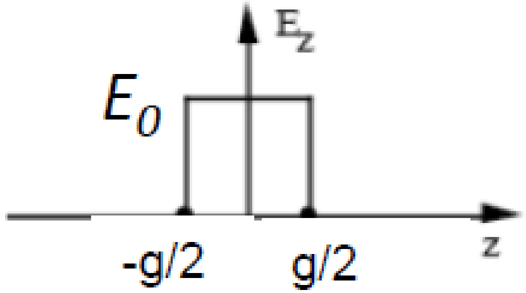
Maximum voltage at $\varphi=0$

- Transit time factor is the additional reduction of energy gain for particles whose velocity is different from optimal velocity (β_0)

Single Gap Structure

- For an accelerating gap with an accelerating field approximated with a square profile
- Here note that as g→0 the T→1, but small gaps cannot support high fields, so we optimize the gap geometry using a number of considerations





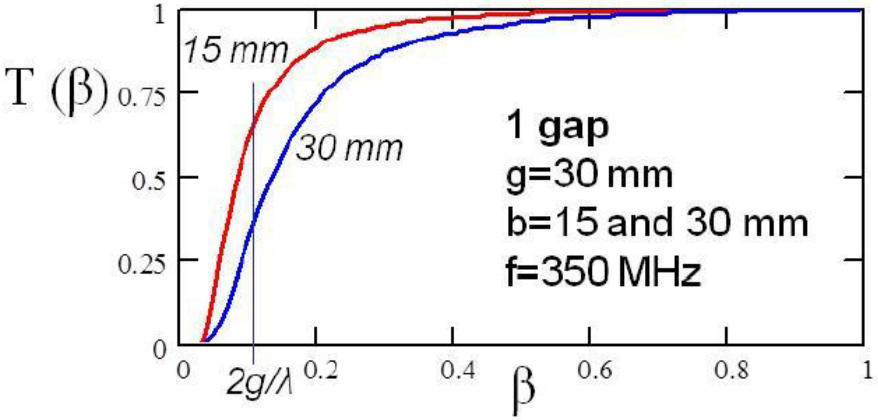
$$E_z(\rho=0,z)=E_0\,\big|_{-g/2}^{g/2}\;\text{ else }\;E_z(0,z)=0\;\text{ so }\;V_0=E_0L=E_0g$$

$$V_c=\int_{-g/2}^{g/2}E_0\cos\left(\frac{2\pi z}{\beta\lambda}\right)dz=E_0\frac{\beta\lambda}{\pi}\sin\left(\frac{\pi g}{\beta\lambda}\right)\qquad T=\frac{V_{eff}}{V_0}=\frac{\beta\lambda}{\pi g}\sin\left(\frac{\pi g}{\beta\lambda}\right)\;\text{ and }\;V_{eff}=E_0TL$$

$$T(\beta)\cong\frac{\sin\left(\dfrac{\pi g}{\beta\lambda}\right)}{\dfrac{\pi g}{\beta\lambda}}$$

Aperture b contributes to the effective gap length:

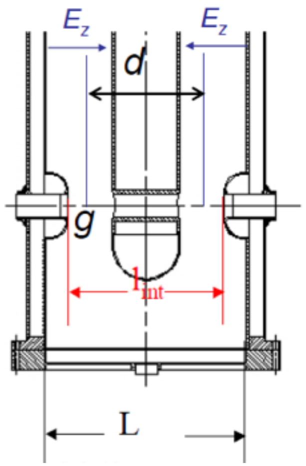
$$g_{eff}\approx\sqrt{g^2+(2b)^2}$$

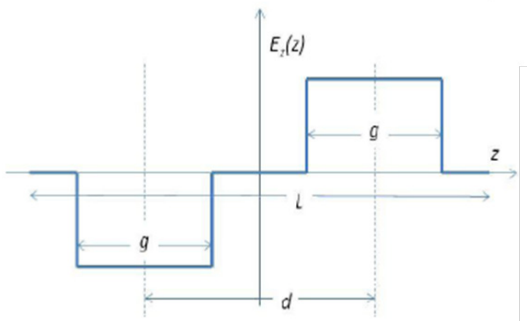


Rule of thumb: $g_{eff}<\beta\lambda/2\rightarrow T(\beta)>0.63$

Two Gap Structure

- For a two gap with an accelerating field approximated with a square profile in the π mode
- Slower or faster particles will get less acceleration due to poor synchronization with the rf phase

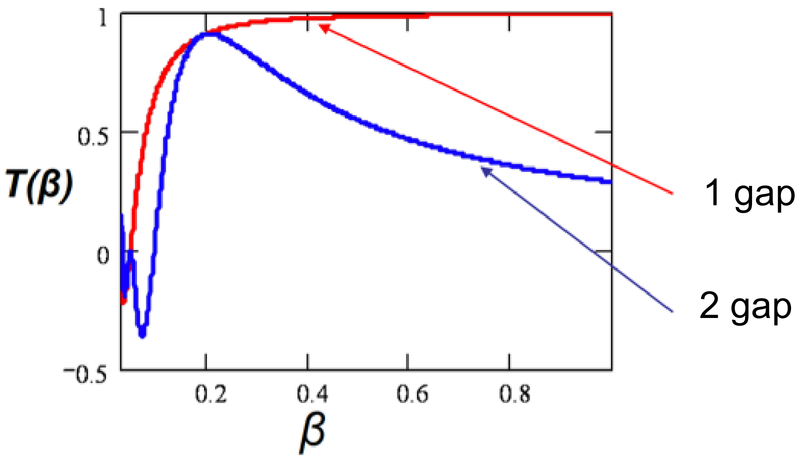




$$T_{2gap}(\beta,g)=\frac{\int_{cav} E(0,z)\sin\left(\frac{2\pi z}{\beta\lambda}\right)dz}{\int_{cav}\left|E(0,z)dz\right|}=\frac{\sin\left(\frac{\pi g}{\beta\lambda}\right)}{\frac{\pi g}{\beta\lambda}}\sin\left(\frac{\pi\beta_0}{2\beta}\right)\text{ where }\beta_0=\frac{2L}{\lambda}$$

$$\Delta W=qV_0T\cos\varphi=qV_{eff}\frac{T(\beta)}{T(\beta_0)}\cos\varphi$$

$$T(\beta)\cong\underbrace{\frac{\sin\left(\frac{\pi g}{\beta\lambda}\right)}{\frac{\pi g}{\beta\lambda}}}_{\substack{\text{1 gap term}\\ \textit{high if } g<\beta\lambda/2}}\underbrace{\sin\left(\frac{\pi d}{\beta\lambda}\right)}_{\substack{\text{2 gap term}\\ \textit{high if } d\sim\beta\lambda/2}}$$



Only 2nd term changes for more than 2 equal gaps in π mode

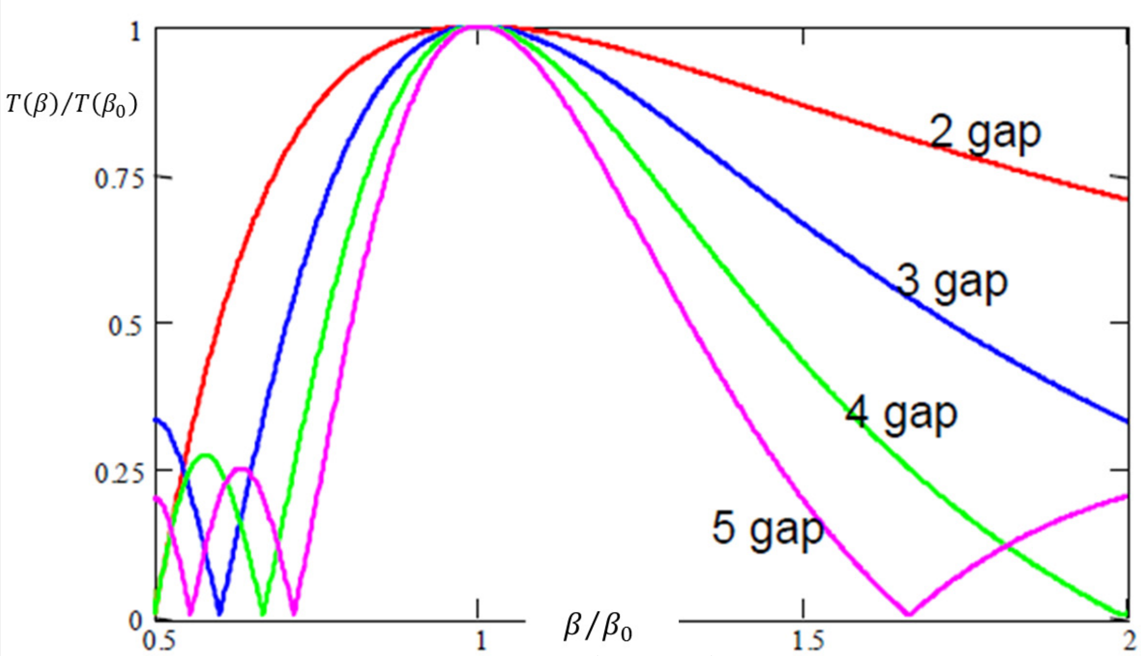
Multi Gap Structures

- Larger the number of gaps (n) larger the energy gain at a given gap voltage V_g
- But larger the number of gaps (n), narrower the velocity acceptance
  - Constant β calls for large n
  - Fast varying β calls for small n
- Higher number of gaps will provide more energy gain over a smaller velocity range

$V_{eff}=E_a*L$

Pay attention to definition of L

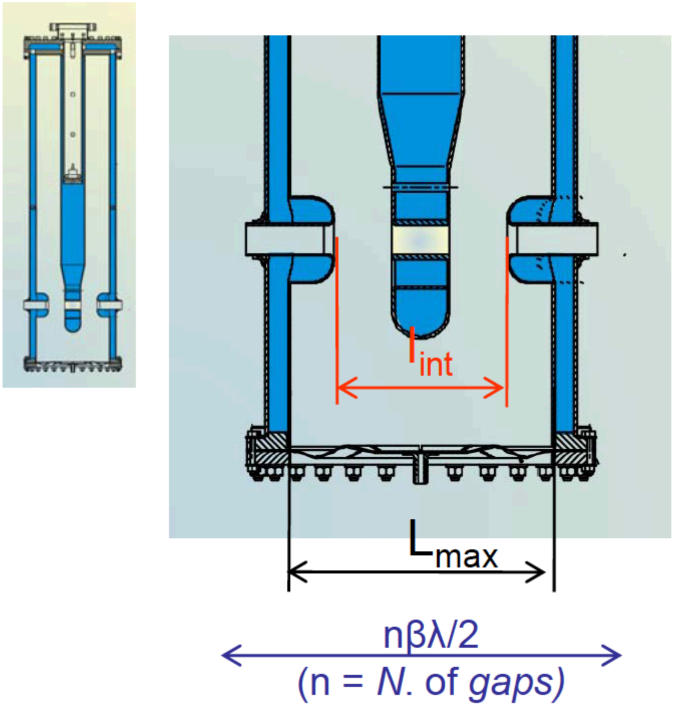
Normalized Transit Time Factor

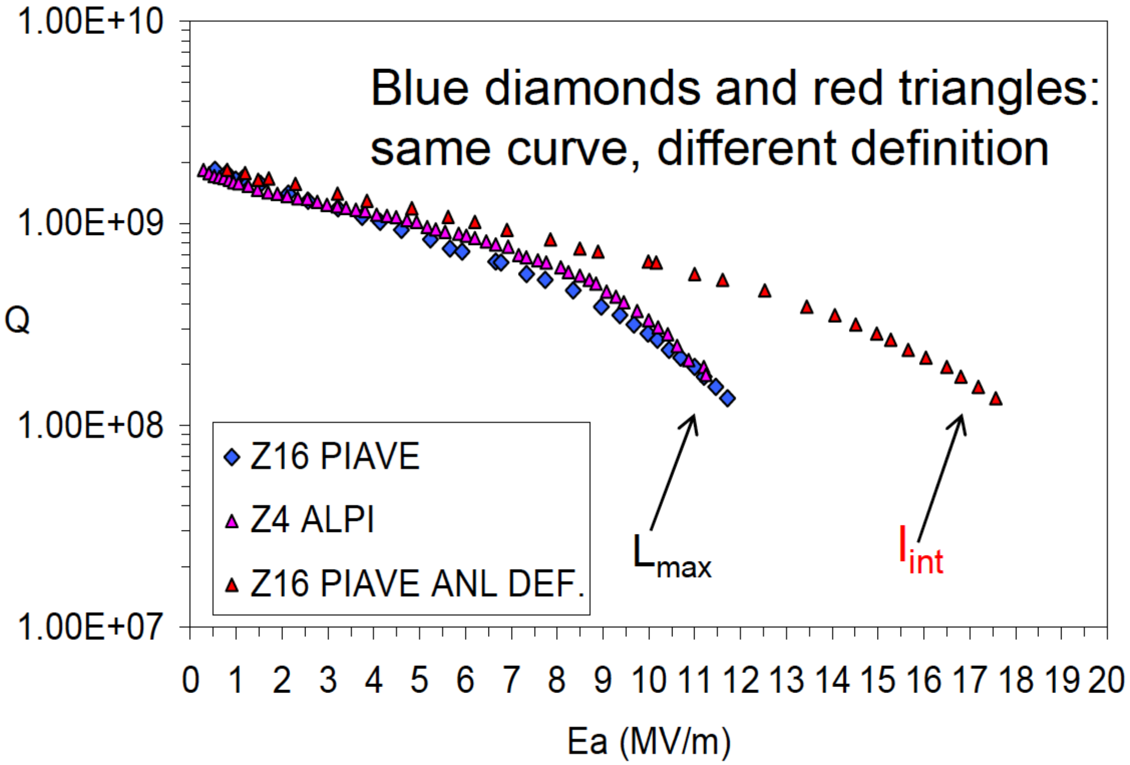


Normalized Velocity

Different Definitions of Accelerating Gradient

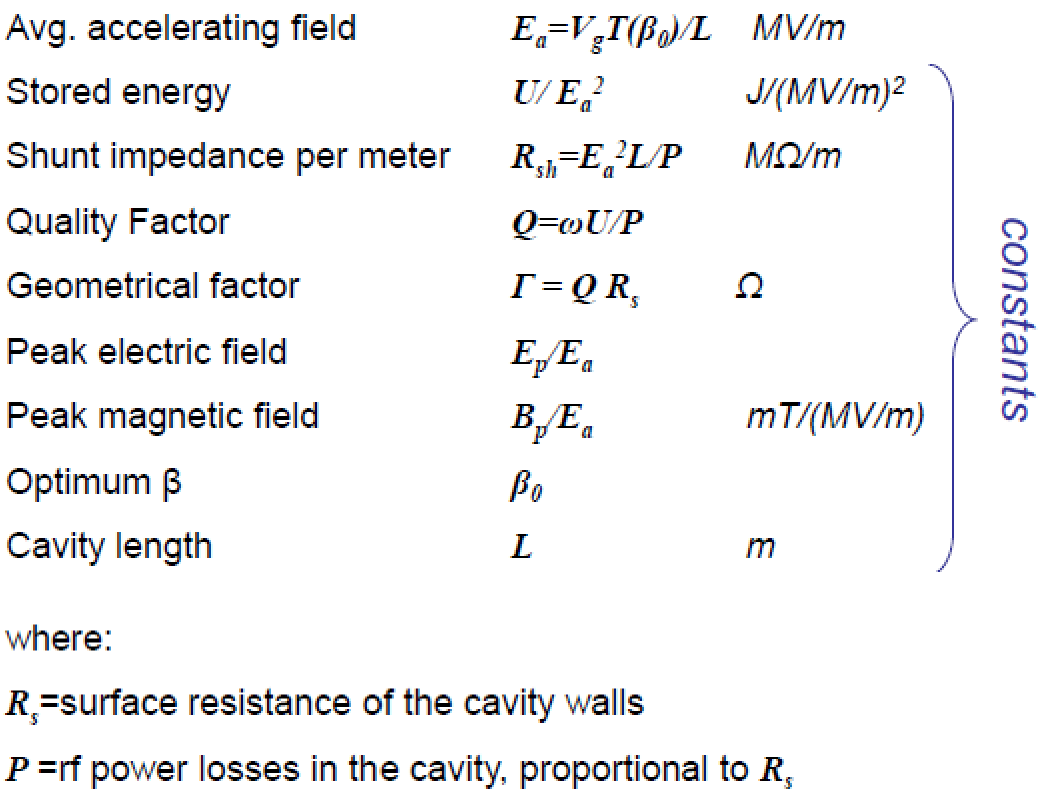
- Sometimes it is difficult to decide on the definition of L: l_{int}, $\boldsymbol{L_{max}}$, $n\beta\lambda/2$
- Shorter ***L*** is defined, larger E_a appears in Q vs E_a curves
- However, energy gain is always the same and all definitions are constant




Blue diamonds and red triangles: same curve, different definition

Important Parameters

- Parameters are the same for elliptical cavities
- Important to specify the cavity reference length in defining E_a



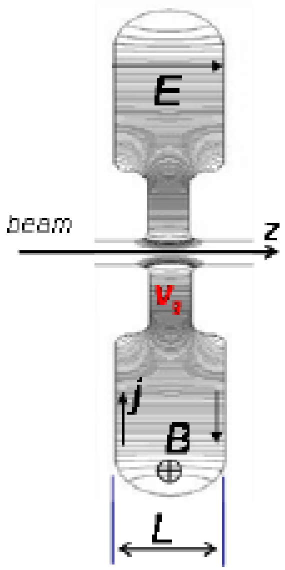
Avg. accelerating field	$E_a=V_gT(\beta_0)/L$	MV/m
Stored energy	U/E_a^2	J/(MV/m)²
Shunt impedance per meter	$R_{sh}=E_a^2L/P$	MΩ/m
Quality Factor	$Q=\omega U/P$	
Geometrical factor	$\Gamma = Q\,R_s$	Ω
Peak electric field	E_p/E_a	
Peak magnetic field	B_p/E_a	mT/(MV/m)
Optimum β	β_0	
Cavity length	L	m

constants

where:

R_s=surface resistance of the cavity walls

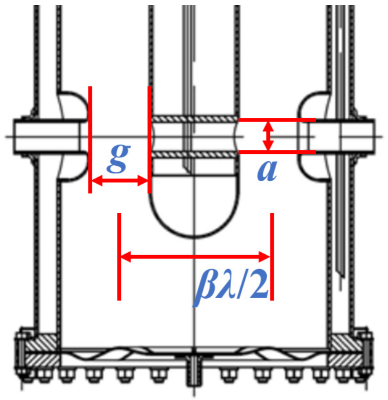
P =rf power losses in the cavity, proportional to R_s

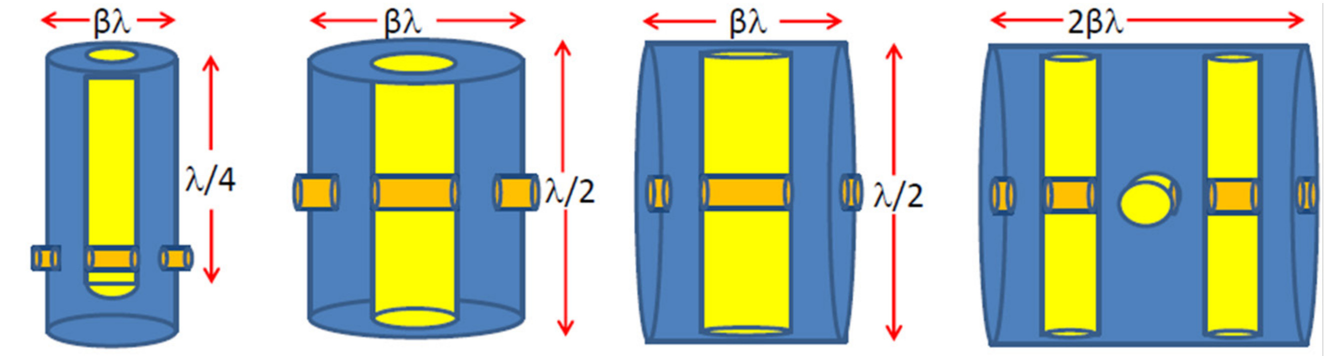


TYPES OF
NON-ELLIPTICAL ACCELERATING CAVITIES

TEM-Type Cavities

- Transverse Electro Magnetic (TEM) mode cavities
  - Mode is related to the cavity symmetry axis
  - Produce accelerating voltages across the coaxial gap with variable gap distance and with transverse dimensions ~2-4 times smaller than an elliptical cavities for the same frequency
- Acceleration typically uses π mode with rf phase advance of 180 deg
  - Requires distance of $\beta\lambda/2$ between gaps for synchronism
- Good transverse acceptance requires a large aperture - efficient rf acceleration requires a gap (g) to aperture (a) ratio g/a>1 and a gap size ~50% of the cell length
  - Low velocities require low frequencies with large wavelengths
  - Low frequency cavities have large accelerating gaps

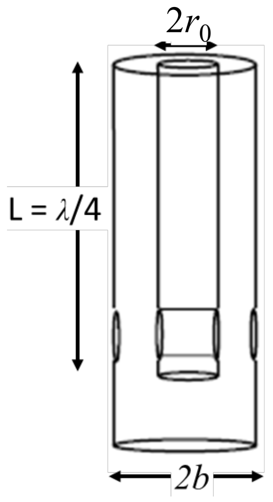


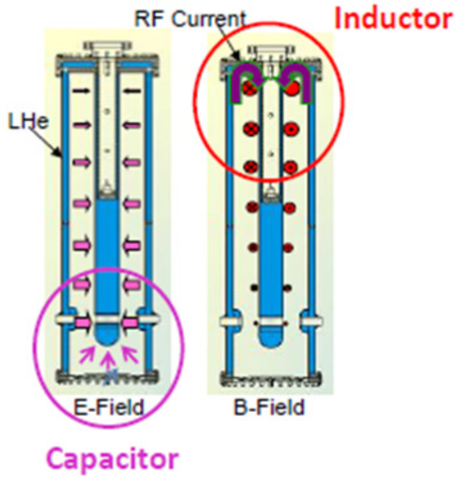


Quarter Wave - QWR
Half Wave - HWR
Single spoke - SSR
Multi-spoke - MSR

Quarter Wave Resonator (QWR)

- QWR → A capactively loaded $\lambda/4$ transmission line
- Maximum voltage builds up on the open tip and maximum current at the plate connecting the center conductor
- Beam tube is placed near the end of the tip to produce a high voltage double gap acceleration geometry





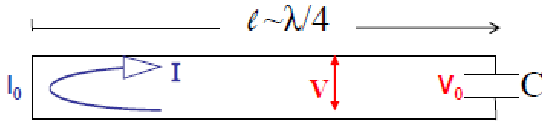
Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{r_0}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$

Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$

$$\rho_0 = \frac{r_0}{b}$$

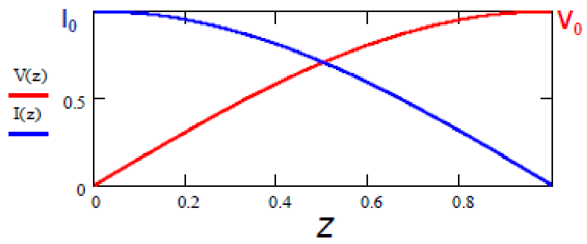


Center conductor voltage

$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda}z\right)$$

Center conductor current

$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda}z\right)$$



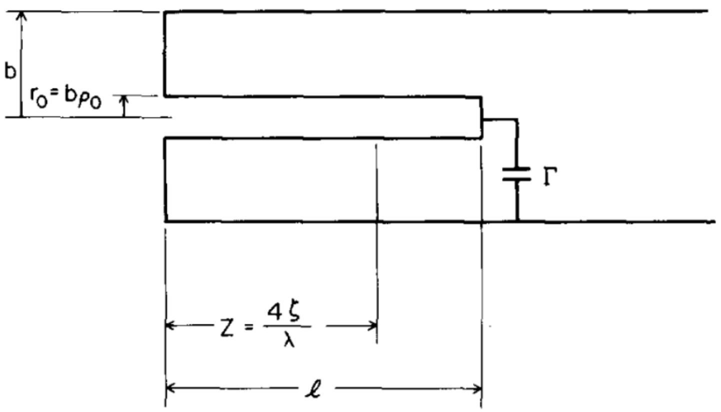
Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi}\ln\left(\frac{1}{\rho_0}\right), \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

Quarter Wave Resonator (QWR)

Optimizing the expected performance of a resonator for given frequency and β

V_p – voltage on the center conductor with outer conductor at ground



Peak Magnetic Field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \sin\left(\frac{\pi}{2}\zeta\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

$B \simeq 9$ mT at 1 MV/m

Geometrical Factor

$$G = QR_s = 2\pi\,\eta \frac{b}{\lambda} \frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

R_s – surface resistance

Energy Content

$$U = V_p^2 \frac{\pi\varepsilon_0}{8} \lambda \frac{1}{\ln(1/\rho_0)} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

Power Dissipation (Ignore losses in the shorting end plate)

$$P = V_p^2 \frac{8}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta\,\lambda^2$$

Shunt Impedance $\left(4V_p^2/P\right)$

$$R_{sh} = \frac{\eta^2}{R_s} \frac{32}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1+1/\rho_0} \frac{\sin^2 \frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin \pi\zeta}$$

$$R_{sh}\, R_s \propto \eta^2\,\beta$$

Other notations R_{sh} , R_a

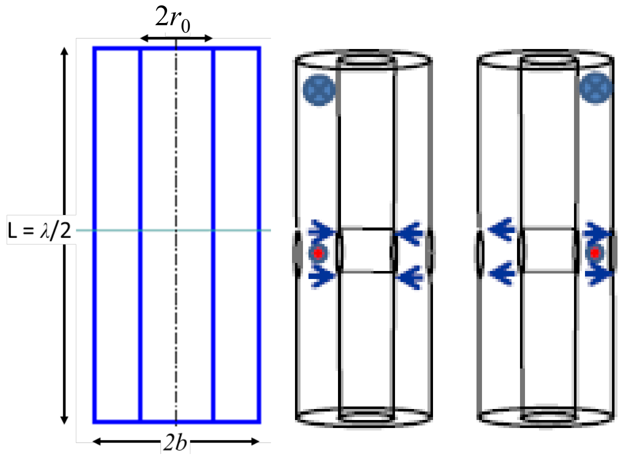
R/Q

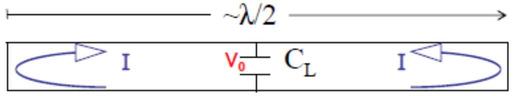
$$\frac{R_{sh}}{Q} = \frac{16}{\pi^2} \eta \ln(1/\rho_0) \frac{\sin^2 \frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin \pi\zeta}$$

$$\frac{R_{sh}}{Q} \propto \eta$$

Half Wave Resonator (HWR)

- HWR → A $\lambda/2$ transmission line
- Equivalent to 2 QWR facing each other and connected
- Magnetic field loop around the inner conductor with peak fields at the shorted ends
- Beam tube is placed at the center of the inner conductor to coincide with the maximum voltage
- Same accelerating voltage is obtained at about 2 times larger power in QWR ($P_{HWR} \sim 2P_{QWR}$)





Capacitance per unit length

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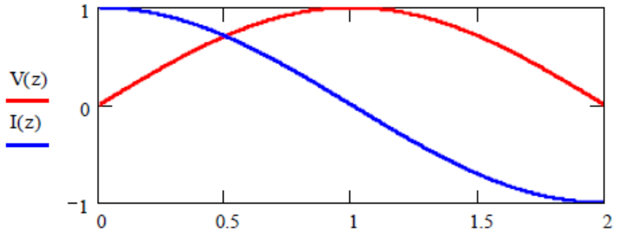
Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$

$$\rho_0 = \frac{r_0}{b}$$



Center conductor voltage

$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda}z\right)$$

Center conductor current

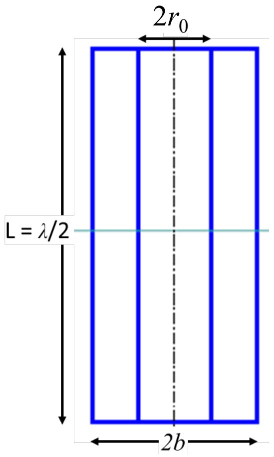
$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda}z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi}\ln\left(\frac{1}{\rho_0}\right), \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

Half Wave Resonator (HWR)

Optimizing the expected performance of a resonator for given frequency and β



V_p – voltage on the center conductor with outer conductor at ground

Peak Magnetic Field

$$\frac{V_p}{b} = \left\{\begin{array}{ll} \eta & H \\ c & B \\ 300 & B \end{array}\right\} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \qquad \left\{\begin{array}{l} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{array}\right\}$$

V_p: Voltage across loading capacitance

$B \simeq 9$ mT at 1 MV/m

Geometrical Factor

$$G = QR_s = 2\pi\,\eta \frac{b}{\lambda} \frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

R_s – surface resistance

Energy Content

$$U = V_p^2 \frac{\pi\varepsilon_0}{4} \lambda \frac{1}{\ln(1/\rho_0)}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

Power Dissipation (Ignore losses in the shorting end plates)

$$P = V_p^2 \frac{16}{\pi} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

Shunt Impedance

$$R_{sh} = \frac{\eta^2}{R_s} \frac{16}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1+1/\rho_0}$$

$$R_{sh}\, R_s \propto \eta^2\, \beta$$

R/Q

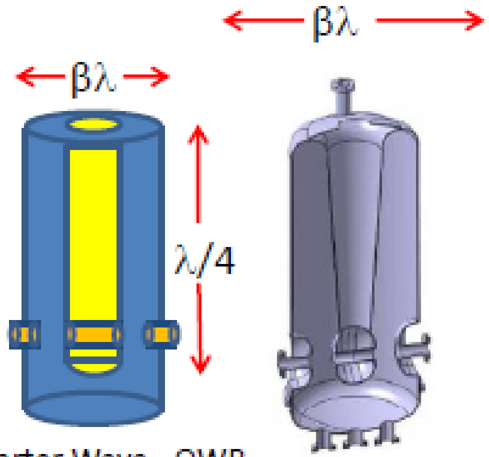
$$\frac{R_{sh}}{Q} = \frac{8}{\pi^2} \eta \ln(1/\rho_0)$$

$$\frac{R_{sh}}{Q} \propto \eta$$

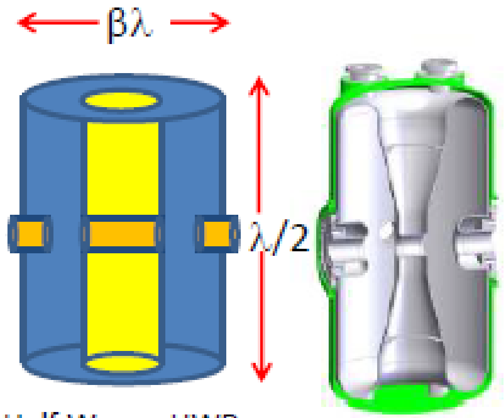
QWR vs HWR

- QWR is the choice for low applications where a low frequency is needed
  - Requires ~50% less structure compared to HWR for same frequency
  - RF power loss is ~50% of HWR for same frequency and β_0
  - Allows low frequency cavities with larger voltage acceptance $(R_{sh}/Q_{QWR}) = 2(\,R_{sh}/Q_{HWR})$
  - Asymmetric field pattern introduces vertical steering especially for light ions that increase with velocity (Avoid using for β_0>0.2)
  - Mechanically less stable than HWR due to unsupported end

- HWR is chosen for mid velocity range (β_0>0.2) or where steering must be eliminated (ie. High intensity light ion applications)
  - Produces 2× more rf losses for the same frequency and β_0
  - 2× longer for the same frequency
  - Symmetric field pattern and increased mechanical rigidity



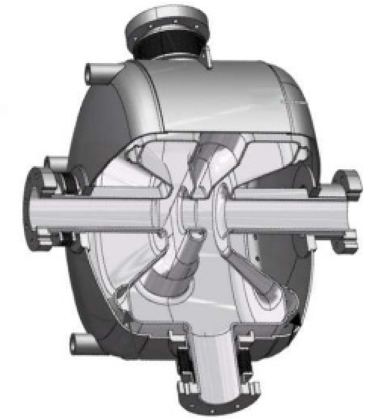
Quarter Wave - QWR

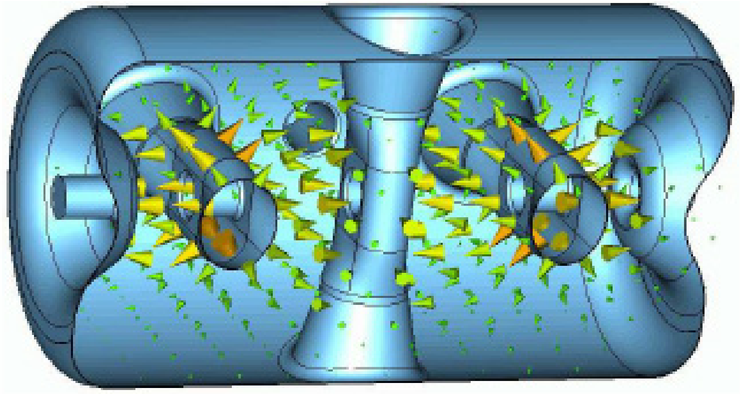


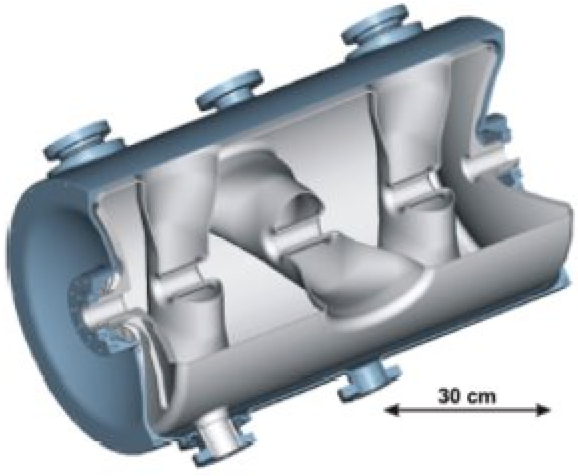
Half Wave - HWR

Single and Multi Spoke Cavities

- Supposed to cover ranges β=0.1-0.6 with *f*=300-900 MHz
- Spoke cavities are also designed at high β and β =1
- Single spoke cavities are same as TEM-like HW cavities with respect to the spoke axis
- Single spoke geometries allow extension along the beam path to provide multipole spoke
  - In multi spoke cavity spokes are rotated 90 deg from cell to cell
  - Higher effective voltage with low velocity acceptance
  - Strong cell-to-cell coupling with cells linked by the magnetic field



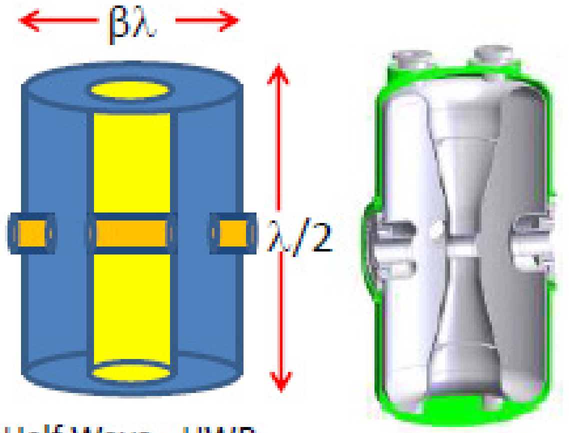




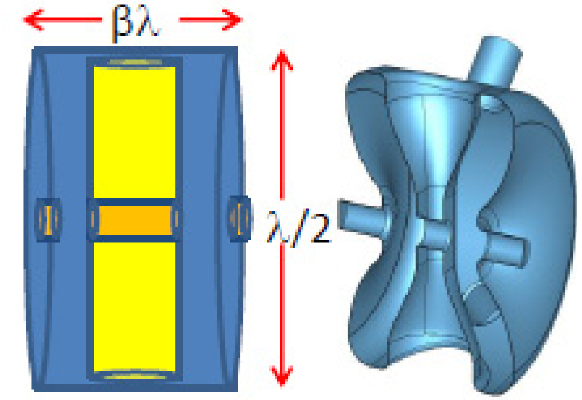
HWR vs Single Spoke Cavities

- Single spoke resonator (SSR) is another variant of the half wave TEM mode cavities
- In HWR the outer conductor (with diameter $\beta_0 \lambda$) is coaxial with the inner conductor
- In SSR the outer cylinder (with diameter $\lambda/2$) is coaxial with beam pipes
  - For $\beta_0 < 0.5$ the SSR has larger overall physical envelop than the HWR for the same frequency

- Cavity choices:
  - For low β applications (β_0 =0.1–0.25)
    - HWR is chosen for ~160 MHz
    - SSR is chosen for ~320 MHz
  - For higher β applications (β_0 =0.25–0.5)
    - HWR and SSR are chosen for ~320 MHz



Half Wave - HWR

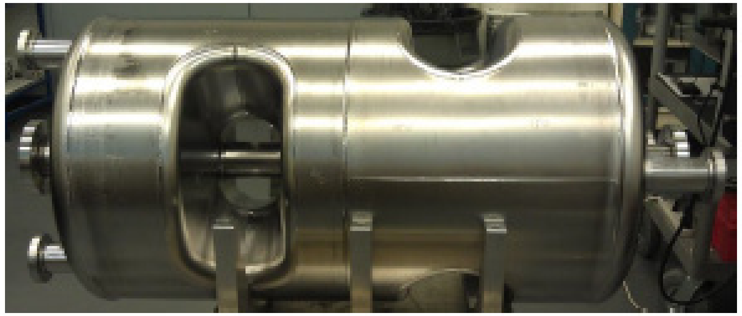


Single spoke - SSR

High β Spoke Cavities

- High velocity spoke cavities with $\beta_0 > 0.8$ are being designed as an alternative to high β elliptical cavities
- Cavity features
  - Cavities are relatively compact
    - Between 20% - 50% smaller (radially) than a TM cavity of the same frequency and β_0
    - For high β_0 cavities diameter is close to TM counterparts
  - Allows low frequency at reasonable size with high longitudinal acceptance
  - Mechanically stable
  - Allows possible 4 K operation
  - Can achieve high shunt impedance
- Possible applications
  - For pulsed spallation neutron sources
  - Compact light sources

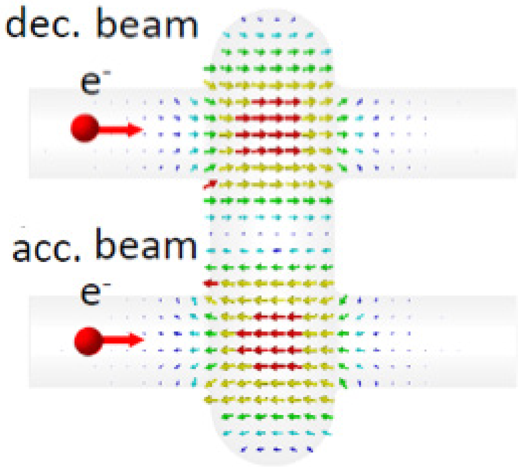
325 MHz β=0.82 Single Spoke Cavity

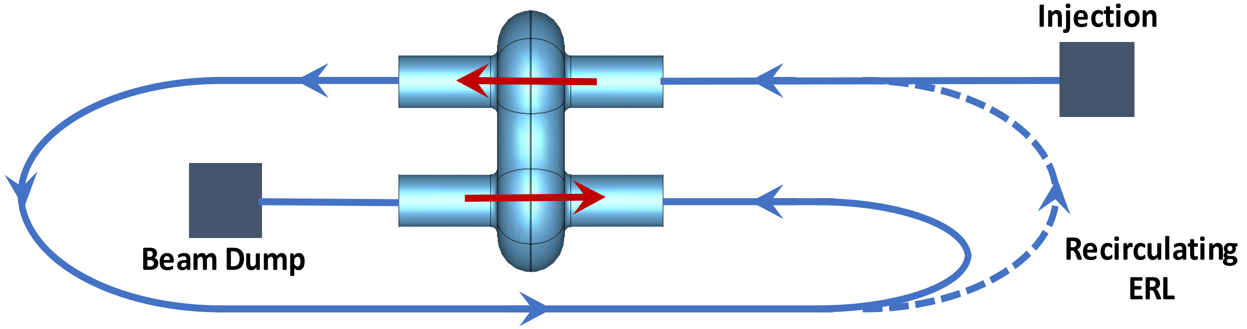


500 MHz β=1.0 Double Spoke Cavity

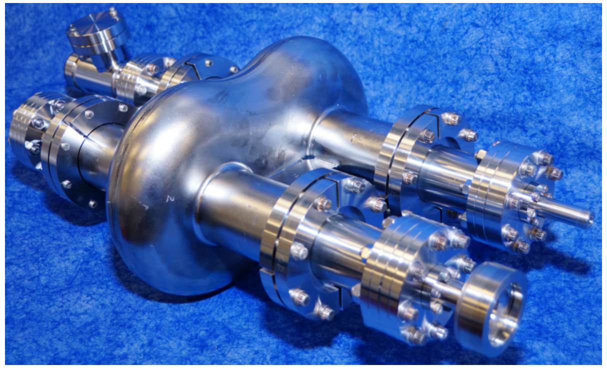
TM-Type Non-Elliptical Cavities

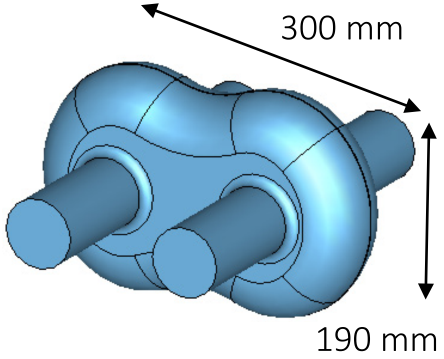
- Cavity with two beam pipes → Twin axis cavity / dual axis cavity
- A superconducting cavity designed to accelerate and decelerate two electron beams in the same cavity
- Used in energy recovery with two separated beams traversing the cavity at the same time



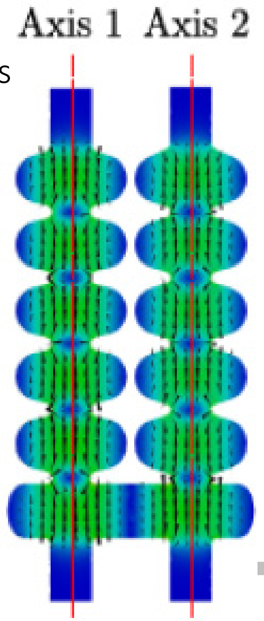


Twin axis cavity





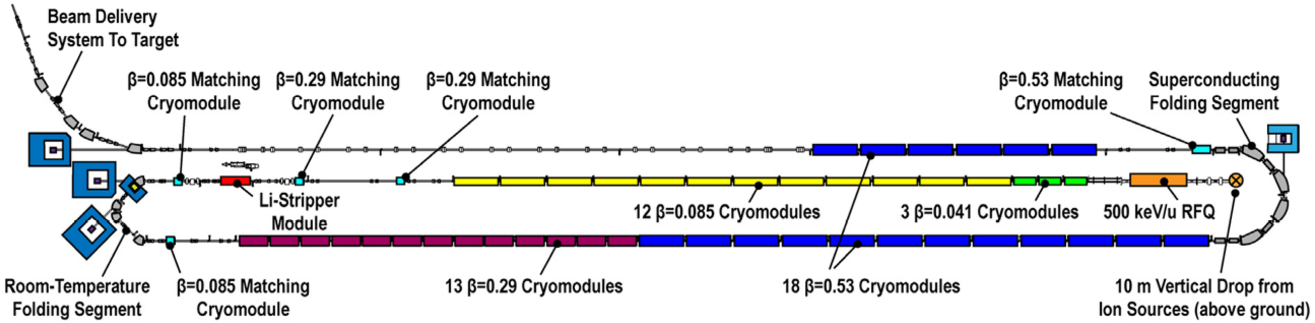
Dual axis cavity

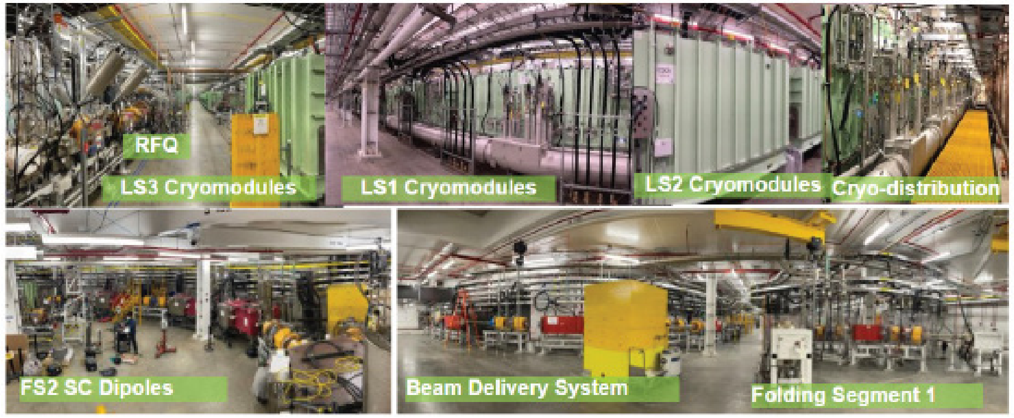


REAL CAVITIES AND ACCELERATOR APPLICATIONS

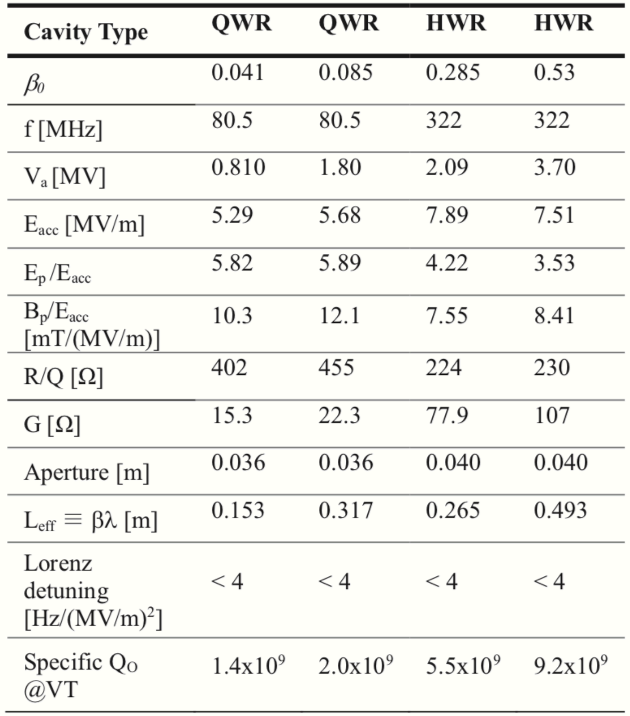
Facility of Rare Isotope Beams (FRIB)

- A high intensity, heavy ion linac
- Accelerate ion species up to ^{238}U (A/q=7) with energies of no less than 200 MeV/u

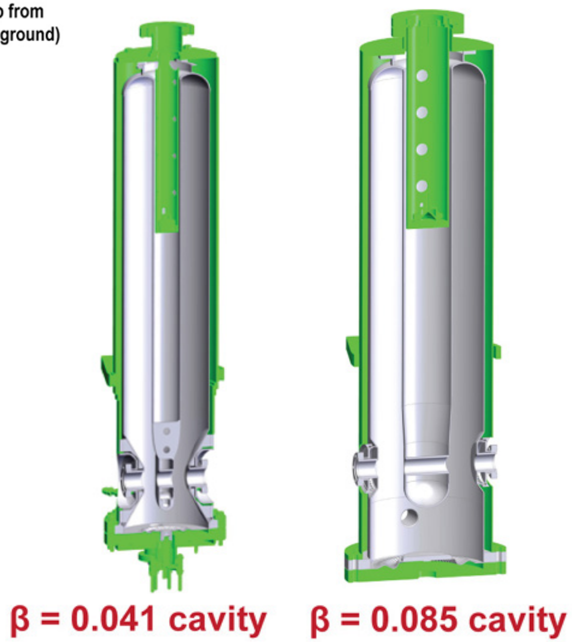




- No. of cavities:
  - QWR (β_0=0.041) → 12
  - QWR (β_0=0.085) → 88
  - HWR (β_0=0.285) → 72
  - HWR (β_0=0.53) → 144



Cavity Type	QWR	QWR	HWR	HWR
β_0	0.041	0.085	0.285	0.53
f [MHz]	80.5	80.5	322	322
V_a [MV]	0.810	1.80	2.09	3.70
E_{acc} [MV/m]	5.29	5.68	7.89	7.51
E_p/E_{acc}	5.82	5.89	4.22	3.53
B_p/E_{acc} [mT/(MV/m)]	10.3	12.1	7.55	8.41
R/Q [Ω]	402	455	224	230
G [Ω]	15.3	22.3	77.9	107
Aperture [m]	0.036	0.036	0.040	0.040
$L_{eff} \equiv \beta\lambda$ [m]	0.153	0.317	0.265	0.493
Lorenz detuning [Hz/(MV/m)^2]	< 4	< 4	< 4	< 4
Specific Qo @VT	1.4x10^9	2.0x10^9	5.5x10^9	9.2x10^9



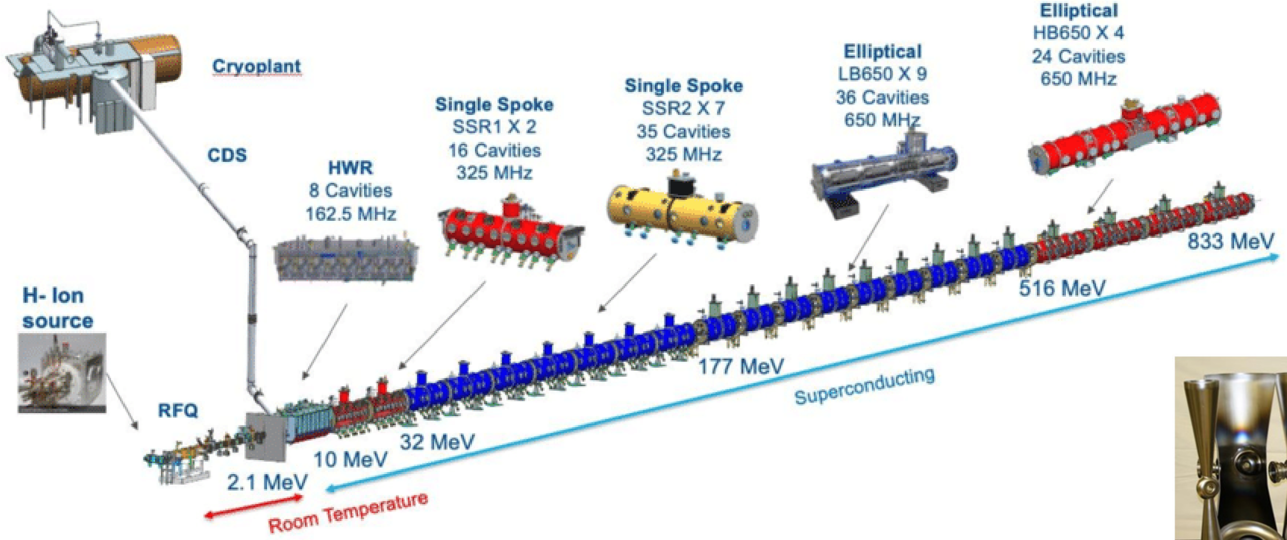
FRIB β=0.041, 0.085 80.5 MHz

FRIB β=0.29, 0.53 322 MHz

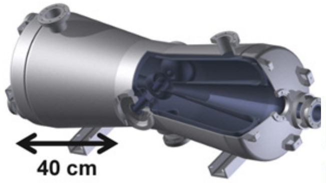
Proton Improvement Plan II (PIP II) at Fermilab

- 800 MeV linac to accelerate protons to generate intense neutrino beam for the Long Baseline Neutrino Facility (LBNF) and Deep-Underground Neutrino Experiment (DUNE)

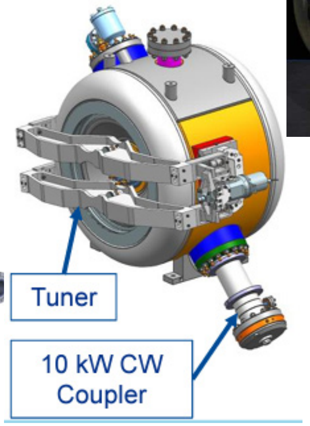
Cavity Type	**HWR**	**SSR1**	**SSR2**	**LB650**	**HB650**
Frequency [MHz]	162.5	325	325	650	650
Optimal β	0.112	0.22	0.475	0.61	0.97
Effective Length [cm]	20.7	20.3	43.8	70.3	106.1
Aperture [mm]	33	30	40	41.5	59
E_{peak}/E_{acc}	4.7	3.84	3.38	2.43	2.1
B_{peak}/E_{acc} [mT/(MV/m)]	5.0	5.81	5.93	4.6	3.94
G [Ω]	48	84	115	187	260
R_{sh}/Q [[Ω]	272	242	297	327.4	576



Section	Freq	Energy (MeV)	Cav/mag/CM	Type
RFQ	162.5	0.03-2.1		
HWR (β_{opt}=0.11)	162.5	2.1-10.3	8/8/1	HWR, solenoid
SSR1 (β_{opt}=0.22)	325	10.3-35	16/8/ 2	SSR, solenoid
SSR2 (β_{opt}=0.47)	325	35-185	35/21/7	SSR, solenoid
LB 650 (β_g=0.61)	650	185-500	33/22/11	5-cell elliptical, doublet*
HB 650 (β_g=0.92)	650	500-800	24/8/4	5-cell elliptical, doublet*



162.5 MHz HWR



325 MHz SSR1

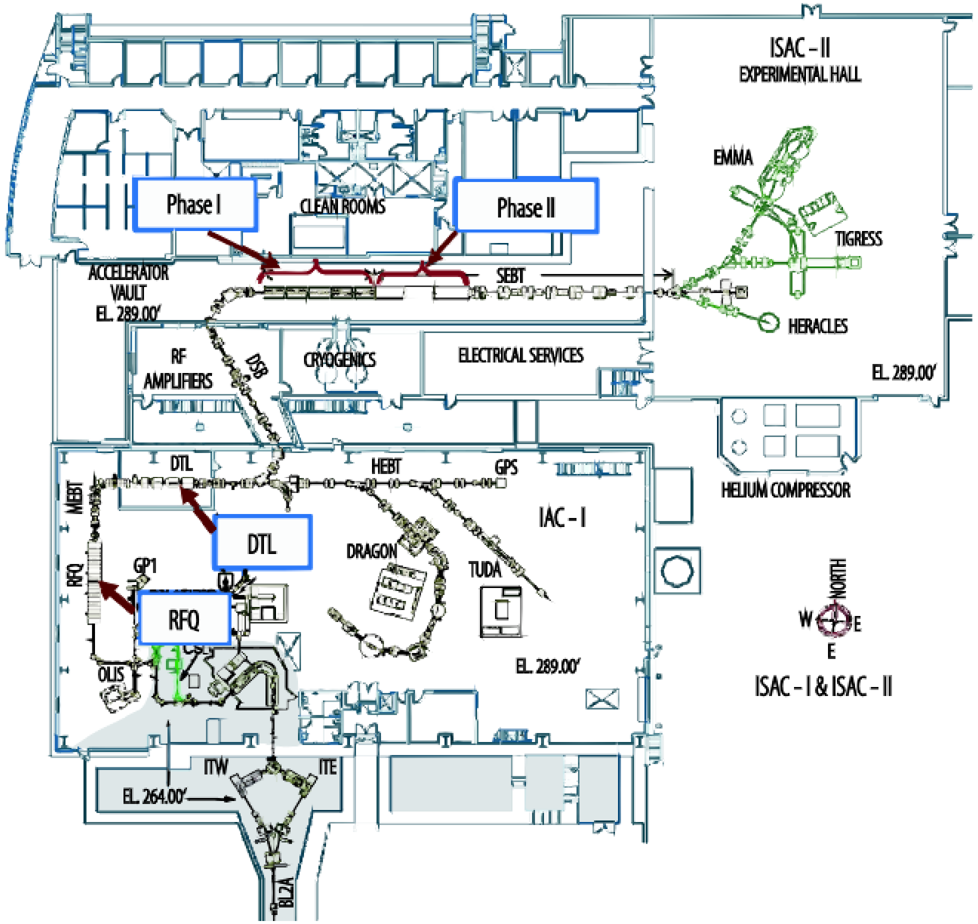
325 MHz SSR2

HB650 5-cell Elliptical Cavity

LB650 5-cell Elliptical Cavity

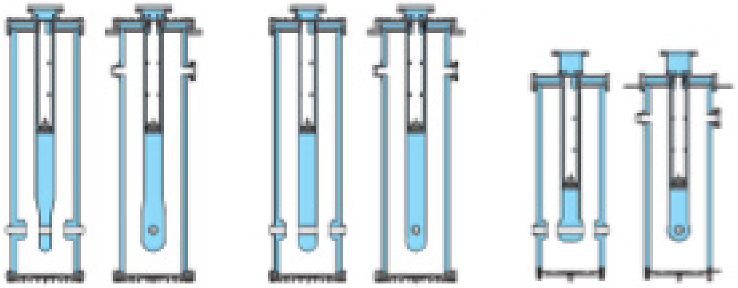
Isotope Separator and Accelerator (ISAC II) and TRIUMF

- To produce rare isotope beams



- Consists of 40 QWRs with three different cavity types of β_0=5.7%, 7.1% and 11% with N = 8, 12 and 20
- All cavities have same outer and inner conductor diameter to reduce fabrication cost

TRIUMF ISAC-II Resonators

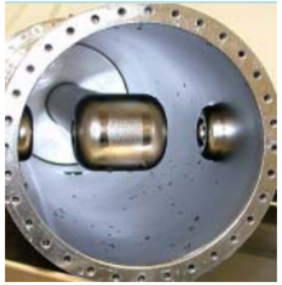


SCB low β=0.057
106.08 MHz

SCB medium β=0.071
106.08 MHz

SCB high β=0.11
141.44 MHz

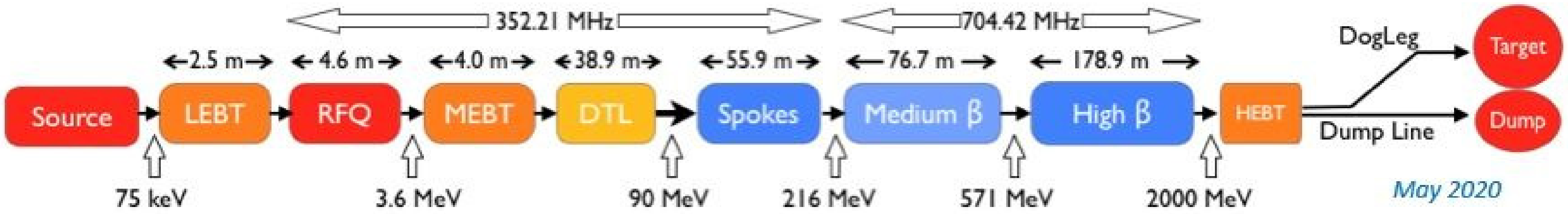
106.08 MHz β=0.057



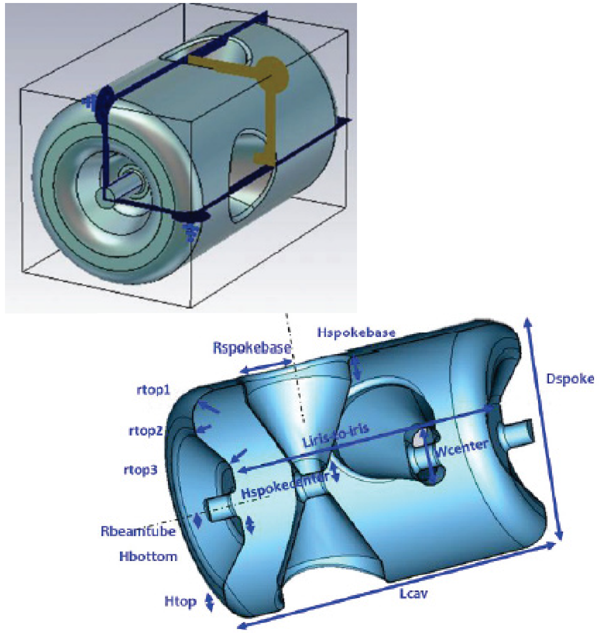
141.44 MHz β=0.11

European Spallation Source (ESS)

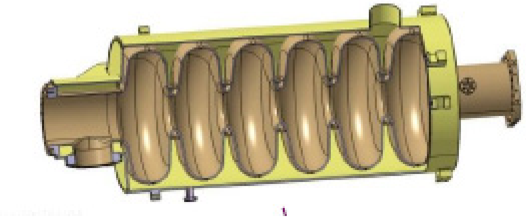
- 5 MW long pulse proton accelerator



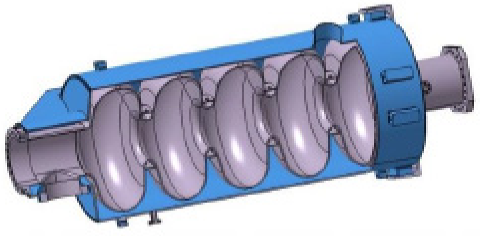
- Cavity types:
  - Spoke cavities – β_0=0.5
  - Medium β elliptical cavities – β_0 =0.67
  - High β elliptical cavities – β_0 =0.86



β=0.5 352 MHz Double Spoke Cavity



β=0.67 704 MHz Medium-β Elliptical Cavity

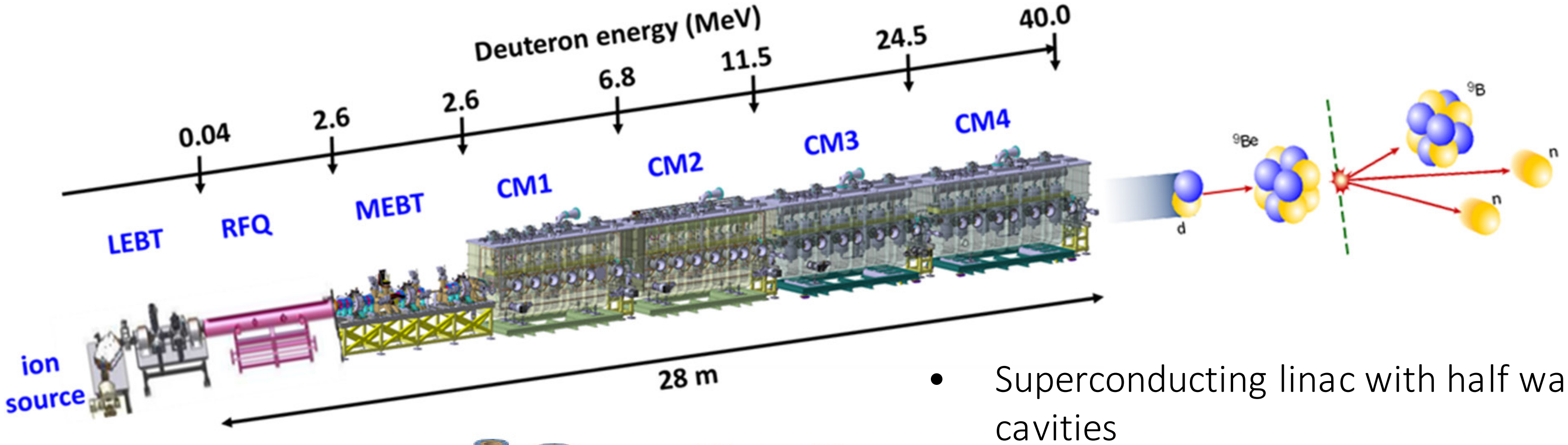


β=0.86 704 MHz High-β Elliptical Cavity

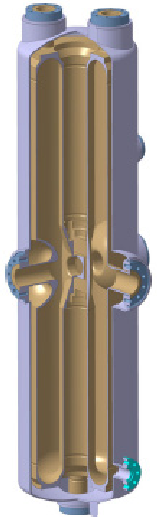
Style	Spoke	Medium-β	High-β
Freq. (MHz)	352.21	704.42	704.42
Cavity #	26	36	84
Velocity range	0.42 to 0.58	0.58 to 0.78	0.78 to 0.95
Nom. Acc. Voltage (MV)	5.74	14.3	18.2
Loaded quality factor	2.85×10^5	8×10^5	7.6×10^5
Dynamic heat load (W)	0.8	4.9	5.5
Max. forward power (kW)	335	1100	1100
RF (baseline)	Tetrode	Klystron	IOT

Soreq Applied Research Accelerator Facility (SARAF)

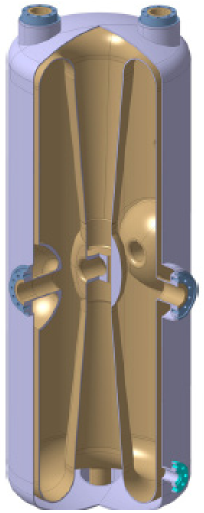
- To accelerate 5 mA beam of protons from 1.3 to 35 MeV or deuterons from 2.6 to 40 MeV



- Superconducting linac with half wave cavities
- Cryomodule layout:
  - β_0=0.09 – CM1 (6 cavities) and CM2 (7 cavities)
  - β_0=0.18 – CM3 (7 cavities) and CM4 (7 cavities)



176 MHz β=0.09



176 MHz β=0.18

	Low β cavity	*High β cavity*
β_{opt}	0.091	0.181
E_{acc}	7 MV/m	8.1 MV/m
U_{acc}	1.07 MV	2.49 MV
E_{pk}	34.5 MV/m	35.8 MV/m
B_{pk}	65.6 mT	65.3 mT
Q_0@45 nΩ	$8\cdot10^8$	$1.2\cdot10^9$
Stored energy	5.7 J	16.8 J
P_{cryo} @45 nΩ	7.9 W	15.5 W

FRIB β=0.041, 0.085 80.5 MHz FRIB β=0.29, 0.53 322 MHz

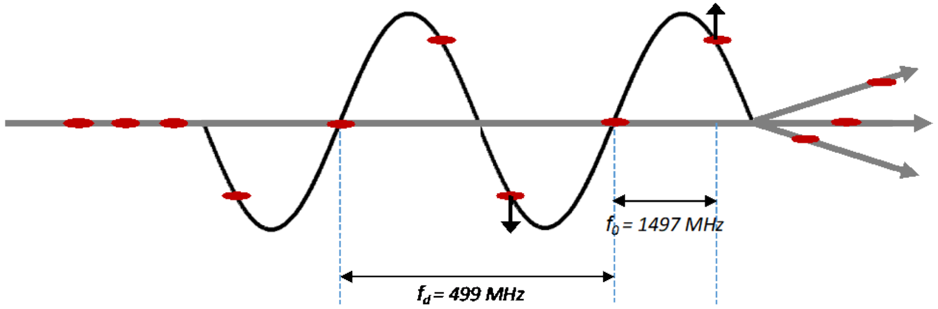
DEFLECTING/CRABBING CAVITIES

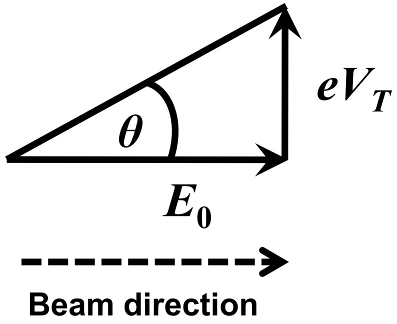
Deflecting/Crabbing Concept

- Deflecting/crabbing resonant cavities are required to generate a transverse momentum

Deflecting Cavities

- To separate a single beam to multiple beams

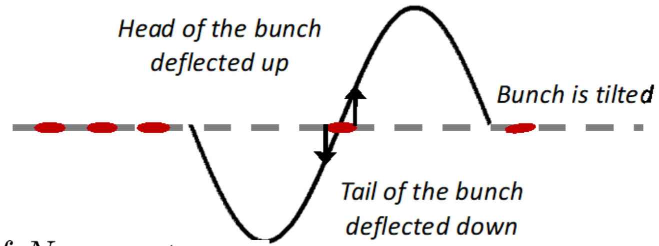




$$\theta = \arctan\left[\frac{eV_t}{E_0}\right] \sim \frac{eV_t}{E_0} \quad \mathrm{V}_t = E_0\,[eV]\,\theta\,[rad]$$

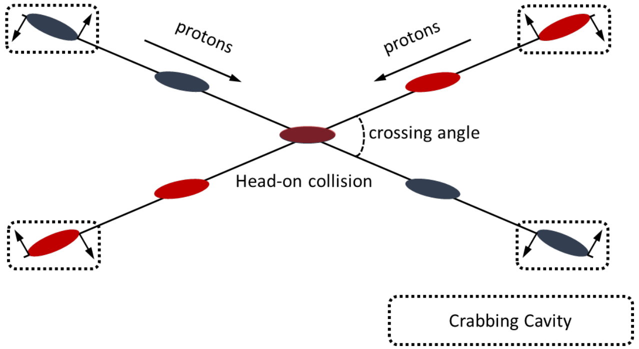
Crabbing Cavities

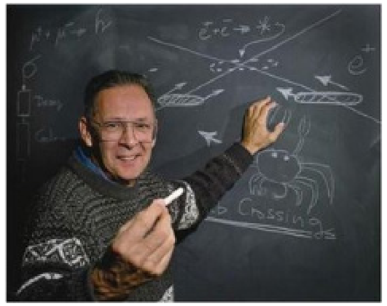
- To increase luminosity in colliding bunches by allowing head-on-collision of beams



$$\mathcal{L} = \frac{N_1 N_2 f_c N_b}{4\pi\sigma_x\sigma_y} F_c = \frac{N_1 N_2 f_c N_b}{4\pi\sigma_x\sigma_y}\frac{1}{\sqrt{1+\left(\frac{\sigma_z\theta_c}{2\sigma_x}\right)^2}}$$

$$V_t = \frac{cE_0\tan(\theta_c/2)}{\omega\sqrt{\beta_{crab}\beta^*}\sin\left(\psi^x_{cc\to ip}\right)}$$





First crabbing concept proposed by R. Palmer (1988)

Deflecting/Crabbing Cavities

- Can be produced by either or by both transverse electric (E_t) and magnetic (B_t) fields
- Lorentz force: $\vec{p}_t = \int_{-\infty}^{\infty} \vec{F}_t \, \mathrm{d}t = \frac{q}{v} \int_{-\infty}^{+\infty} \left[\vec{E}_t + j(\vec{v} \times \vec{B}_t) \right] \, \mathrm{d}z$
- Transverse momentum is related to the gradient of the longitudinal electric field along the beam axis (Panofsky Wenzel theorem)

$$\vec{p}_t = -i\frac{q}{\omega} \int_{-\infty}^{+\infty} \vec{\nabla}_t E_z \, \mathrm{d}z$$

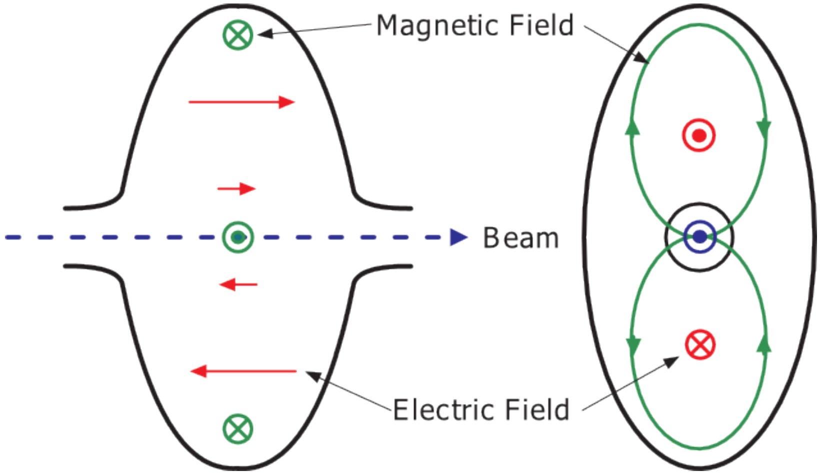
$$p_t = -i\frac{q}{\omega} \lim_{r_0 \to 0} \frac{1}{r_0} \int_{-\infty}^{+\infty} [E_z(r_0, z) - E_z(0, z)] \, \mathrm{d}z$$

- According to the theorem:
  - In a pure TE mode the contribution to the deflection from the magnetic field is completely cancelled by the contribution from the electric field

- Types of designs:
  - TM-type designs → Main contribution from B_t
  - TE-like designs → Main contribution from E_t
  - TEM-type designs → Contribution from both E_t and B_t

TM-Type Deflecting/Crabbing Cavities

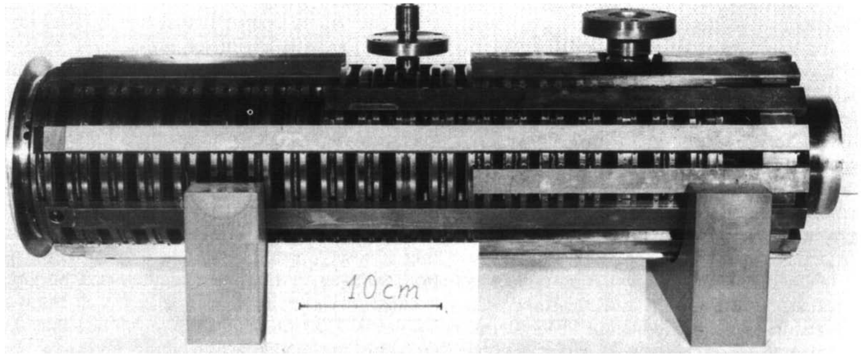
- Operates in TM_{110}-type mode
  - Lowest deflecting mode
- Squashed elliptical geometry: To separate the two polarizations of same frequency
- Contribution to the net deflection is mainly from transverse magnetic field

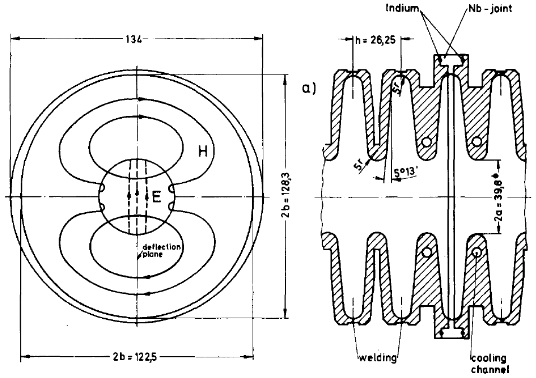


- Requires damping of the lowest mode (TM_{010}) in the design
- Cavity frequency is inversely proportional to transverse dimensions
- Cavity length ~ $\lambda/2$
- Large with respect to wavelength compared to new designs
  - Disadvantageous for low frequency
  - Advantageous for high frequency
  - Able to accommodate large apertures

1st Deflecting/Crabbing Cavities

- 1st superconducting deflecting cavity
  - Deflecting cavity: 2.865 GHz Karlsruhe/CERN Separator (104 cells)

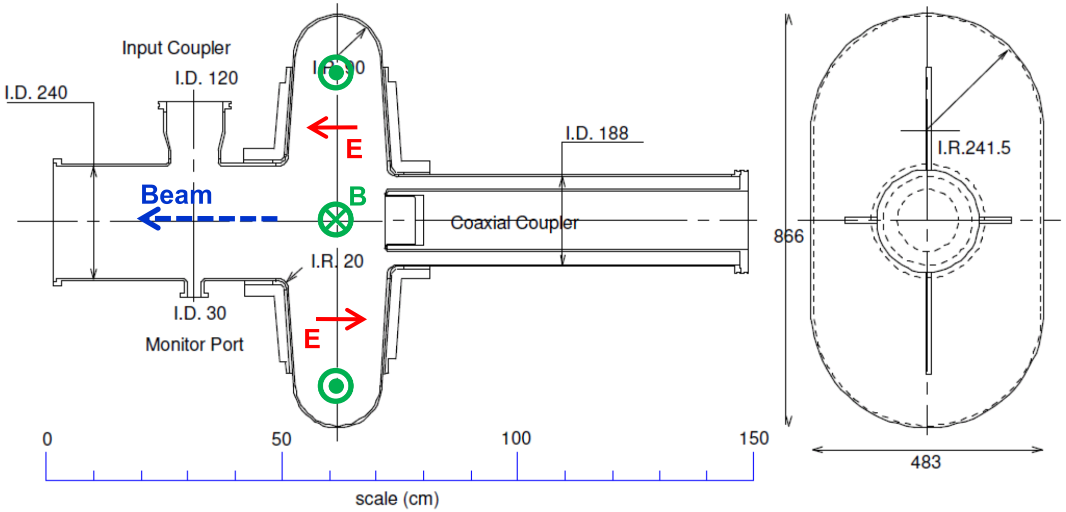




Designed 1970, operated 1977-1981
At IHEP since 1998

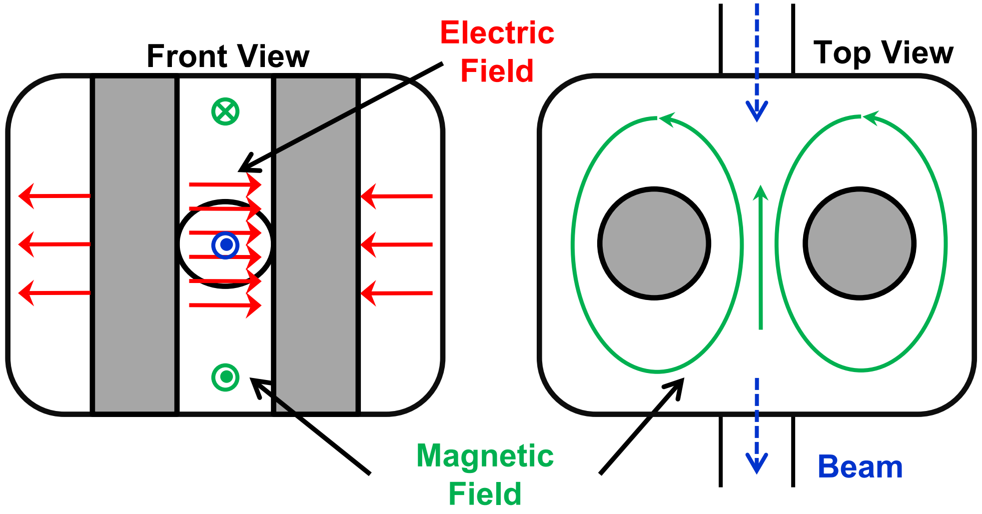
- 1st superconducting crabbing cavity
  - Crabbing cavity: 508.9 MHz cavity for SuperB Factory at KEK

Crab cavities operated from 2007-2010



TEM-Type Deflecting/Crabbing Cavities

- Use both electric and magnetic fields to produce the net deflection

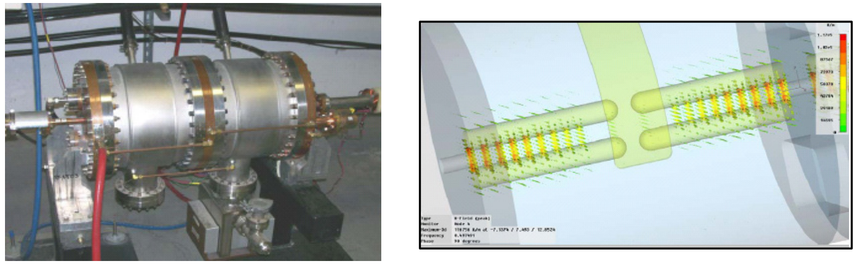


- At low operating frequencies gives:
  - Compact designs
  - Low surface fields and high shunt impedance
  - Some designs have no lower order modes
- New <u>compact</u> deflecting/crabbing designs are originated from TE-like or TEM-type designs

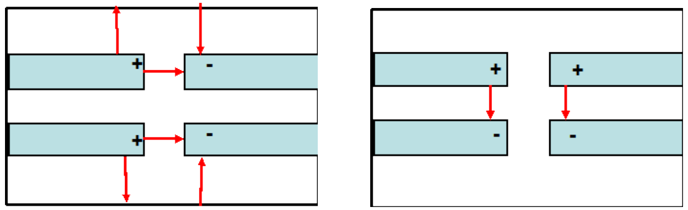
4-Rod Crabbing Cavity

- 4-Rod crabbing cavity – University of Lancaster / Cockcroft Institute
- Adapted from JLab normal conducting cavity
- Proposed for LHC high luminosity upgrade

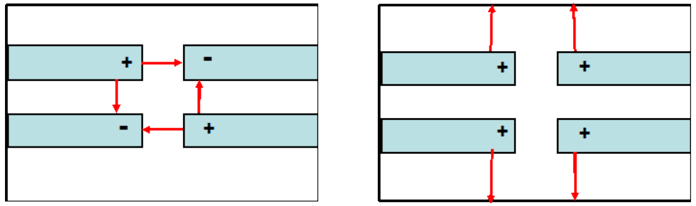
- Operates in a TEM-like mode
  - Uses both electric and magnetic fields
  - Deflecting mode is not the lowest mode



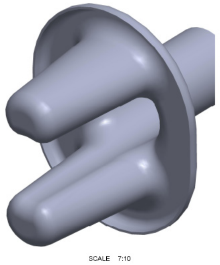
499 MHz normal conducting rf separator at JLab

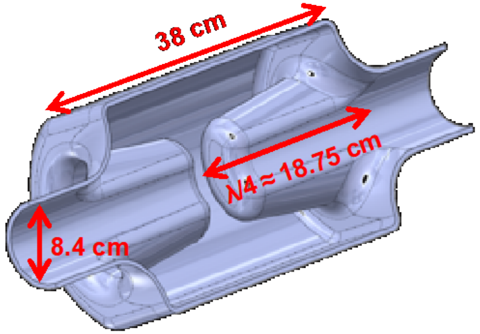


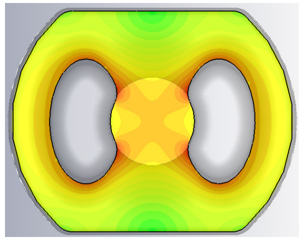
Accelerating lower order mode



Fundamental deflecting mode





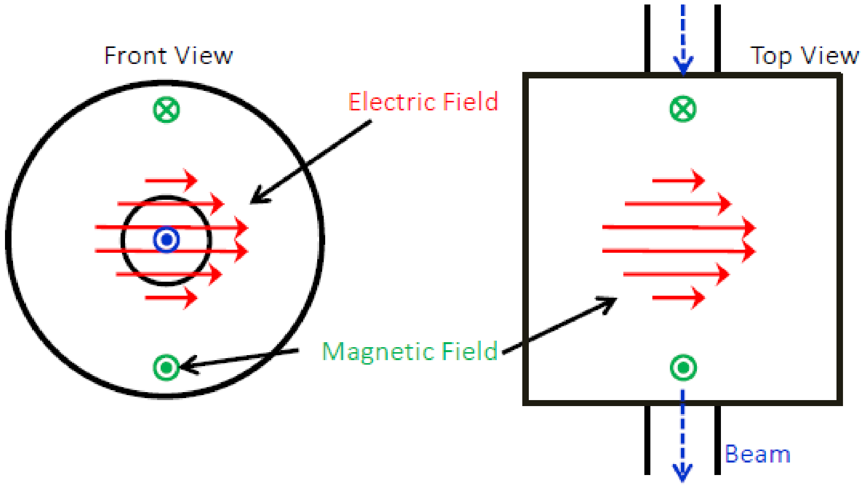


Rod shaping to reduce surface electric and magnetic fields, and the offset nonlinearities

Frequency	**400.0**	**MHz**
LOM	375.2	MHz
Nearest HOMs	436.6, 452.1	MHz
E_p^*	4.0	MV/m
B_p^*	7.56	mT
B_p^*/E_p^*	1.89	mT/ (MV/m)
$[R/Q]_T$	915.0	Ω
Geometrical Factor (G)	62.8	Ω
$R_T R_S$	5.7×10^4	Ω^2
At $E_T^* = 1$ MV/m		

TE-Like Deflecting/Crabbing Cavities

- Operate in TE_{111}-like mode
  - Cannot be a pure TE_{111} mode where the contribution from electric and magnetic fields cancel each other
  - Main contribution to the transverse voltage is from transverse electric field

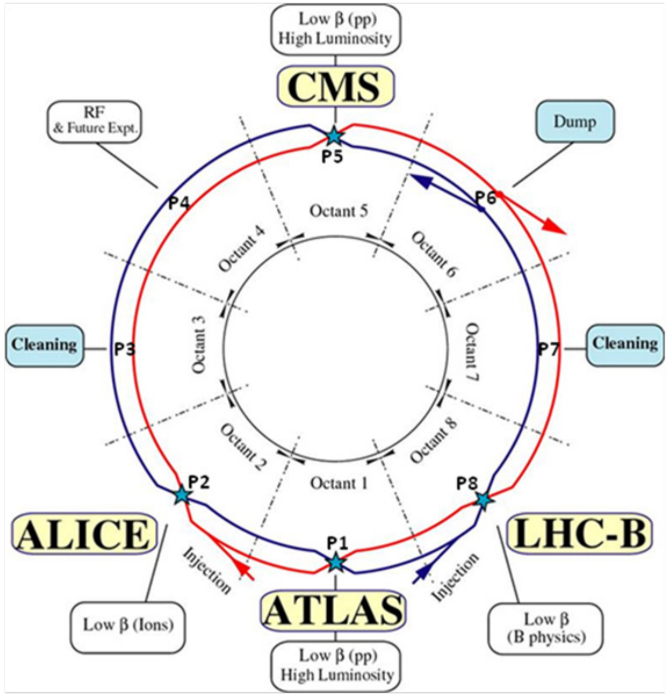


- Pure cylinder would cancel the contribution from E and B fields
- So need deformed shapes

- Has similar rf properties as TEM-type cavities
  - Compact designs
  - Favorable for low frequencies (length ~ $\lambda/2$ and diameter ~$1/f$)
  - No lower order mode (TE_{111} is the lowest mode)
  - Have demonstrated transverse voltages at high peak surface fields

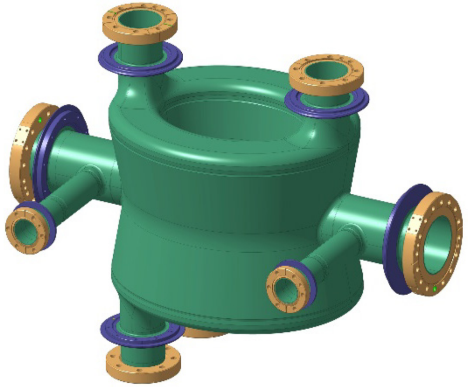
TE-Like Cavities for LHC High Luminosity Upgrade

- Crabbing cavities for LHC high luminosity upgrade – Operate at 400 MHz

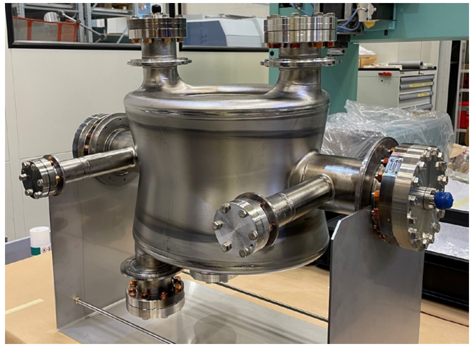


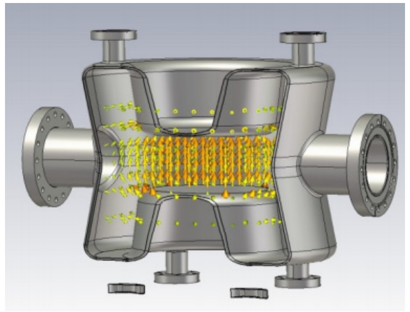
- Two interaction points (P1-ATLAS and P5-CMS)
- 8 cavities per interaction point
- Crossing angle – 285 μrad

Double Quarter Wave Cavity

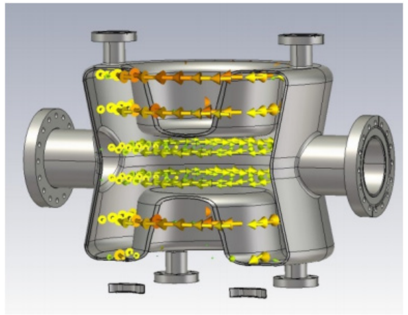


For vertical crabbing





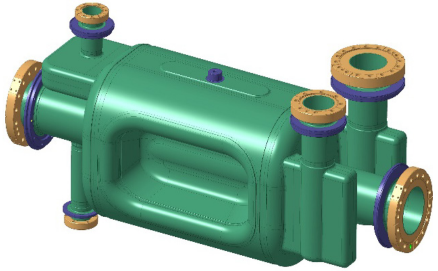
E Field

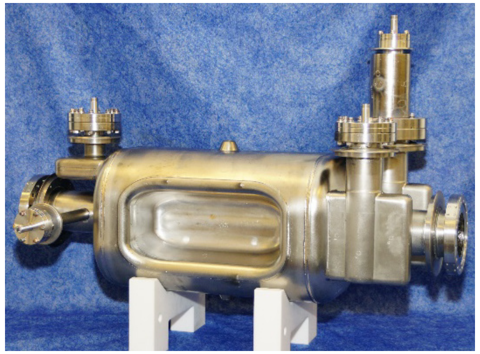


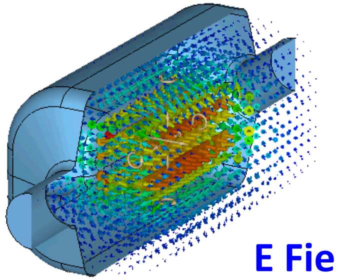
B Field

RF-Dipole Cavity

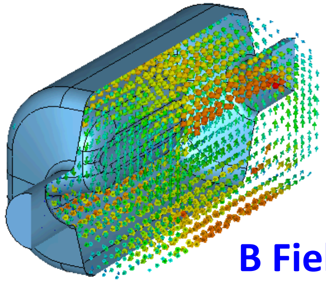
For horizontal crabbing







E Field



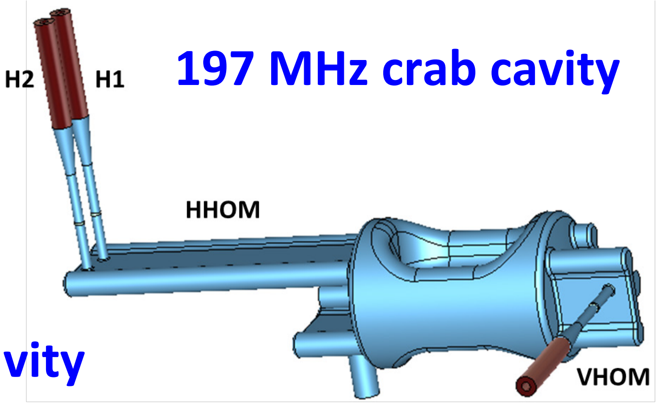
B Field

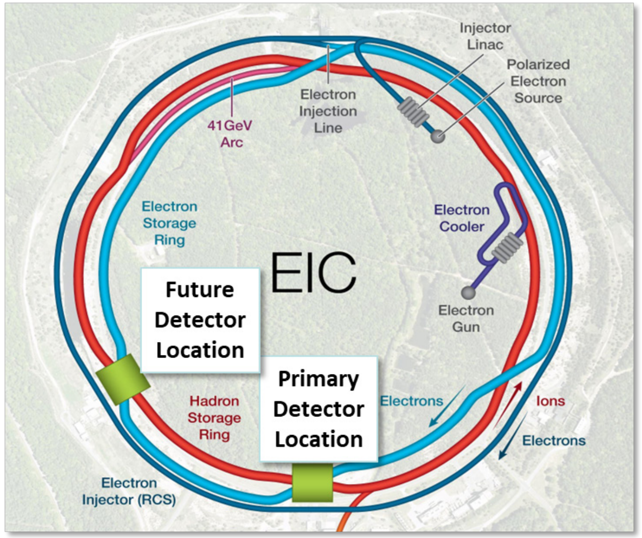
Electron–Ion Collider (EIC) at BNL

- Collisions of electrons and protons/ions for center mass of ~30-140 GeV
- Two rings: Electron storage ring (ESR) and Hadron storage ring (HSR)

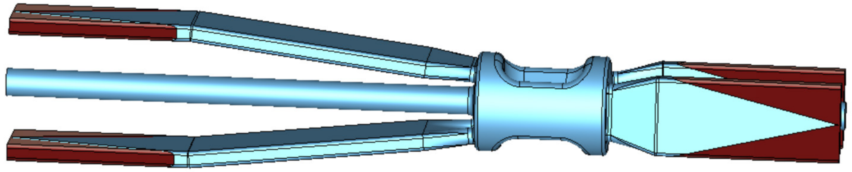
	HSR		ESR
Cavity frequency [MHz]	197	394	394
Crossing angle [mrad]	25.0	25.0	25.0
β_{cc} [m]	1300	1300	150
β_{IP} [m]	0.8	0.8	0.45
Piwinski angle [rad]	7.9	7.9	2.4
Crabbing voltage [MV]	33.83	–4.75	2.9

197 MHz crab cavity



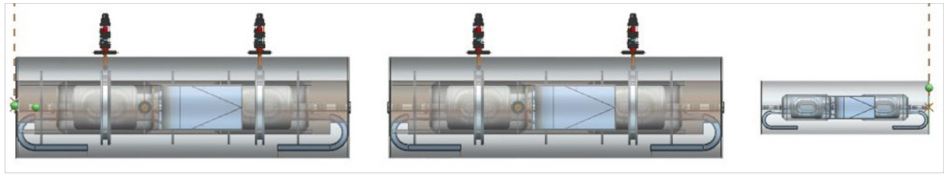


394 MHz crab cavity



	V_t [MV]		No. of cavities	
System	**HSR**	**ESR**	**HSR**	**ESR**
197 MHz	33.83	–	4	–
394 MHz	4.75	2.90	2	1

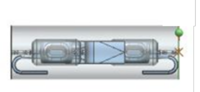
Crabbing Systems for HSR (per side per interaction point)



197 MHz Cryomodule 197 MHz Cryomodule 394 MHz Cryomodule

15 m

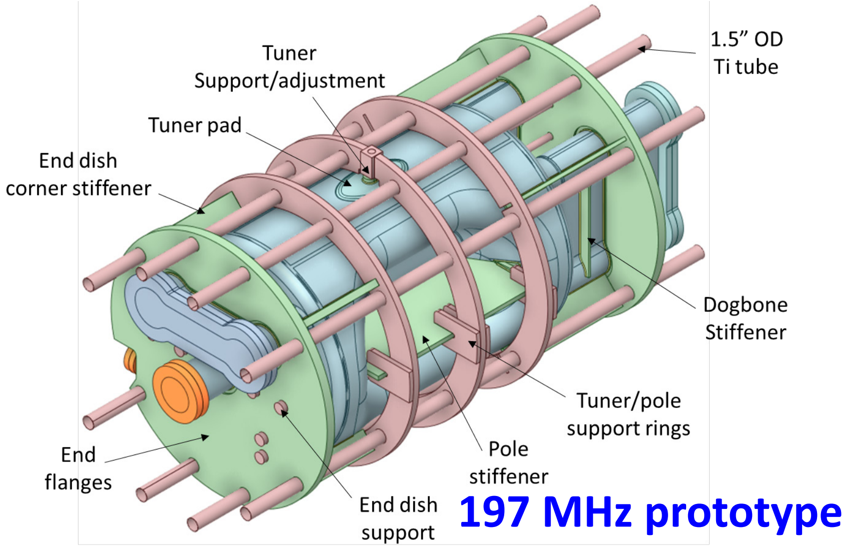
Crabbing System for ESR (per side per interaction point)



394 MHz Cryomodule

4.5 m

🟦	Nb
🟩	NbTi
🟥	Ti
🟧	SS
🟨	Weld

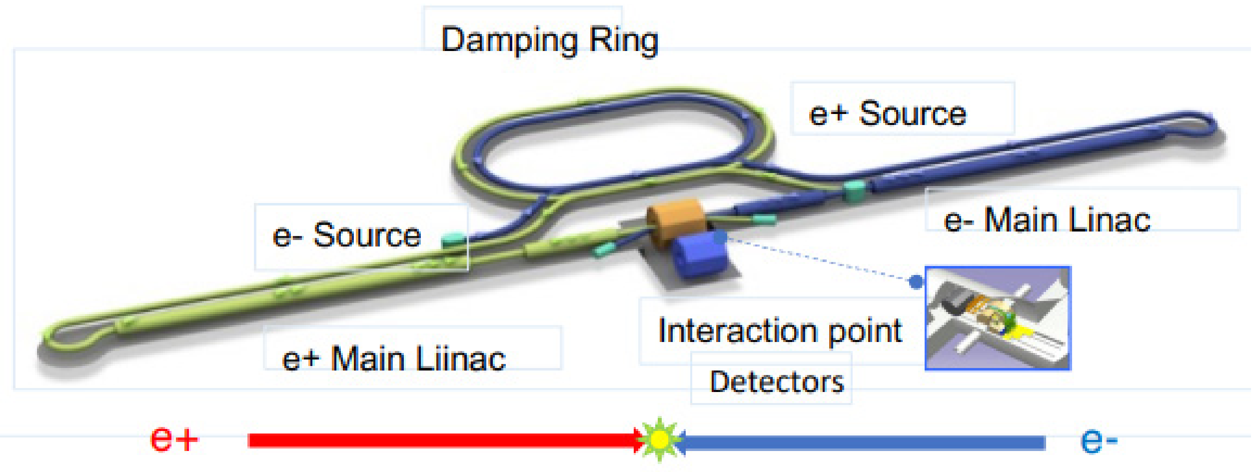


197 MHz prototype crab cavity

Size and weight: 5’ overall long, 30” OD, ~800 lb

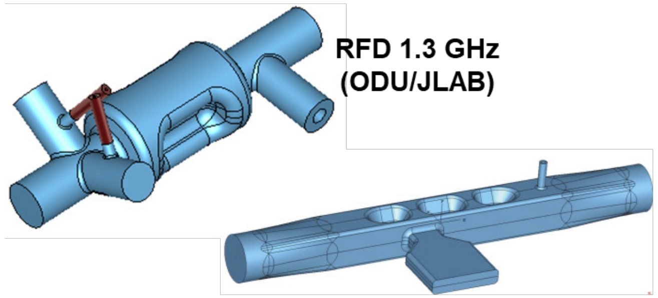
International Linear Collider (ILC)

- $e^{*}$and e^{+} collision in a linear collider
- Crossing angle 14 mrad
- Luminosity: $1.35/2.7\times10^{10}$ $cm^{2}s^{-1}$



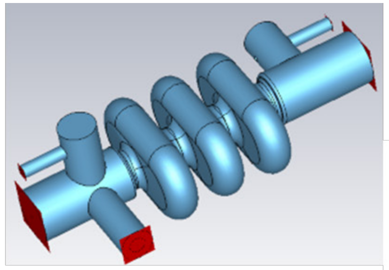
Parameter	Value
Center of mass energy [GeV]	250/1000
Crossing angle [mrad]	14.0
β_{cc} [m]	23.2
β_{IP} [mm]	13.0/22.6
RMS bunch length (σ_z) [mm]	0.3/0.25
RMS horizontal beam size (σ_x^{*}) [nm]	729.0/481.0

Beam Energy	250 GeV			1 TeV		
f_0 [GHz]	**1.3**	2.6	3.9	**1.3**	2.6	3.9
Tot. V_t [MV	**1.845**	0.923	0.615	**7.4**	3.7	2.5



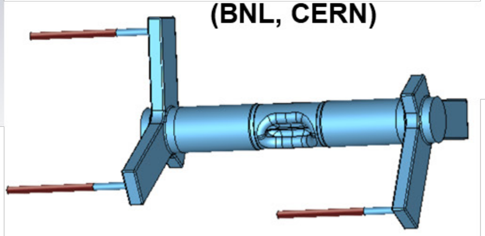
RFD 1.3 GHz (ODU/JLAB)

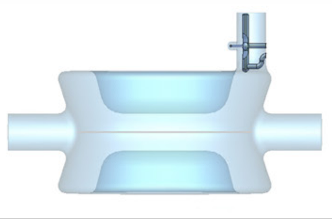
QMIR 2.6 GHz (FERMILAB)



RACETRACK 3.9 GHz (LANCASTER UNIV. /COCKTROFT)

WOW 1.3 GHz (BNL, CERN)





DQW 1.3 GHz (BNL, CERN)

DESIGN CONSIDERATIONS OF NON-ELLIPTICAL ACCELERATING CAVITIES

(1) DESIGN ASPECTS

(2) MULTIPACTING

(3) MECHANICAL DESIGN

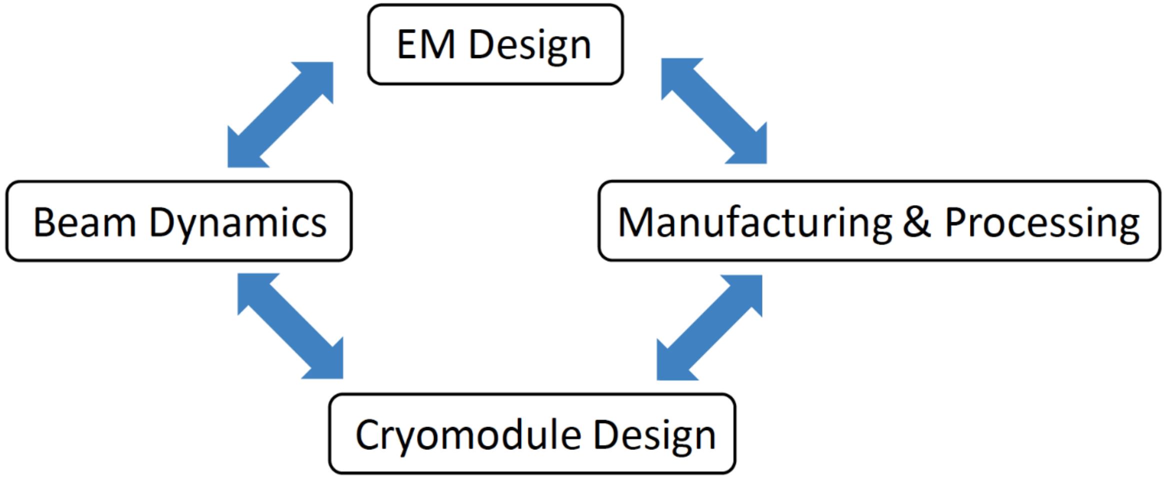
(4) FABRICATION

(5) CHEMICAL ETCHING

(6) CRYOMODULE DESIGN

Cavity Design Considerations

- Beam dynamics:
  - Cavity frequency
  - Beam aperture
  - Voltage acceptance



EM Design

Beam Dynamics

Manufacturing & Processing

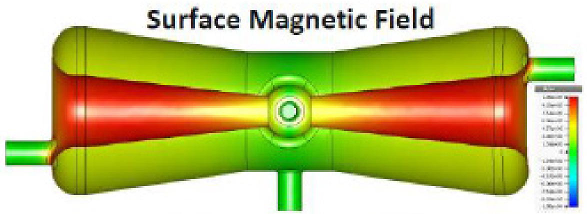
Cryomodule Design

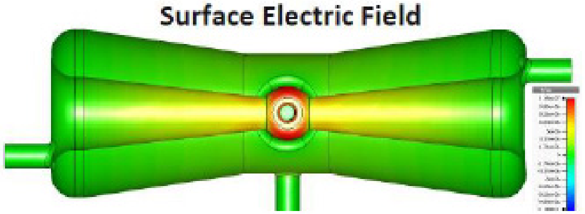
- Ideally cavities should have:
  - Large accelerating gradient (E_a) / Energy gain (W)
  - Large shunt impedance ($R_{sh}=G*(R/Q)$) for low losses to reduce power consumption
  - Shapes that reduce peak fields (E_p, B_p) for given E_a
  - Efficient energy transfer to the beam (β)
  - Reduce multipacting levels

- In addition, related practical issues of these cavities
  - Reduce pressure sensitivity (df/dp)
  - Microphonics
  - Operation: cw or pulsed
  - Cavity tuning
  - Cavity fabrication
  - Chemical processing, cleaning, and assembly

Designing Non-Elliptical Cavities

- Cavity frequency:
  - To minimize unique number of cavity designs
- Cavity β: Number of cavity designs also depend on required velocity range
  - T(β) is efficient over a range of velocities from $0.7\beta_0 < \beta < 2\beta_0$ (Especially for QWR)
  - For β>0.5 possible to consider multi-spoke cavities where the reduced transit time factor is compensated by the higher voltage
  - Maintain a certain cavity type until T(β) lowers the voltage below the voltage of the next cavity series
  - For post accelerators with different ion acceleration – β profile should be chosen that all ions can be accelerated near the maximum gradient
- Peak surface fields: $E_p \leq$35 MV/m and $B_p \leq$70 mT
  - Dominates the optimization between practicality and complexity
- Multipacting analysis: Multipacting levels may not be eliminated, but reduced by optimizing the design
  - Advanced simulation tools exist in simulating multipacting resonance levels in cavities that have matched with measurements


Surface Magnetic Field


Surface Electric Field

Designing Non-Elliptical Cavities

- Baseline mechanical and fabrication model
  - Consider ports: FPC, HOM couplers, Rinsing and surface treatment
  - Choose material thickness: Based on stress analysis
    - Maintain safe limits in terms of stress
  - Check all pressure differential throughout cavity life cycle
  - Minimize Lorentz force detuning due to radiation pressure → Stiffeners
  - Tuner designs

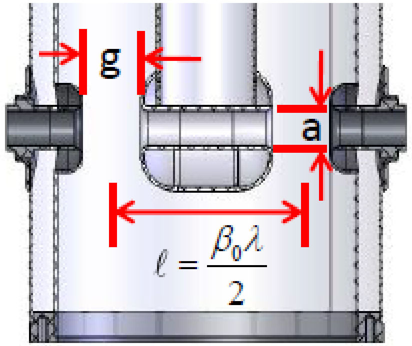
- Integrating design for fabrication variables include:
  - Cavity performance
  - Complexity in geometry
  - Operational requirements: 4 K or 2 K
  - Stress analysis
  - Material cost vs machining cost

RF Frequency Considerations

- Smaller structures are cheaper to fabricate so favor small λ (high frequency)
- Lower frequencies increase size of stable region in longitudinal phase space and increase cavity active length so increase the effective voltage

- Lower velocities require lower frequencies
- Typically, $\beta\lambda/2$ between gaps (one accelerating cell)
- Gap is typically half a cell or $g \sim \beta\lambda/4$

- Aperture should be less than the gap to improve acceleration efficiency → so $a < \beta\lambda/4$



g=gap, a=aperture

Example:

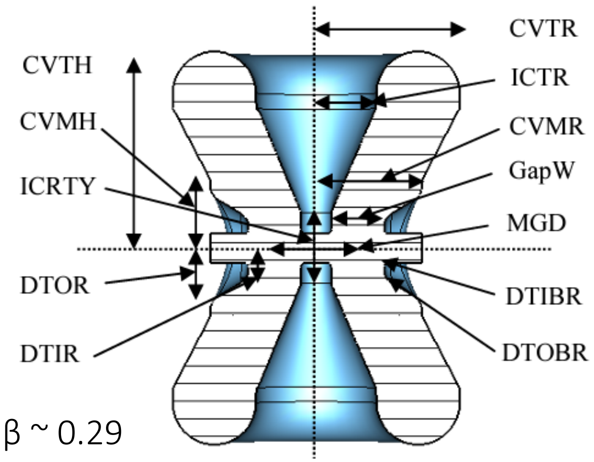
for a=30mm and β =4%

λ>3m or f < 100MHz

Design Optimization

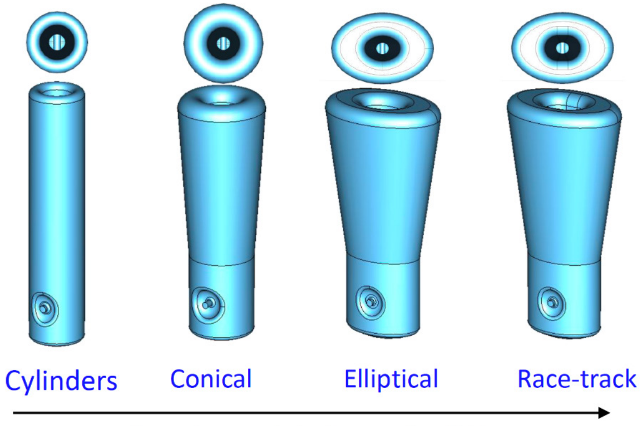
- Good simulation tools exist both on the EM and mechanical aspects (CST, HFSS, COMSOL, ANSYS, SLAC ACE3P Code Suite, …)
- Many geometrical parameters of optimization compared to elliptical cavities
- Optimize cavity parameters to:
  - Minimize peak surface field ratios – E_p/E_{acc} and B_p/E_{acc} (E_p/E_t and B_p/E_t for deflecting and crabbing cavities)
  - Maximize $R_{sh} = G*(R/Q)$

HWR



322 MHz, β ~ 0.29

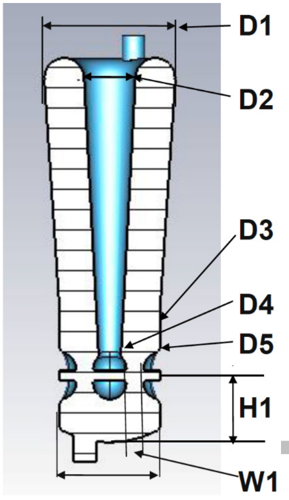
QWR

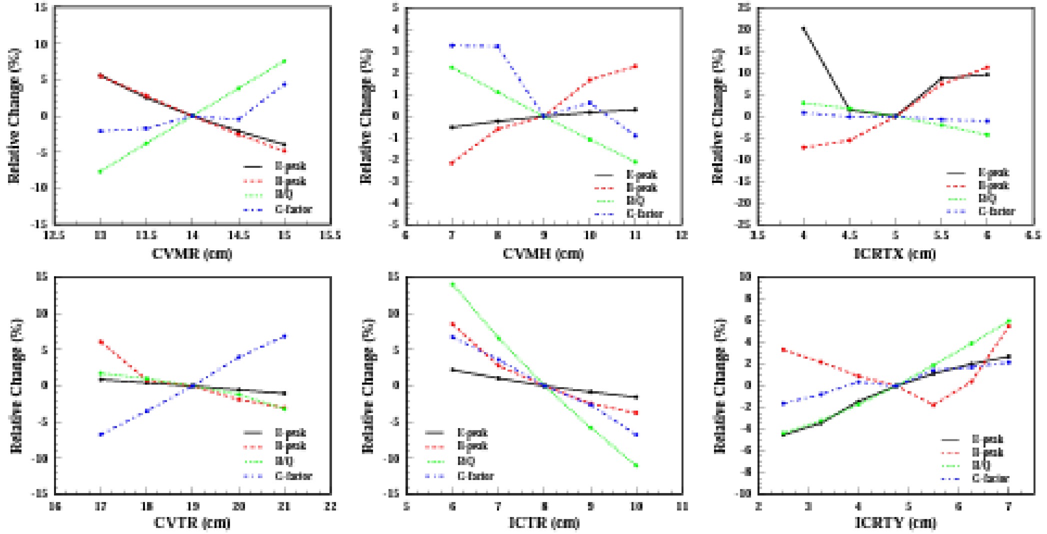


Increasing complexity, lower surface fields

Some primary cavity geometrical parameters:

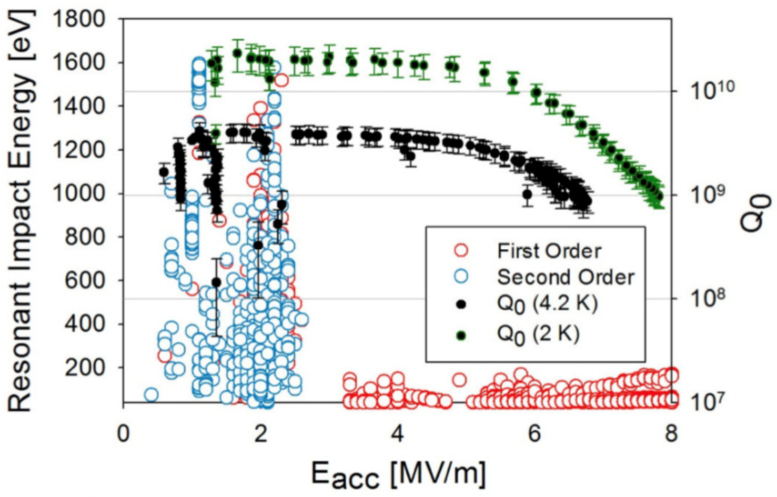
Cavity Top Diameter	(D1)
Stem Top Diameter	(D2)
Cavity Lower Diameter	(D3)
Stem Bottom Diameter	(D4)
Drift Tube Outer Diameter	(D5)
Drift Tube Gap Width	(W1)
Cavity Bottom Height	(H1)



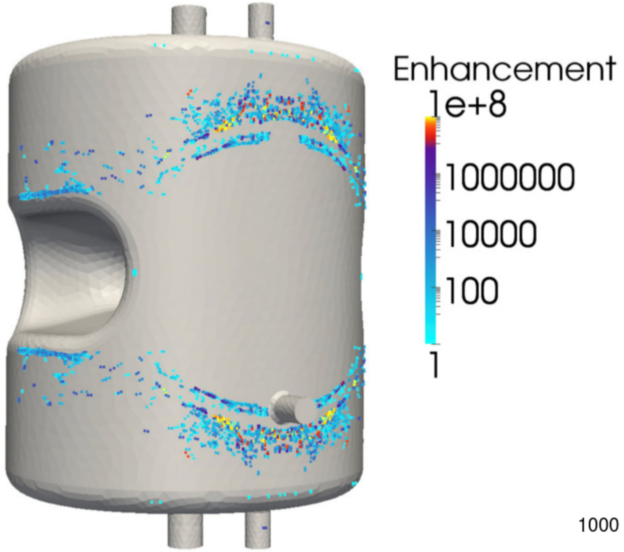


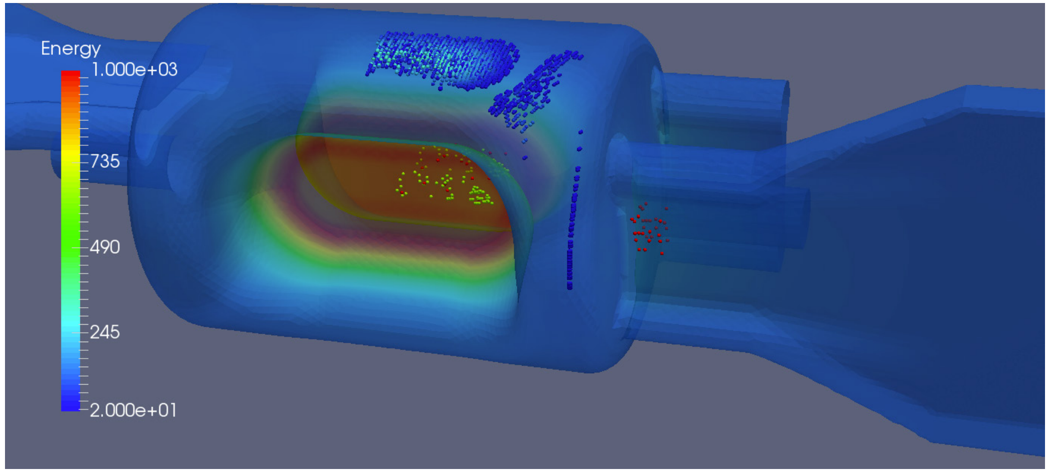
Multipacting in Non-Elliptical Cavities

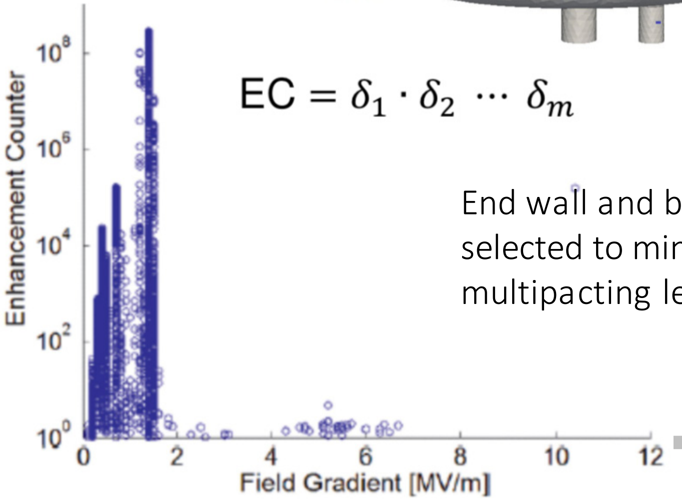
- Multipacting always occur in non-elliptical cavities due to complex geometries → But not a showstopper
- Now reliable tools exist that can model multipacting resonant levels



Multipacting experienced up to 2.1 MV/m

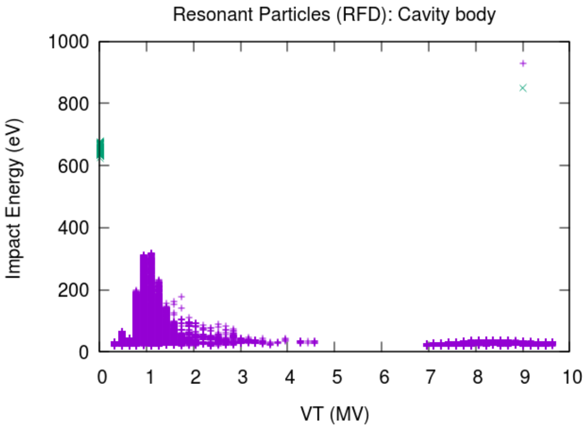


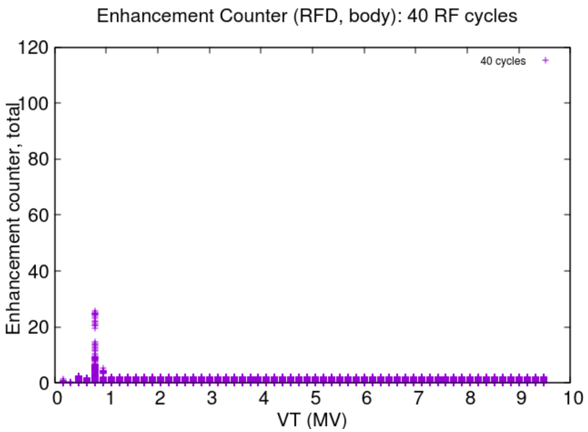




$$EC = \delta_1 \cdot \delta_2 \cdots \delta_m$$

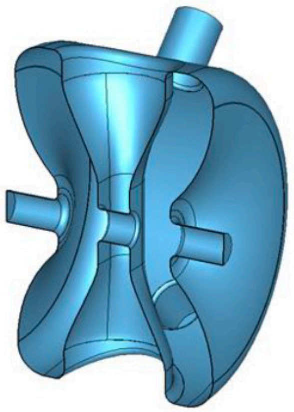
End wall and bend radius selected to minimize multipacting levels

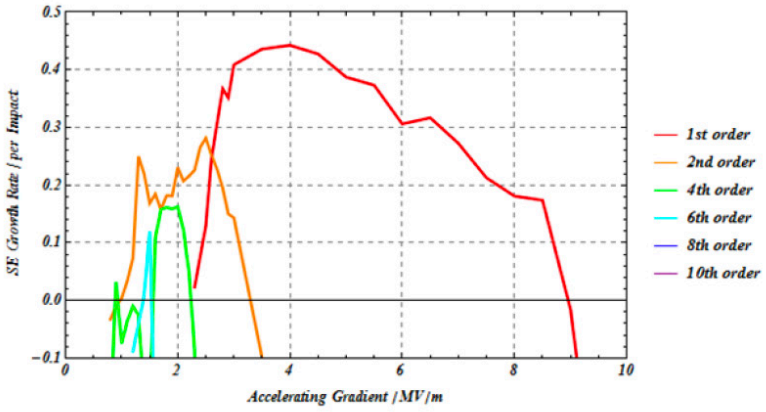


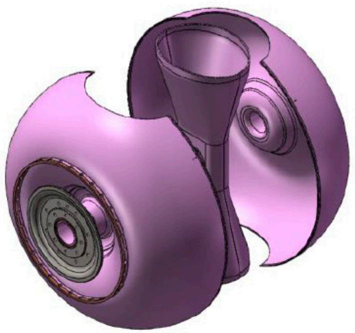


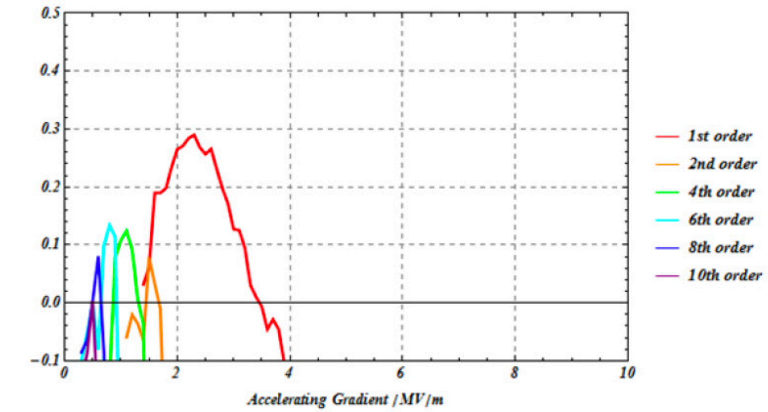
Multipacting in Non-Elliptical Cavities

- Multipacting can be reduced careful cavity design
- Example – single spoke resonator designed by TRIUMF for RISP
- Balloon variant reduces serious multipacting in operating region



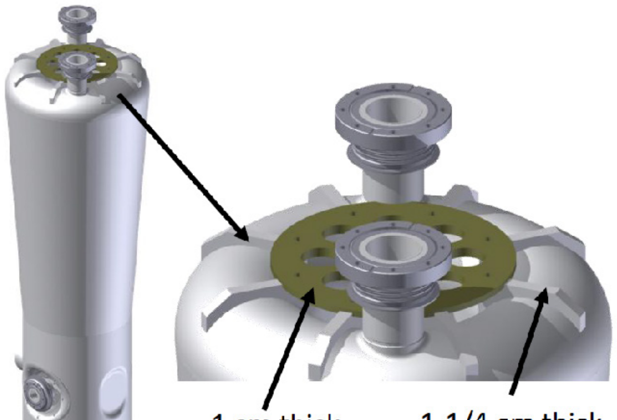


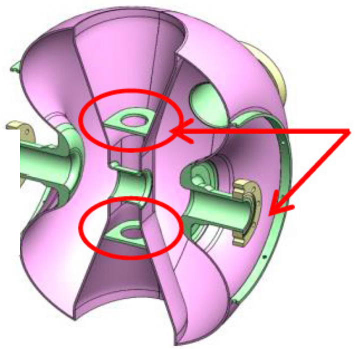


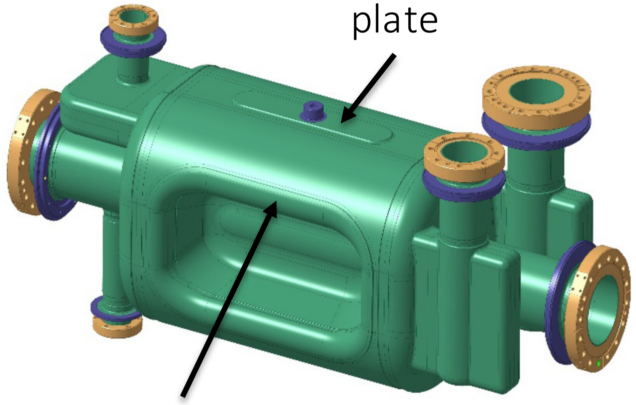


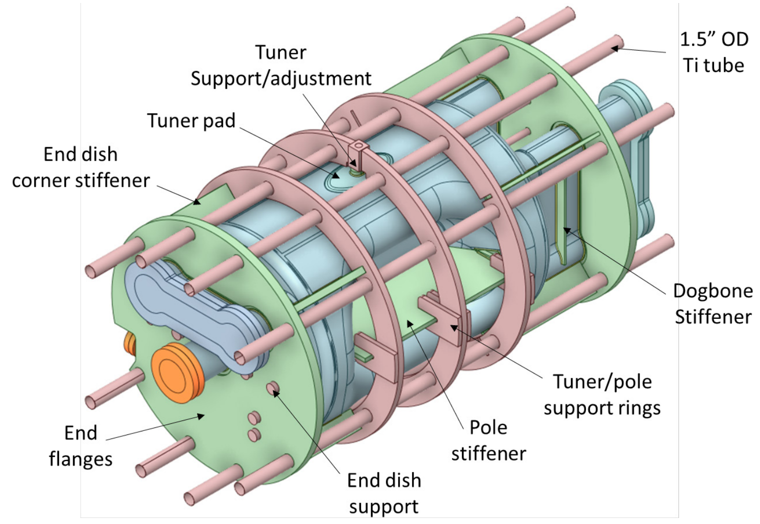
Mechanical Designs of Non-Elliptical Cavities

- Mechanical design focuses on reducing internal stresses under the external pressure load for various operational conditions
- Study mechanical stability to microphonics, pressure fluctuations (df/dp), Lorentz force detuning
- Stiffeners can be added strategically to reduce and improve mechanical stability
- Consider thermal performance when considering thicker material to reduce stresses
- Consider tuning range and tuning force required and cavity stresses for maximum tuning range







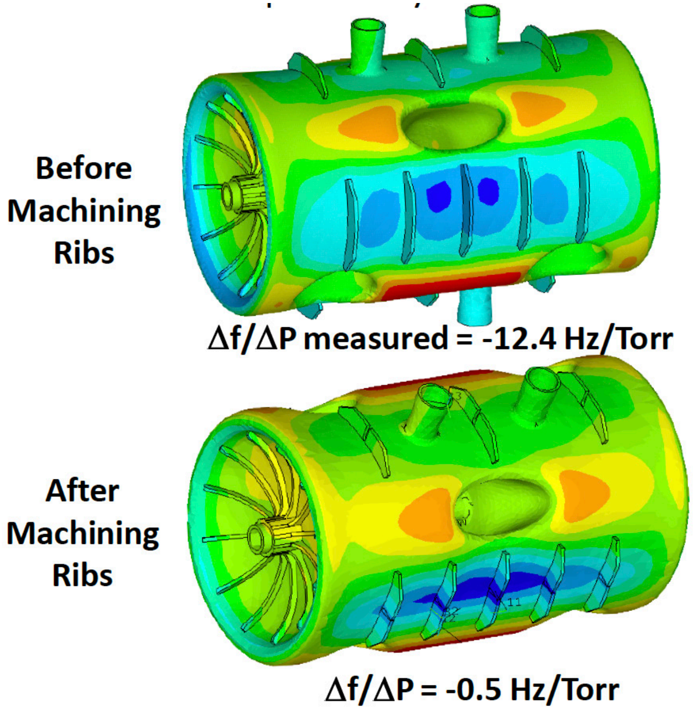


Mechanical Designs of Non-Elliptical Cavities

- Response of the cavity to external pressure changes can be dominant contribution at 4 K operation
- Lorentz force detuning: inward pressure in E field and H field regions produce frequency shifts in opposite direction

$$\Delta f = k\int\left(\varepsilon_0 E^2 - \mu_0 H^2\right)dV$$

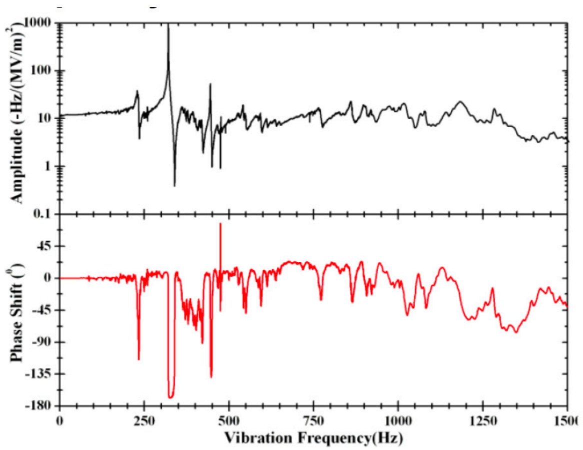
- Support ribs in E field and H field to balance deflections so that frequency shift can be cancelled

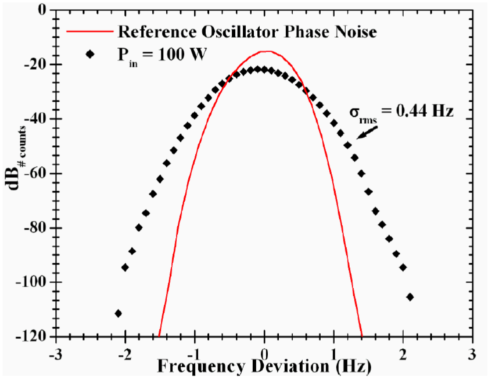



β=0.5 triple-spoke w/ support ribs

Mechanical Designs of Non-Elliptical Cavities

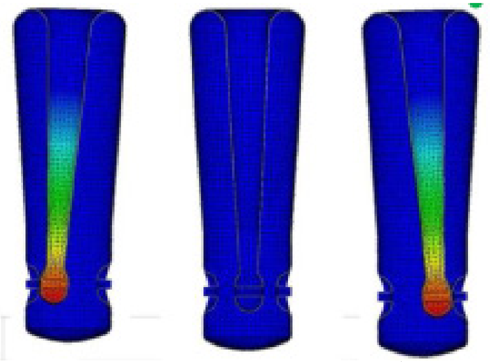
- Driven by mechanical vibration in the environment
- QWRs inner conductor can have low mechanical frequencies (50-100Hz)
- Need to reduce the RMS detuning to <<10% of the design bandwidth to avoid nuisance unlocks
- Mitigations
  - Stiffening during the design/manufacture to raise the mechanical frequency of the lowest modes
  - Centering the inner conductor by plastic deformation so df/dx=0
  - Adding passive dampers in QWR
  - Reducing environmental noise in and around the cryomodule -operation at 2K is quieter than 4K

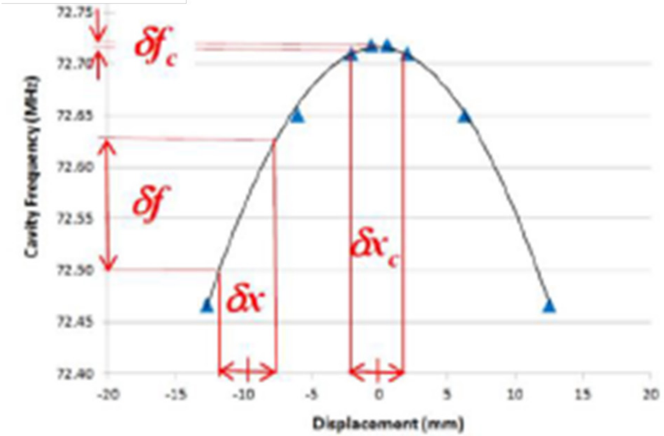


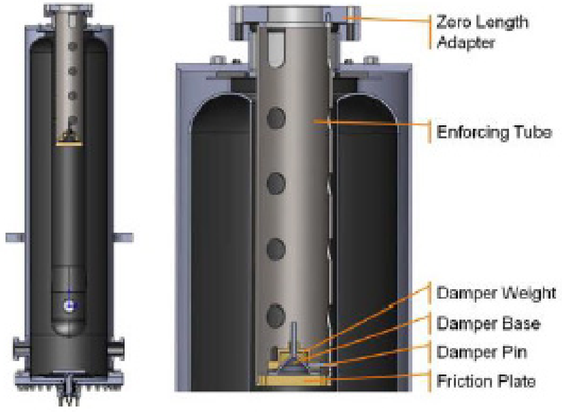


df/dp = -0.4 Hz/mbar

345 MHz β=0.5 triple spoke cavity







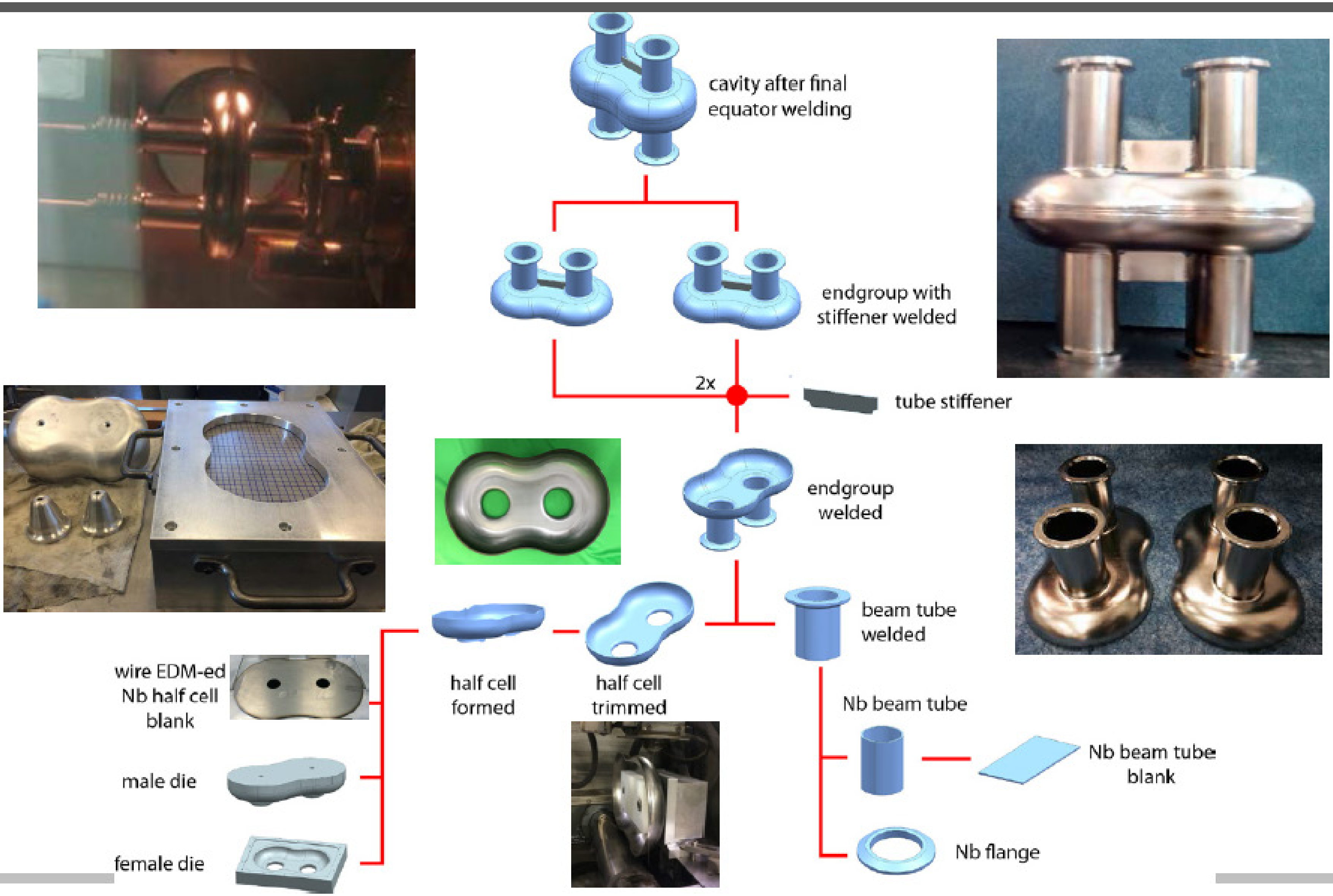
Cavity Fabrication

- Fabrication of TEM mode cavities must follow the same standards as for elliptical cavities – the differences can come from the novel shapes and unique forming steps
  - Consists of many more subcomponents compared to elliptical cavities
  - Requires many more machining/trimming fixtures at different stages of the fabrication process
  - Frequency tuning requires different fixturing compared to elliptical cavities
  - May need additional plastic deformation of welded subcomponents and post fabrication tuning
- New procedures can be qualified in Copper as a `cheap’ substitute for Nb or RG Nb
- Fabrication steps can involve
  - Machining of parts from bulk – cost optimization between material cost and fabrication cost – EDM, milling, lathe
  - Forming –deep drawing from sheet –good for production quantities
  - Spinning from sheet –fast, not as precise as forming typically
  - EB welding in high vacuum for RF surfaces to maintain RRR value of Nb
  - TIG in controlled atmosphere for jacket

SRF 2023 Tutorial –

Cavity Fabrication and Preparation, Rongli Geng

Cavity Fabrication



Cavity Fabrication

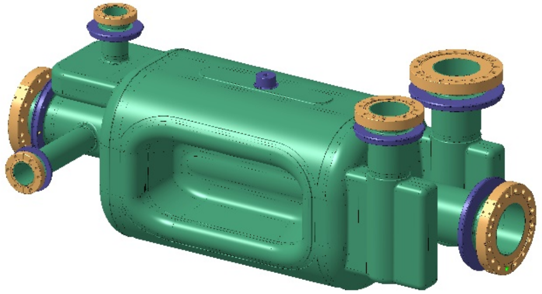
325 MHz Single-Spoke

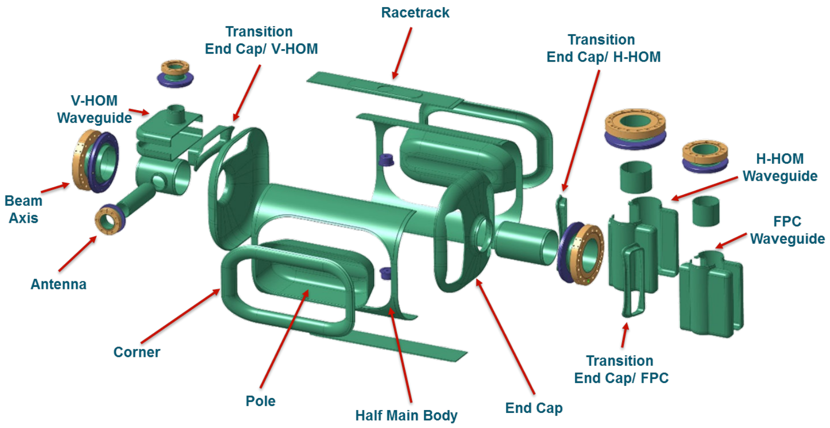
Fabricated at Niowave Inc.

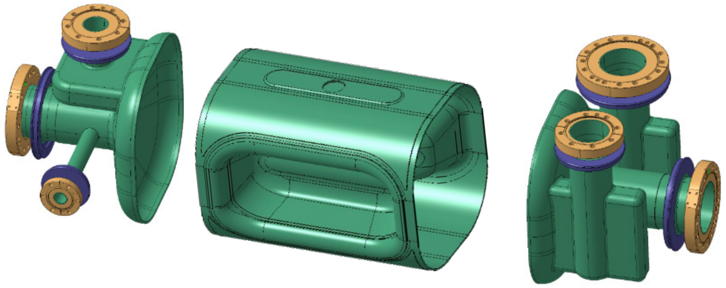
500 MHz Double-Spoke

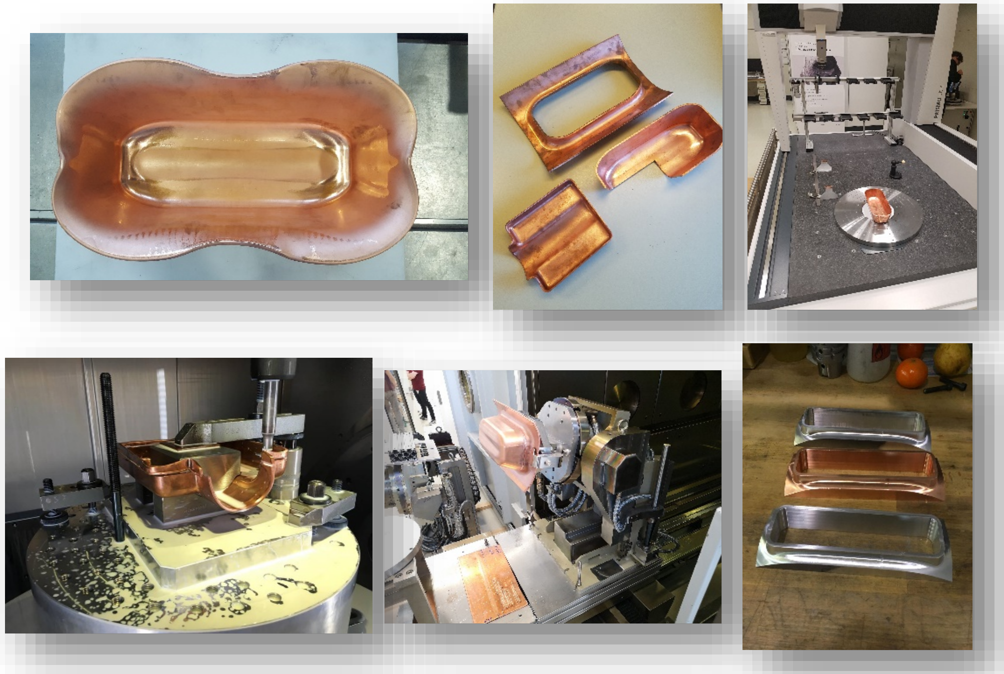
Fabricated at Jefferson Lab
(HyeKyoung Park)

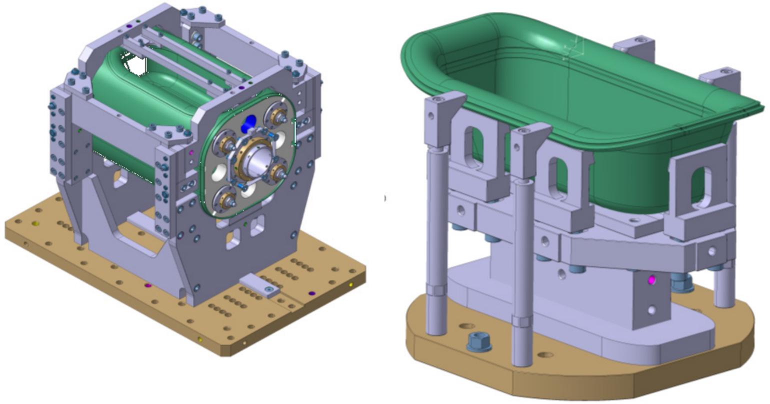
Fabrication of Non-Elliptical Cavities









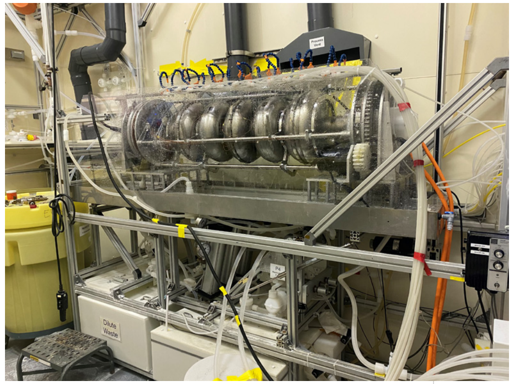


Cavity Surface Treatment, Processing and Assembly

- A standard recipe

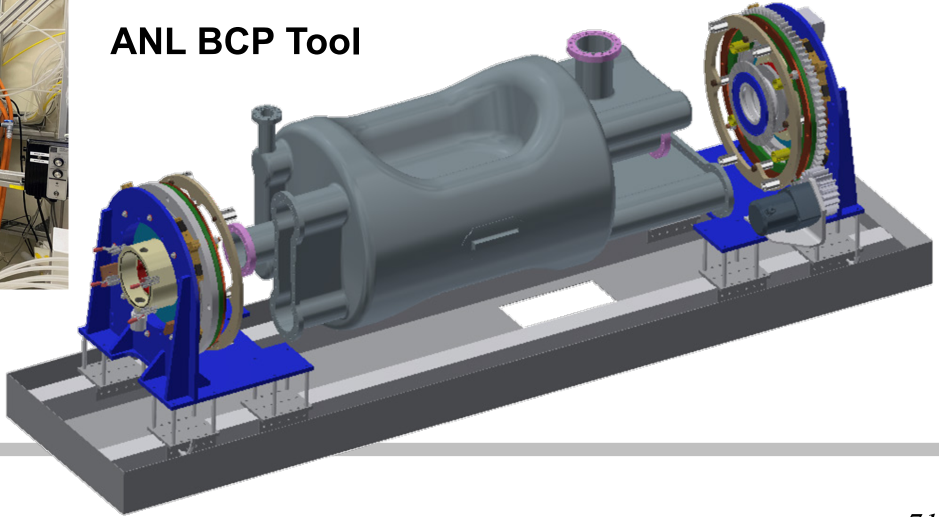
1	Bare Cavity Inspection – Visual, Dimensional, Vacuum, RF
2	US cleaning and rinse
3	BCP 120-150 µm (flip half-way)
4	High-Pressure Rinse
5	Hydrogen Degassing 600 °C, 10 h
6	RF Tuning
7	BCP 20-30 µm
8	HPR (horiz + vert)
9	Clean Room Assembly
10	Low Tem Bake 120 °C, 48 h
11	Vertical Test @ 2K
12	Helium Vessel Welding
13	US cleaning
14	BCP 20-30 µm
15	HPR
16	Clean Room Assembly
17	Low Temp Bake 120 °C, 48 h
18	Horizontal Test @ 2K
19	Ready for String Assembly

High Temp Furnace - JLAB



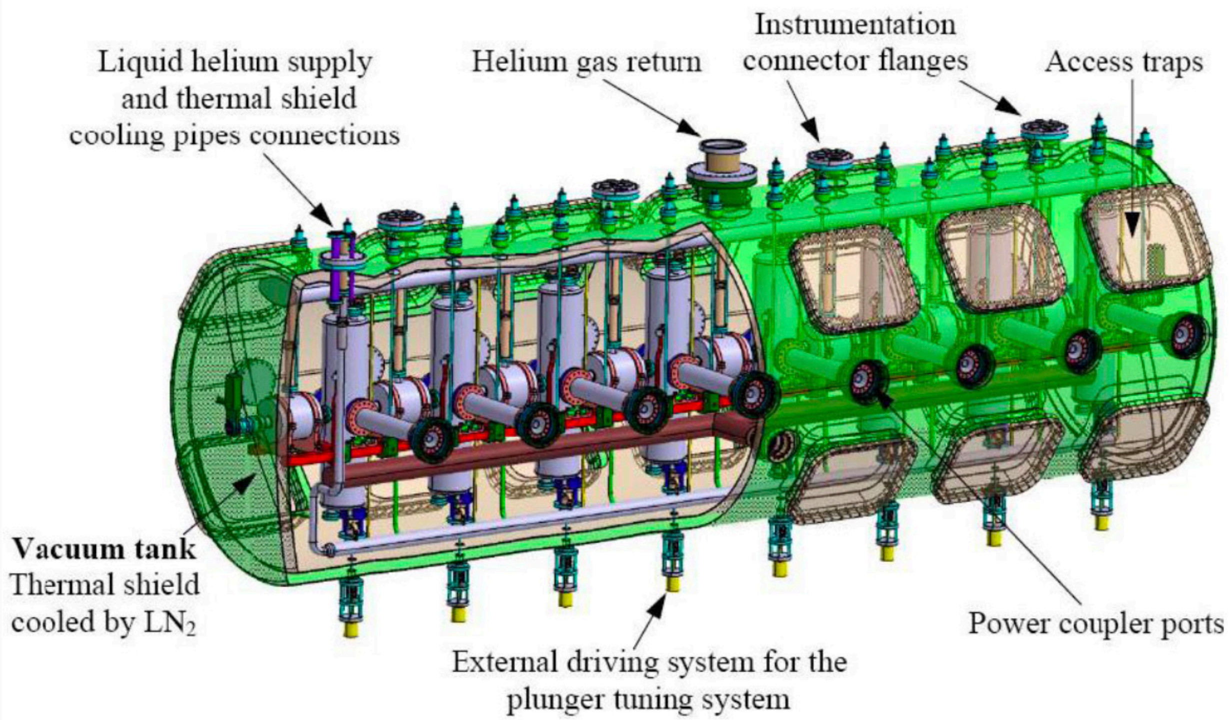
Low Temp Bake Box

ANL BCP Tool



Cryomodule Designs

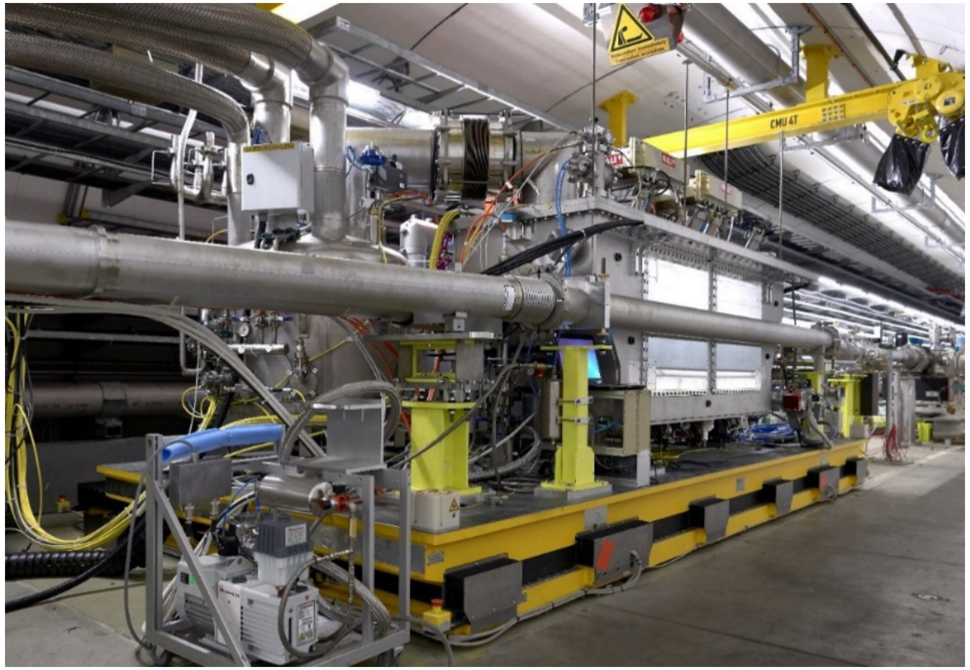
- Non-elliptical cavities require more complicated cryomodule structures compared to elliptical cavities
- Different solutions investigated for the same cavity types
- Couplers, tuners, and rf lines are often dominant components



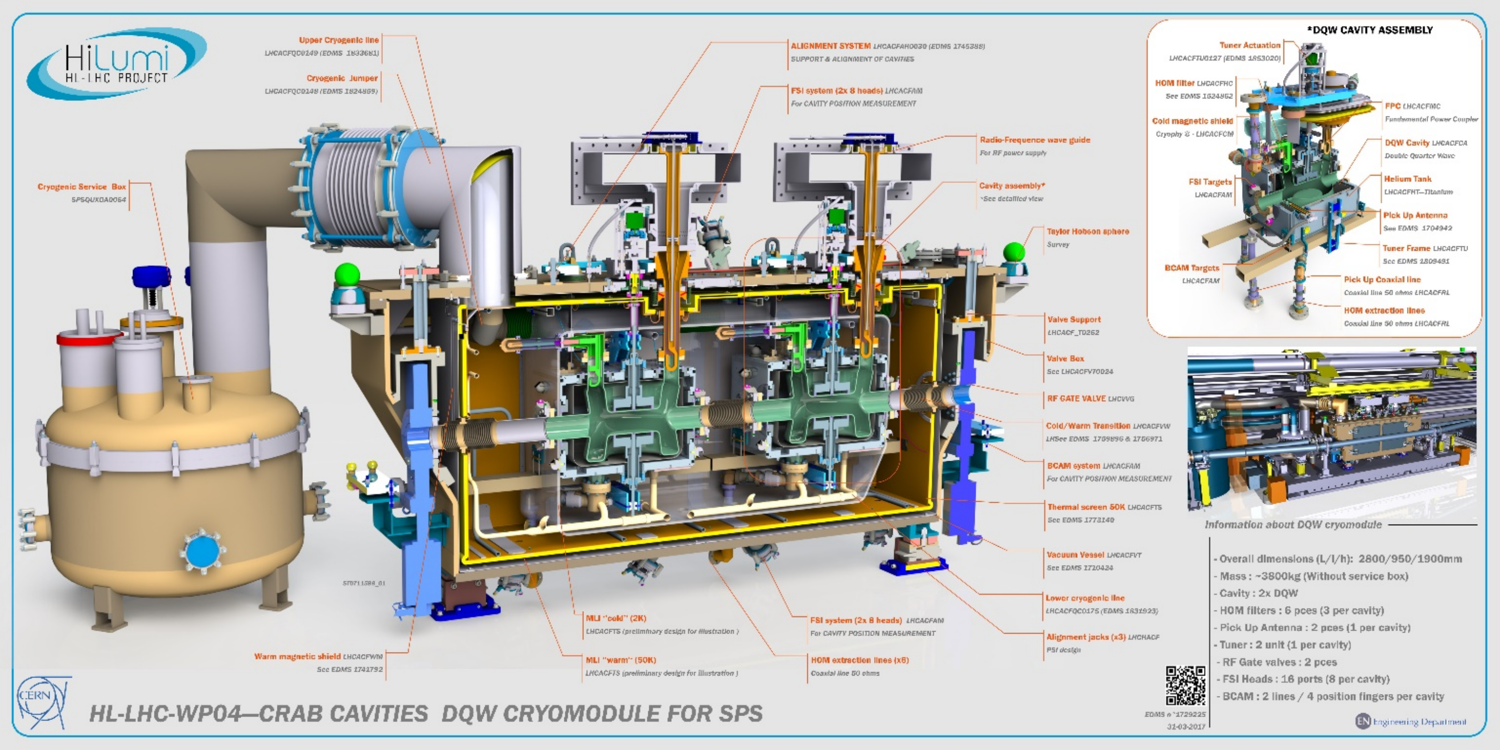
SRF 2023 Tutorial –

Cryomodule Design, Nicolas Bazin

Cryomodule Designs



400 MHz Double Quarter Wave
Cryomodule for SPS, CERN



Information about DQW cryomodule

- Overall dimensions (L/l/h): 2800/950/1900mm
- Mass : ~3800kg (Without service box)
- Cavity : 2x DQW
- HOM filters : 6 pces (3 per cavity)
- Pick Up Antenna : 2 pces (1 per cavity)
- Tuner : 2 unit (1 per cavity)
- RF Gate valves : 2 pces
- FSI Heads : 16 ports (8 per cavity)
- BCAM : 2 lines / 4 position fingers per cavity

HL-LHC-WP04—CRAB CAVITIES DQW CRYOMODULE FOR SPS

Final Comments

- Non-elliptical cavities have evolved in the past several decades → Tremendous global interest in superconducting cavities for high energy hadron acceleration, compact high beta cavities, and deflecting/crabbing cavities

- Many accelerators are using non-elliptical cavities such as QWR, HWR, Spoke that have moved the technologies to industrial production aiming high performance and reliability

- Compact crabbing cavity designs have advanced particle colliders in achieving high luminosities

- Parameter, tradeoff, and option space available to the designer is large → No universal design where designs are application specific

- Design process is not reduced to a few simple rules of recipes → Room for pushing the performance of the non-elliptical cavities to higher accelerating/crabbing voltages

- Look out for non-elliptical contributions at SRF 2023

References

- Past SRF Tutorials:
  - SRF 2021 – Non-Elliptical Cavities – R. Laxdal
  - SRF 2019 – Non-Elliptical Cavities – S. De Silva
  - SRF 2017 – Non-Elliptical Cavities – R. Laxdal
  - SRF 2015 – Non-Elliptical Resonators – A. Facco
  - SRF 2013 – TEM Class Cavity Design – M. Kelly
  - SRF 2007 – Low and Medium β Superconducting Cavities and Accelerators - J. Delayen

- Non-Elliptical Accelerating Cavities
  - Design of Low-Velocity Superconducting Accelerating Structures Using Quarter-Wavelength Resonant Lines – J. R. Delayen, NIM, A259, 341-357 (1987)
  - Longitudinal Transit Time Factors of Short Independently-Phased Accelerating Structures for Low Velocity Ions – J. R. Delayen, NIM, A258, 15-25 (1987)
  - Superconducting Spoke Cavities – M. Kelly, 39th ICFA Advanced Beam Dynamics Workshop on High Intensity High Brightness Hadron Beams (2006), http://accelconf.web.cern.ch/AccelConf/abdwhb06/PAPERS/THAY07.PDF
  - Medium-β Superconducting Accelerating Structures – J. R. Delayen, SRF 2001, https://accelconf.web.cern.ch/accelconf/srf01/papers/fa007.pdf
  - Applications of Spoke Cavities – J. R. Delayen, LINAC 2010, https://accelconf.web.cern.ch/accelconf/LINAC2010/papers/tu302.pdf
  - Superconducting RF Development For FRIB At MSU - K. Saito, et. al., LINAC 2014, http://inspirehep.net/record/1363521/files/thioa02.pdf

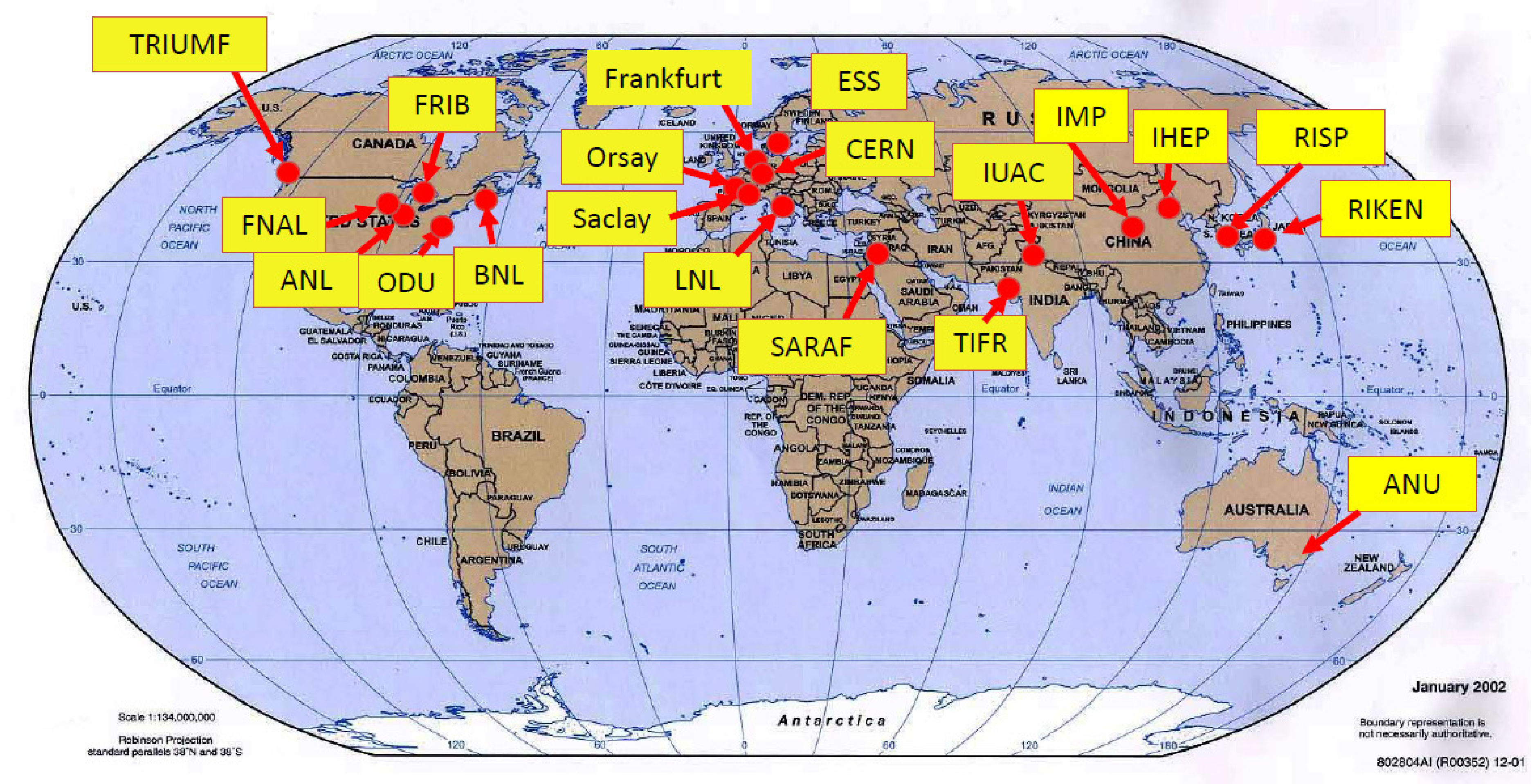
References

- Non-Elliptical Accelerating Cavities
  - Superconducting Spoke Cavities for High-Velocity Applications – C. S. Hopper and J. R. Delayen, Phys. Rev. ST Accel. Beams, 16, 102001, (2013)
  - High-Velocity Spoke Cavities – C. S. Hopper, SRF 2015, https://digitalcommons.odu.edu/cgi/viewcontent.cgi?article=1178&context=physics_fac_pubs
  - Multipacting Analysis of High-Velocity Superconducting Spoke Resonators – C. S. Hopper and J. R. Delayen, LINAC 2012
  - STUDY OF BALLOON SPOKE CAVITIES, Z.Y. Yao, et. al., SRF 2013, http://accelconf.web.cern.ch/AccelConf/SRF2013/papers/thp033.pdf

- Non-Elliptical Deflecting/Crabbing Cavities
  - Energy Scaling Crab Crossing and the Pair Problem – R. B. Palmer, SLAC-PUB-4707 (1988)
  - Design Evolution and Properties of Superconducting Parallel-Bar RF- Dipole Deflecting and Crabbing Cavities – S. U. De Silva and J. R. Delayen, Phys. Rev. ST Accel. Beams 16 (2013) 012004
  - Design, Prototyping and Testing of a Compact Superconducting Double Quarter Wave Crab Cavity - B. P. Xiao et al., Phys. Rev. ST Accel. Beams 18, 041004 (2015)
  - Designing the Four Rod Crab Cavity for the High-Luminosity LHC Upgrade - B. Hall, EuCARD Monograph Vol. 25 (2014)
  - SPX Crab Cavity Development and Testing Result – H. Wang, TTC Meeting (2012), https://www.jlab.org/indico/event/24/session/9/contribution/36/material/slides/1.pdf

ADDITIONAL SLIDES

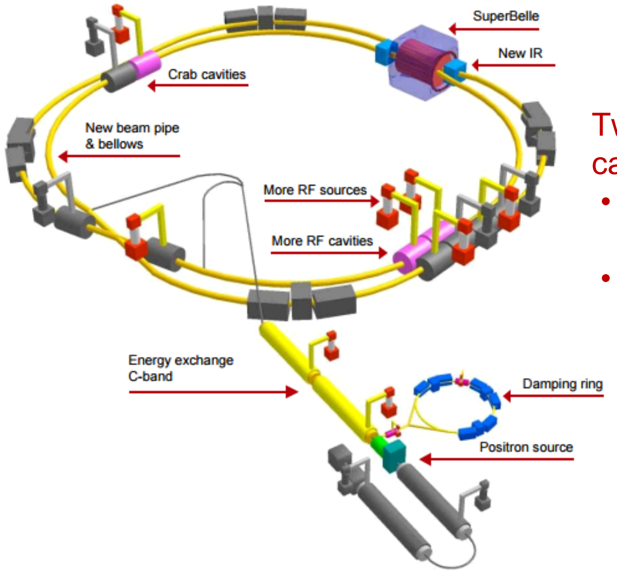
Superconducting Non-Elliptical Cavity Community



Non-Elliptical Facilities and Projects

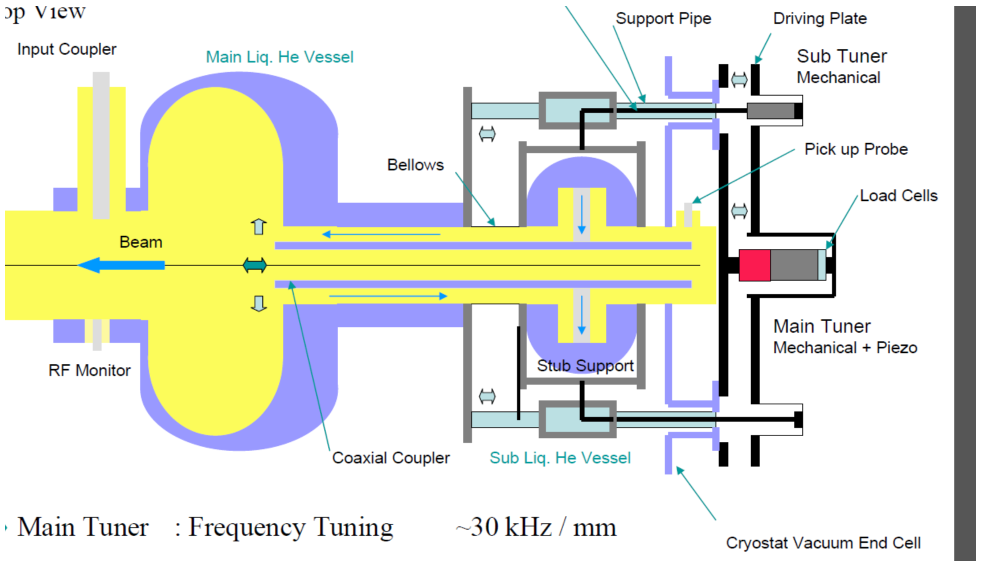
Project	Lab	Driver	Post-accelerator	Particle	Structure
ATLAS	ANL		√	HI	Split-ring, QWR
ALPI	LNL		√	P, d / HI	QWR (sputter, bulk)
ISAC-II	TRIUMF		√	HI	QWR
IUAC	IUAC		√	HI	QWR
ReA3/6	NSCL		√	HI	QWR
HIE-Isolde	CERN		√	HI	QWR (sputter)
SARAF	SOREQ	√		P, d	HWR
SPIRAL-II	GANIL	√		P, d, HI	QWR
IFMIF	Saclay	√		P,d	HWR
FRIB	NSCL	√		HI	QWR, HWR
ESS	ESS	√		P	DSR
RAON	RISP	√		HI	QWR, HWR, SSR
ADS	IMP,IHEP	√		p	HWR, SSR
PIP-2	FNAL	√		P	HWR, SSR
Hi-Lumi	CERN			p	Crab cavities - DQWR, RFD

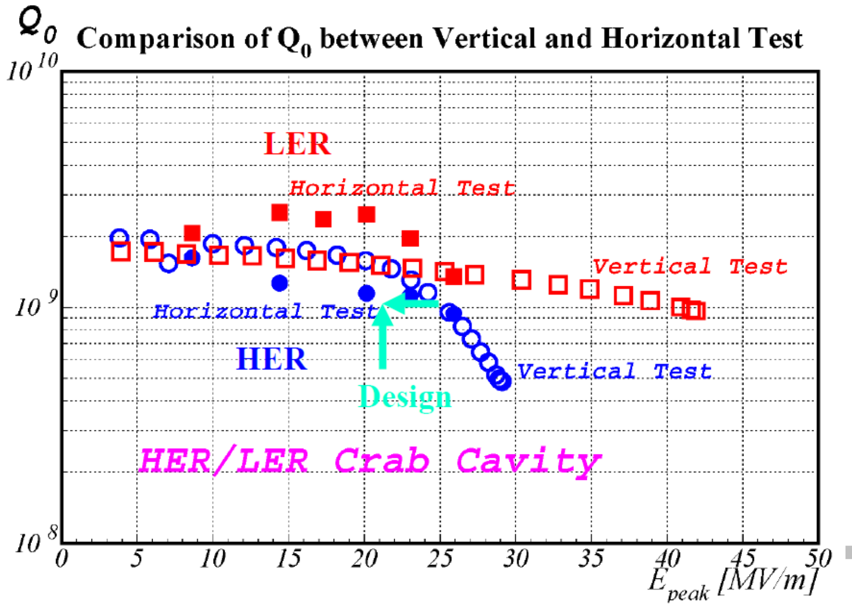
KEK Crabbing Cavity



Two crabbing cavities

- High energy ring (HER)
- Low energy ring (LER)

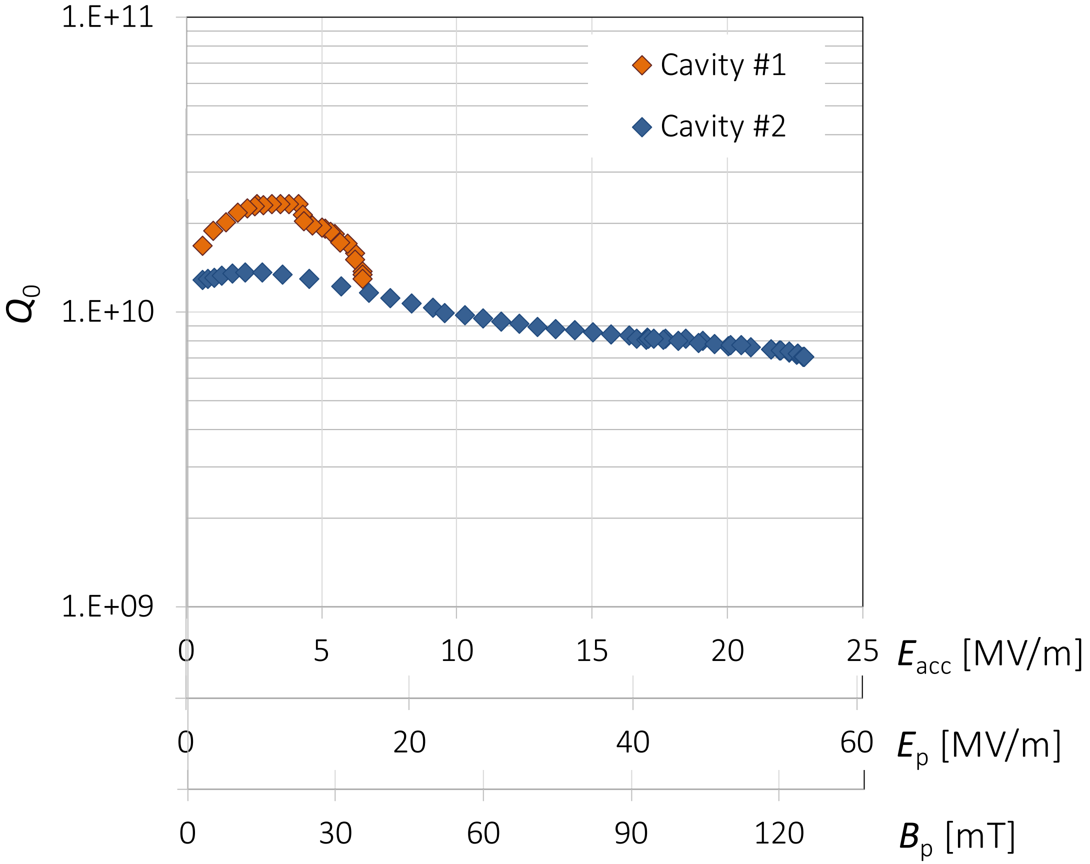




Frequency	**508.9**	**MHz**
LOM (TM_{110})	410.0	MHz
Nearest HOMs	630.0, 650.0, 680.0	MHz
E_p^*	4.24	MV/m
B_p^*	12.23	mT
B_p^*/E_p^*	2.88	mT/(MV/m)
$[R/Q]_T$	48.9	Ω
Geometrical Factor (G)	227.0	Ω
$R_T R_S$	1.11×10^4	Ω^2
At $E_T^* = 1$ MV/m		

RF Test Results of Twin Axis Cavity

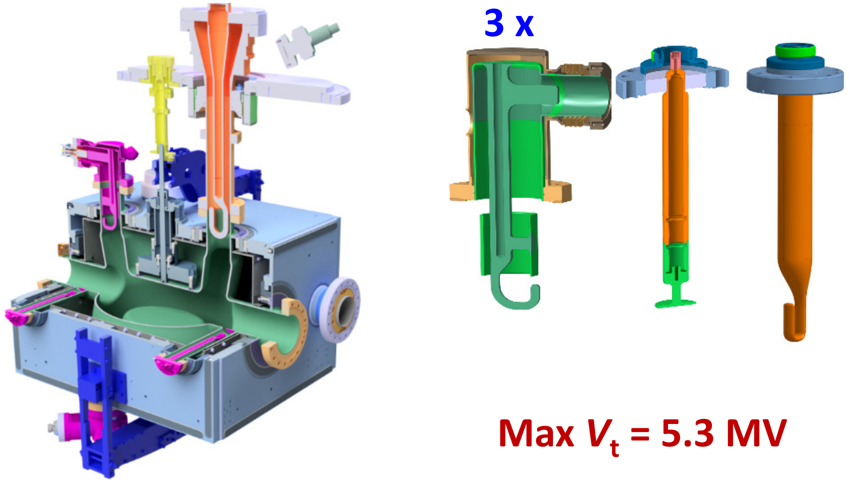
- Cavity 1:
  - Low field Q_0 of 2.3×10^{10}
  - Cavity quenched at 6.5 MV/m
  - Weld defect seems to be the limiting factor according to OST measurements
- Cavity 2:
  - Low field Q_0 of 1.3×10^{10}
  - Achieved an accelerating gradient of 23 MV/m (E_p = 56 MV/m & B_p = 126 mT)
  - No multipacting levels were observed during the test
  - Given minimal surface treatment (BCP only, no EP) cavity reached high Q_0



TE-Like Cavities for LHC High Luminosity Upgrade

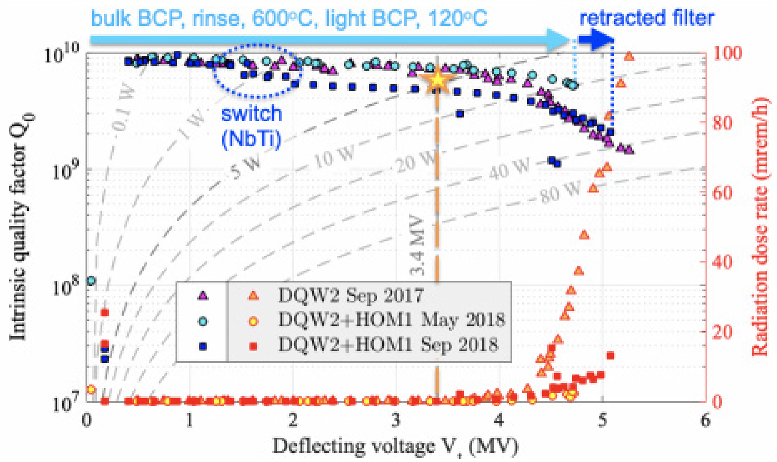
- Crabbing cavities for LHC high luminosity upgrade – Operate at 400 MHz

Double Quarter Wave Cavity

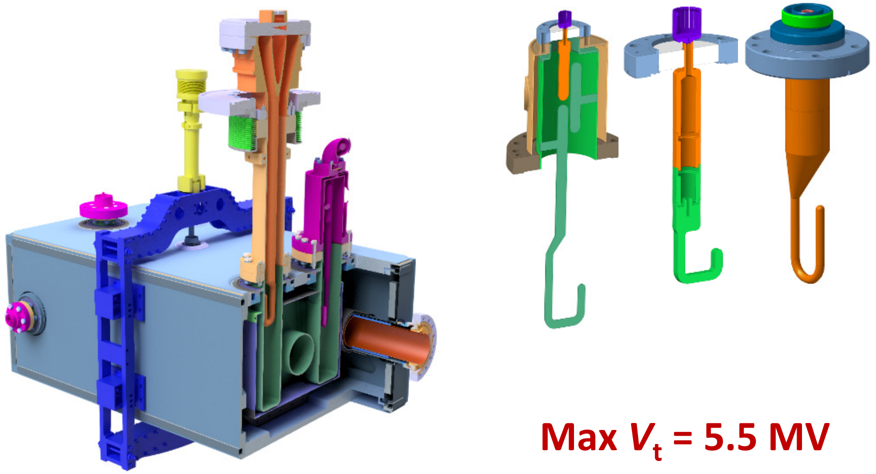


3 x

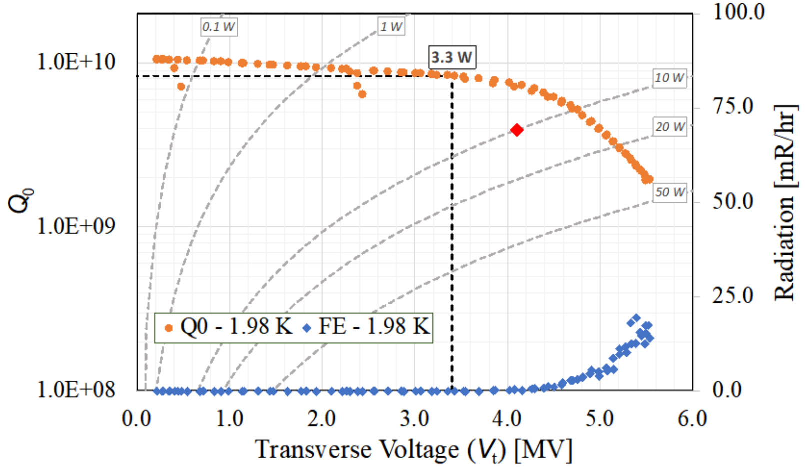
Max V_t = 5.3 MV



RF-Dipole Cavity



Max V_t = 5.5 MV

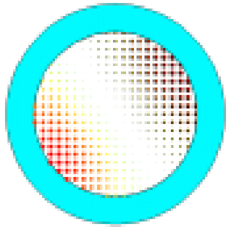


TE-Type Non-Elliptical Cavities

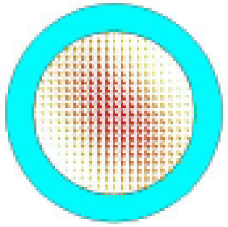
IH 4-rod RFQ

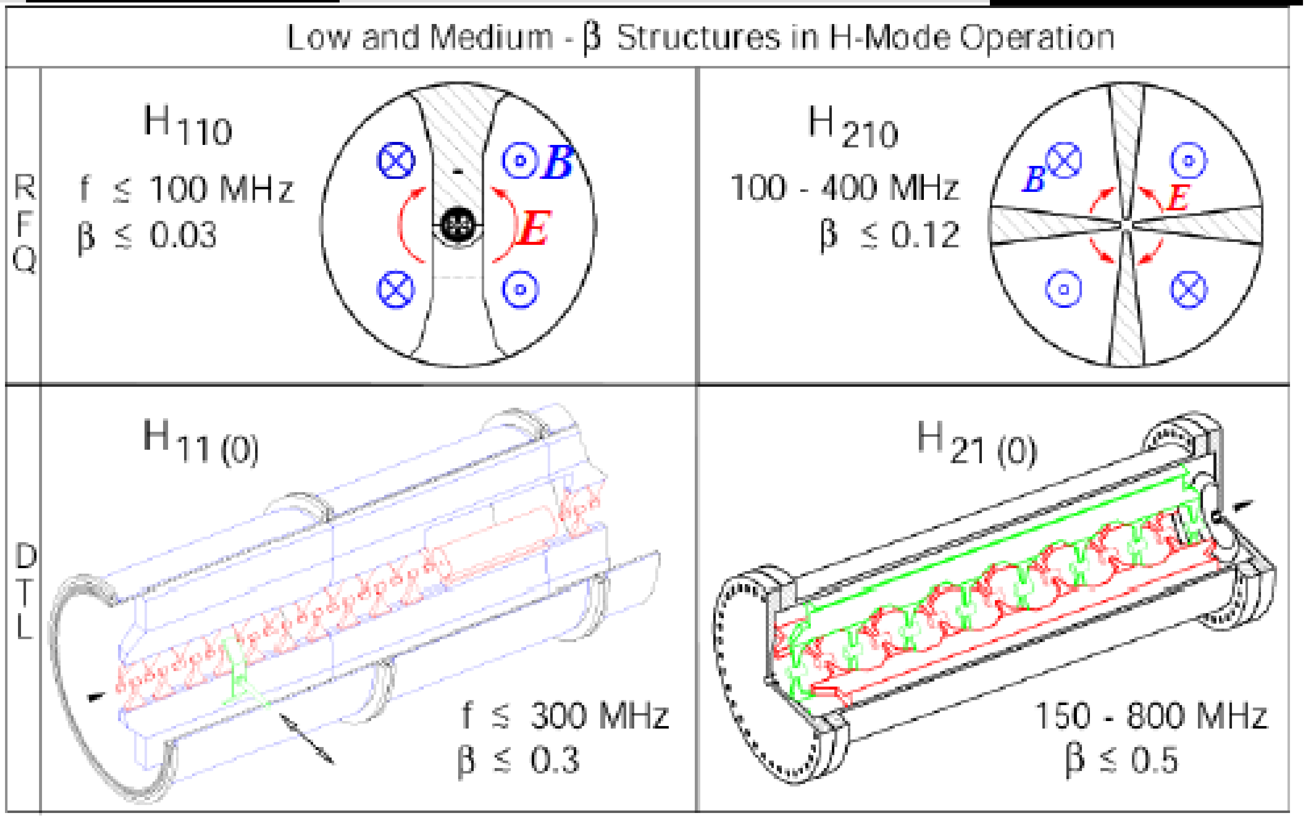
4-vane RFQ

B-Field

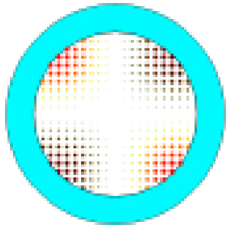


E-Field

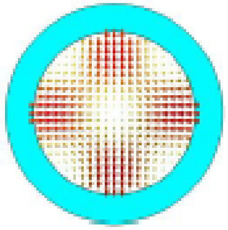




B-Field



E-Field



Courtesy of H. Podlech

IH-Structure

CH-Structure

Normalized T(β)

- It is usually convenient to define the normalized transit time factor and include the gap effect in the accelerating gradient
- Normalized transit time factor: $T^*(\beta) = \frac{T(\beta)}{T(\beta_0)}$
- Average accelerating gradient: $E_a^* = T(\beta_0)E_a$

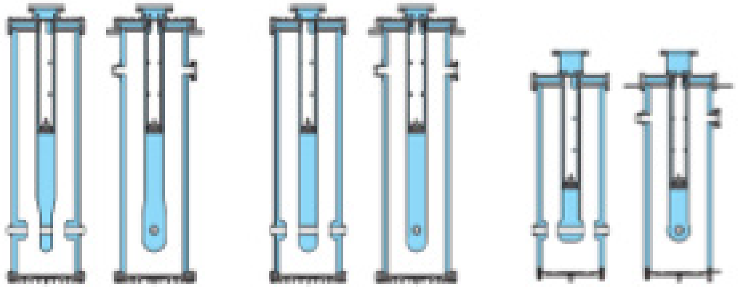
where $\beta_0 \equiv \beta / T(\beta_0) = \max\{T(\beta)\}$ and $T^*(\beta_0) = 1$

- Energy gain definition doesn't change $\Delta W_p = qE_a^* L T^*(\beta)\cos\varphi$

Real QWR Cavities

- Typical range → frequency: 50 MHz – 160 MHz; β_0: 0.04 – 0.2

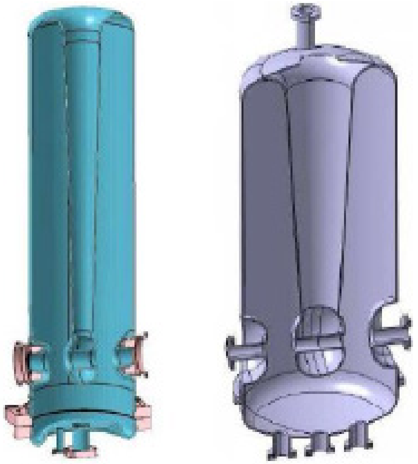
TRIUMF ISAC-II Resonators



SCB low β (5.7%) 106.08 MHz

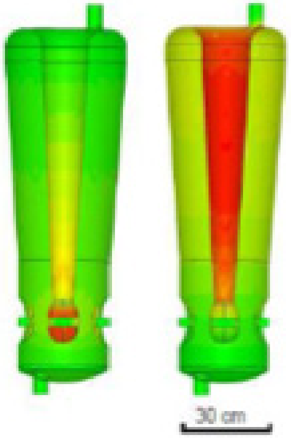
SCB medium β (7.1%) 106.08 MHz

SCB high β (11%) 141.44 MHz



Spiral-2 β=0.007, 0.12
88.05 MHz

RAON β=0.047
81.25 MHz



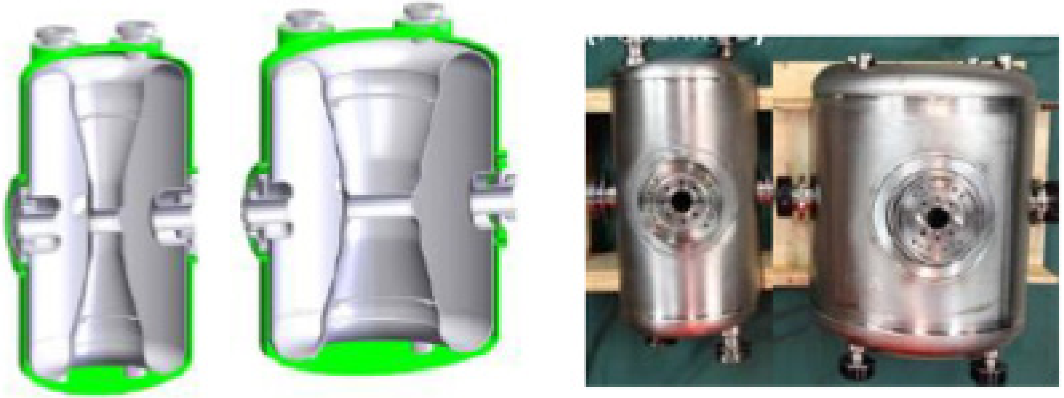
ANL β=0.077, 0.085 72.5 MHz

FRIB β=0.041, 0.085 80.5 MHz

Real HWR Cavities

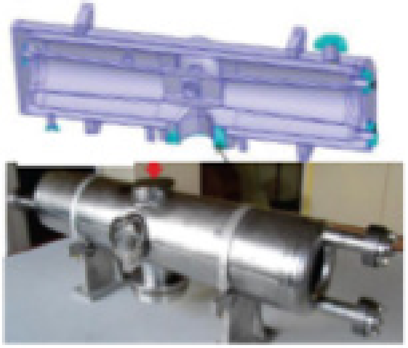
- Typical range → frequency: 140 MHz – 325 MHz; β_0: 0.1 – 0.5

ANL β=0.12 325 MHz

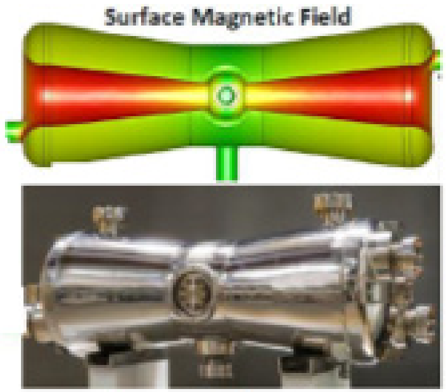


FRIB β=0.29, 0.53 322 MHz

IMP β=0.1 162.5 MHz



IFMIF β=0.11 175 MHz



ANL β=0.112 162.5 MHz

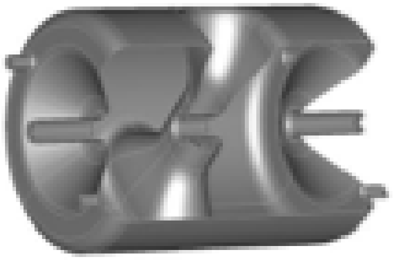
Real Single Spoke Cavities

- Typical range → frequency: 325 MHz – 700 MHz; β_0: 0.15 – 0.7

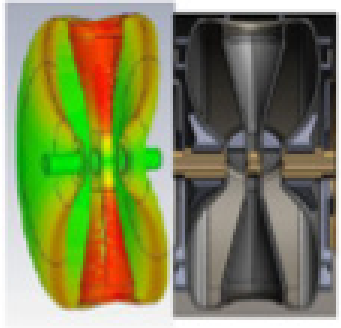
ANL β=0.3 850 MHz

TRIUMF/RISP β=0.3 325 MHz

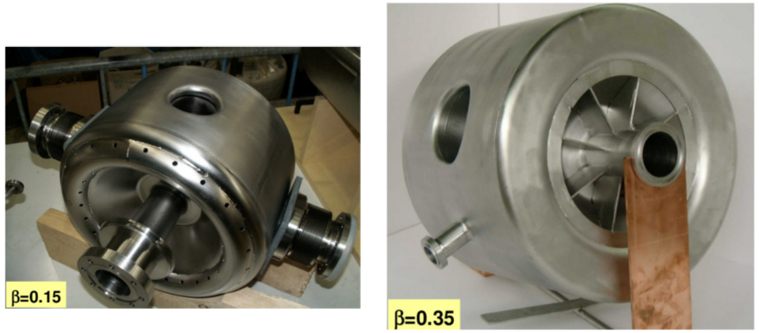
FNAL β=0.215 325 MHz



ODU β=0.82 325 MHz

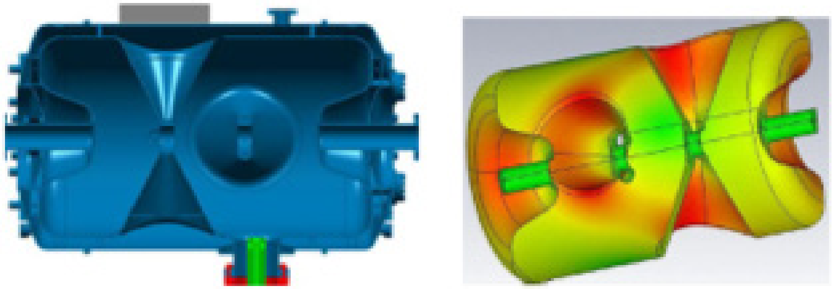


IHEP β=0.12 325 MHz

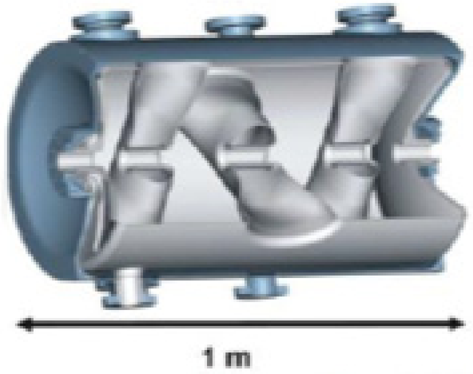


IPN-Orsay β=0.15, 0.35 352 MHz

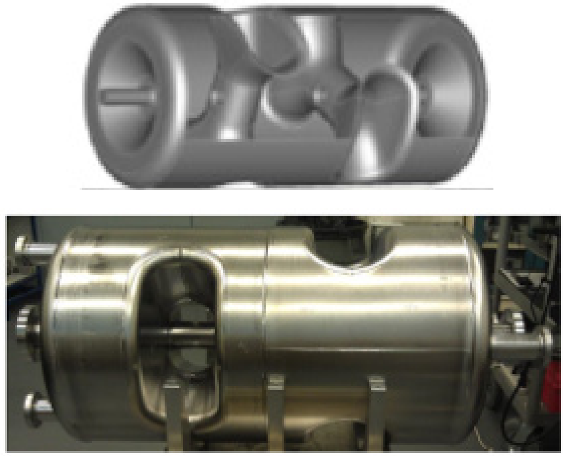
Real Multi Spoke Cavities



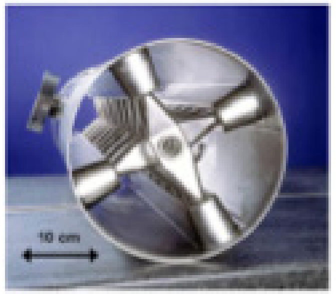
ANL β=0.12 325 MHz



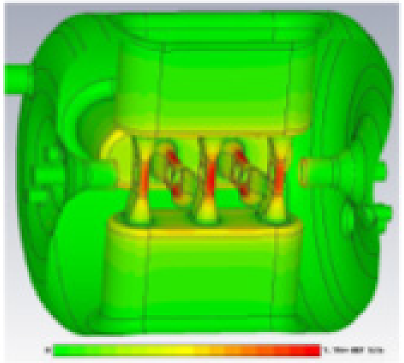
ANL β=0.63 345 MHz



ODU β=1.0 500 MHz

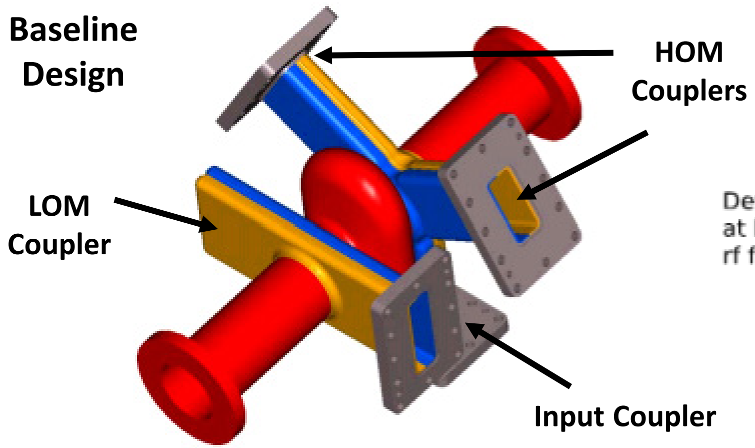


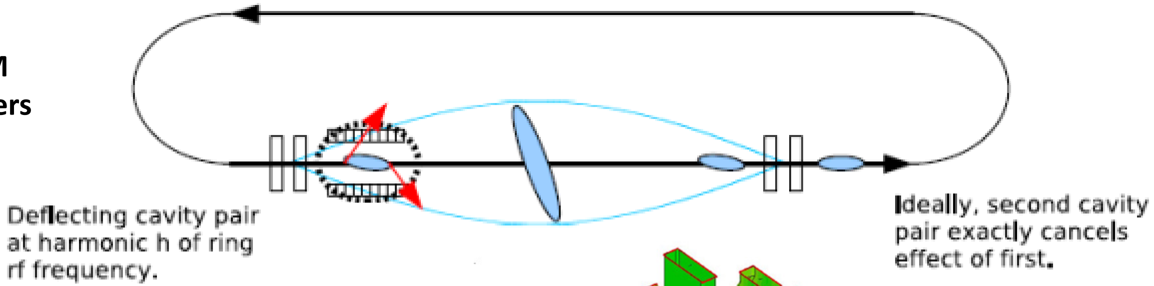
IAP β~0.1 360 MHz
19 gap CH resonator

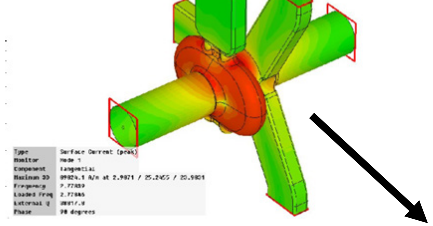


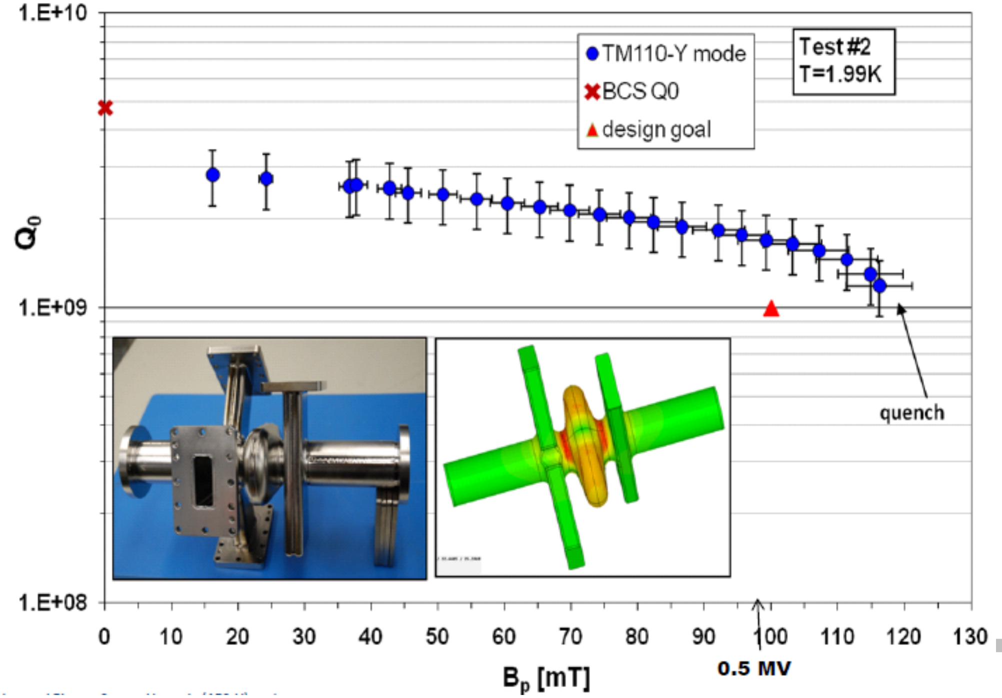
IMP β=0.067 162.5 MHz
CH resonator

Crabbing Cavity for Short Pulse X-ray Project









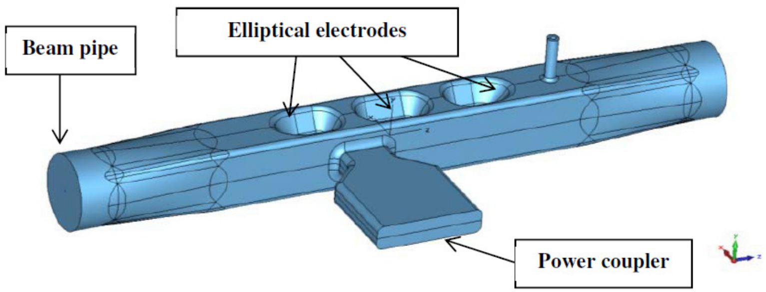
Parameter	Baseline Design	Alternate Design	Units
Frequency	**2.815**		GHz
Beam iris	25		mm
V_t	0.5		MV
B_t	98	100	mT
E_t	41	42	MV
$[R/Q]_t$	35.8	37.1	Ω
G	227.5	227.8	Ω
Material thickness	3 Nb Sheet	4 Nb Block	mm

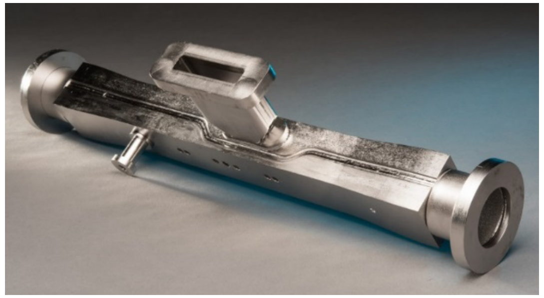
Multi-Cell TE_{11}-Like Cavities

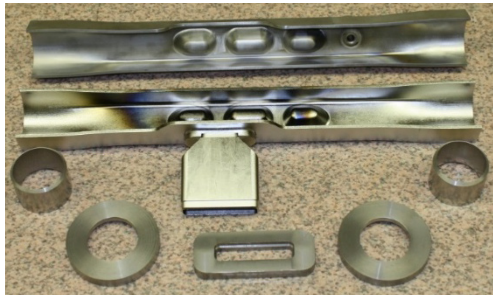
- Multi-cell cavities provide higher gradient with reduced space on the beam line
- No of HOMs multiplies with no. of cells
- Has lower order modes

QMiR (Quasi-waveguide Multi-cell Resonator)

2.815 GHz

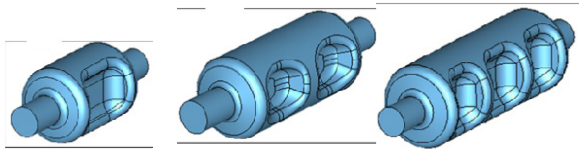






For JLEIC

***e* – 12 GeV** **_p_ – 200 GeV**



	Single-Cell RFD	Two-Cell RFD	Three-Cell RFD	Unit
Frequency	**952.6**			**MHz**
Aperture	**70**			mm
SOM	None	846	756.8, 862.2	MHz
LOM Mode Type	–	Dipole	Dipole	
1st HOM	1411.5	1379.5	1335.4	MHz
E_p/E_t	5.4	5.66	5.6	
B_p/E_t	13.6	11.64	11.4	mT/(MV/m)
$[R/Q]_t$	50	147.5	218.8	Ω
G	165.7	169	178.9	Ω
R_tR_s	8.3×10^3	2.5×10^4	3.9×10^4	Ω^2
Total V_t (*e*/*p*) (per beam per side)	**4.2 / 21.5**			**MV**
V_t (per cavity)	**0.86**	**1.9**	**3.1**	**MV**
No. of cavities (*e*/*p*)	**5 / 25**	**3 / 12**	**2 / 7**	
E_p	30	34	39	MV/m
B_p	**70**	**70**	**70**	**mT**

TRIUMF RF-Deflector

- Due to low performance specifications, fabrication methods include some alternative techniques:
  - Machining from bulk reactor grade Nb
    - RRR of 45 compared to usual ~300
  - Tungsten Inert Gas (TIG) welding
    - Developed as an alternative to electron beam welding

Cavity performance parameters:

- Superconducting Niobium cavity at 4.2 K		
- Resonant frequency:	650 MHz	
- Deflecting voltage:	0.3	(0.6) MV
- Shunt impedance:	625 Ω	
- Geometry factor:	99 Ω	
- Peak electric field:	9.5	(19) MV/m
- Peak magnetic field:	12	(24) mT
- RF power dissipation:	0.35	(1.4) W